Home assignment

Numerical bifurcation analysis of delay differential equations

1. Determination of the stability-determining roots of the characteristic equation:
   write the matrix $M(h, t_0)$ that approximates the solution operator $S(t_0)$ of the variational equation in case the DDE systems consists of 3 equations $(n=3)$ with 2 delays $\tau_1 = 0.5$, $\tau_2 = 2$, assuming that $t_0 = 0$ and that the solution operator is discretized by the simplest LMS method, i.e. the forward Euler method
   $y_1 = y_0 + h\left(A_0 y_0 + \sum_{j=1}^{m} A_j \tilde{y}(t_0 - \tau_j)\right)$ where $\tilde{y}(t_0 - \tau_j)$ denotes an interpolated state.
   Assume a step size h = 0.2.
   What is the dimension of the matrix?

2. Draw the structure (sparsity pattern) of the matrix that arises in the collocation procedure described in [3] for computing a periodic solution with period $T = 100$ of a DDE with 2 equations (n=2) and 2 delays $\tau_1 = 16$, $\tau_2 = 34$.
   Suppose that we use (piecewise) polynomials of degree d=4 and a non-equidistant mesh with mesh points $s_i = t_i/T$, $i = 0 \ldots 7$ with $s = [s_0, \ldots , s_7] = [0, 0.2, 0.3, 0.4, 0.5, 0.6, 0.8, 1]$.

3. Use DDE-BIFTOOL and/or PDDE-CONT to study one of the following problems:
   a) model for the evolution of insulin and glucose quantities: Eq. (3) of [1]
   b) model for the dynamics of n cars on a circular track: Eqs. (44-45) of [2] (see also the orginal papers [3] and [4])

   You should reproduce some of the bifurcation diagrams given in the papers. Describe in detail how you organized your calculations and the results that you obtained (the description in the papers is very compact!).
   Focus on those aspects that are particular for DDEs, not on those aspects that were mainly covered in other courses (such as continuation as such).
   You should also report at least one bifurcation diagram that is not presented in the papers.
   Give enough information about these calculations so that it can be reproduced.
   Discuss your results in the context of the theory of nonlinear dynamical systems, in particular of systems with delays.

   The report should contain ± 10 pages of *text* (figures not included). The report should be sent to Dirk.Roose@cs.kuleuven.be . You can also contact me in case you have questions about this assignment.

[1] K. Engelborghs, V. Lemaire, J. Bélair, and D. Roose, Numerical bifurcation analysis of delay differential equations arising from physiology modeling, Journal of Mathematical Biology 42 (4), pp. 347-360, 2001.

[2] D. Roose, R. Szalai, Continuation and Bifurcation Analysis of Delay Differential Equations, in Numerical Continuation Methods for Dynamical Systems (B. Krauskopf, H.M. Osinga, J. Galan-Vioque, Eds), Springer, 2007.

[3] G. Orosz, R.E. Wilson, B. Krauskopf, Global bifurcation investigation of an optimal velocity traffic model with driver reaction time, Phys. Rev. E 70 (2) 026207, 2004. (can be downloaded from http://www.me.ucsb.edu/~gabor/)

[4] G. Orosz, B. Krauskopf, and R. E. Wilson, Bifurcations and multiple traffic jams in a car following model with reaction delay time, Physica D, 211:277–293, 2005. (can be downloaded from http://www.me.ucsb.edu/~gabor/)

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