Mid-term exam SCI 211, November 1, 2002

1 Our purpose is to compute the integral

$$\int_{-\pi}^{\pi} (\cos x)^4 (\sin x)^2 \, \mathrm{d}x$$

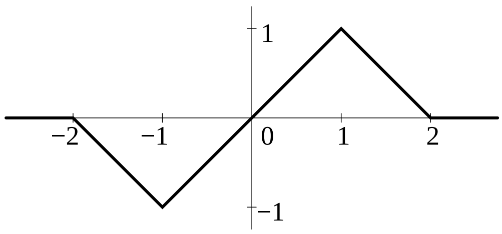
without too much work.

a) Write $(\cos x)^2 \sin x$ first in complex notation, and then as a sine series.

b) Use Parseval's identity in Theorem 2.6 in order to compute the integral.

2 Find the Fourier transform of the function $f : \mathbf{R} \to \mathbf{R}$, which is defined by $f(t) = t\,\mathrm{e}^{-2t}$ when $t > 0$ and $f(t) = 0$ when $t \leq 0$.

3 Let $f(t)$ be a complex valued function of which the Fourier transform $g(\omega) = \widehat{f}(\omega)$ is given by



Make a sketch of the graph of the Fourier transform of the function $f(t)\cos(2t)$. Don't forget to show the scale on the axes!

4 Consider the vector field $F(x, y) = (x, y)$ on the plane $\mathbf{R}^2$.

a) Find a differentiable function $g : \mathbf{R}^2 \to \mathbf{R}$ such that $F = \operatorname{grad} g$.

b) Let $\gamma : [0, T] \to \mathbf{R}^2$ be the curve in the plane which is defined by

$$\gamma(t) = \left(\frac{\cos t}{1+t}, \frac{\sin t}{1+t} \right), \quad 0 \leq t \leq T.$$

Compute the line integral of the vector field F over the curve γ. What happens with this integral when $T \to \infty$?