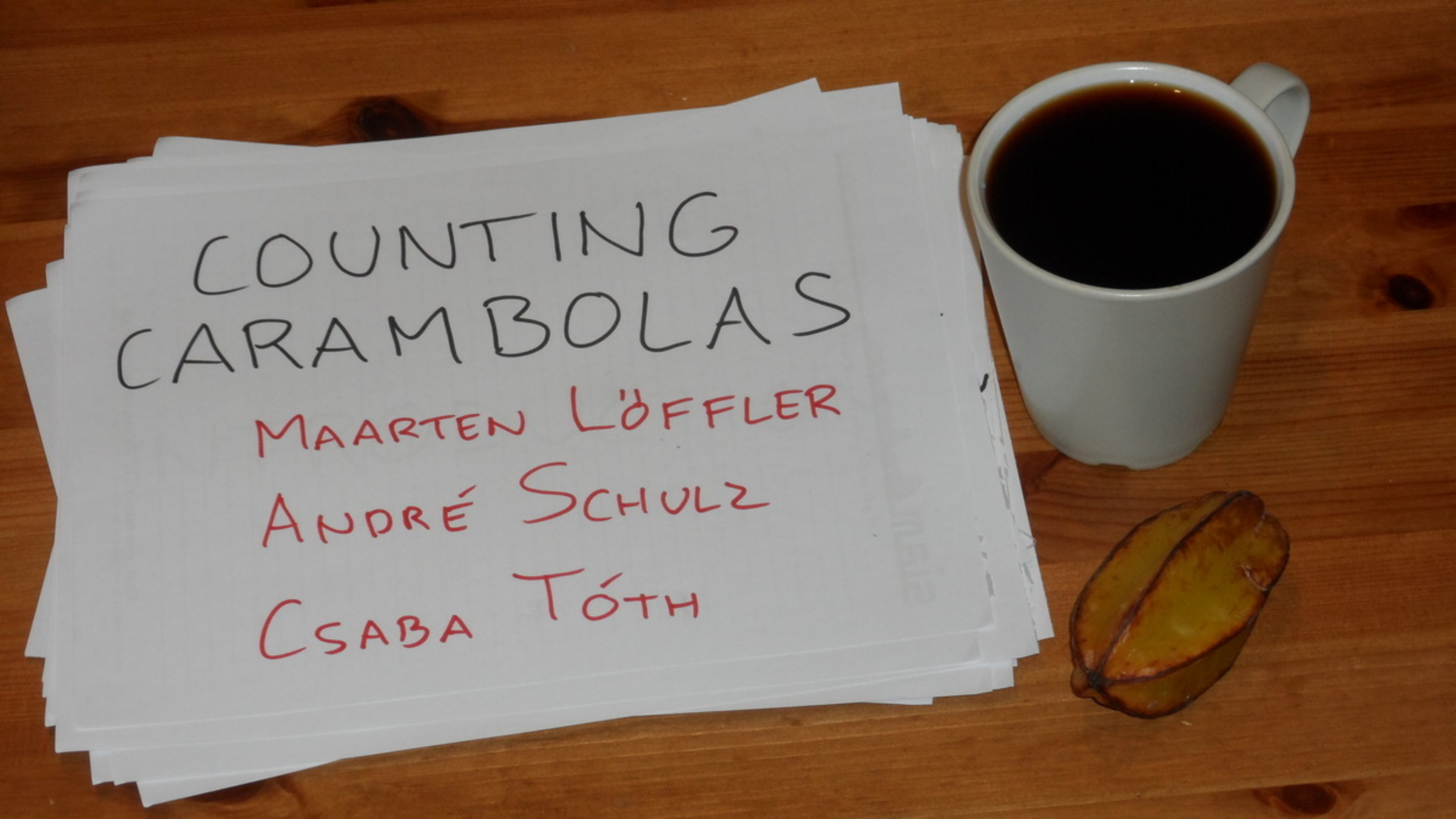
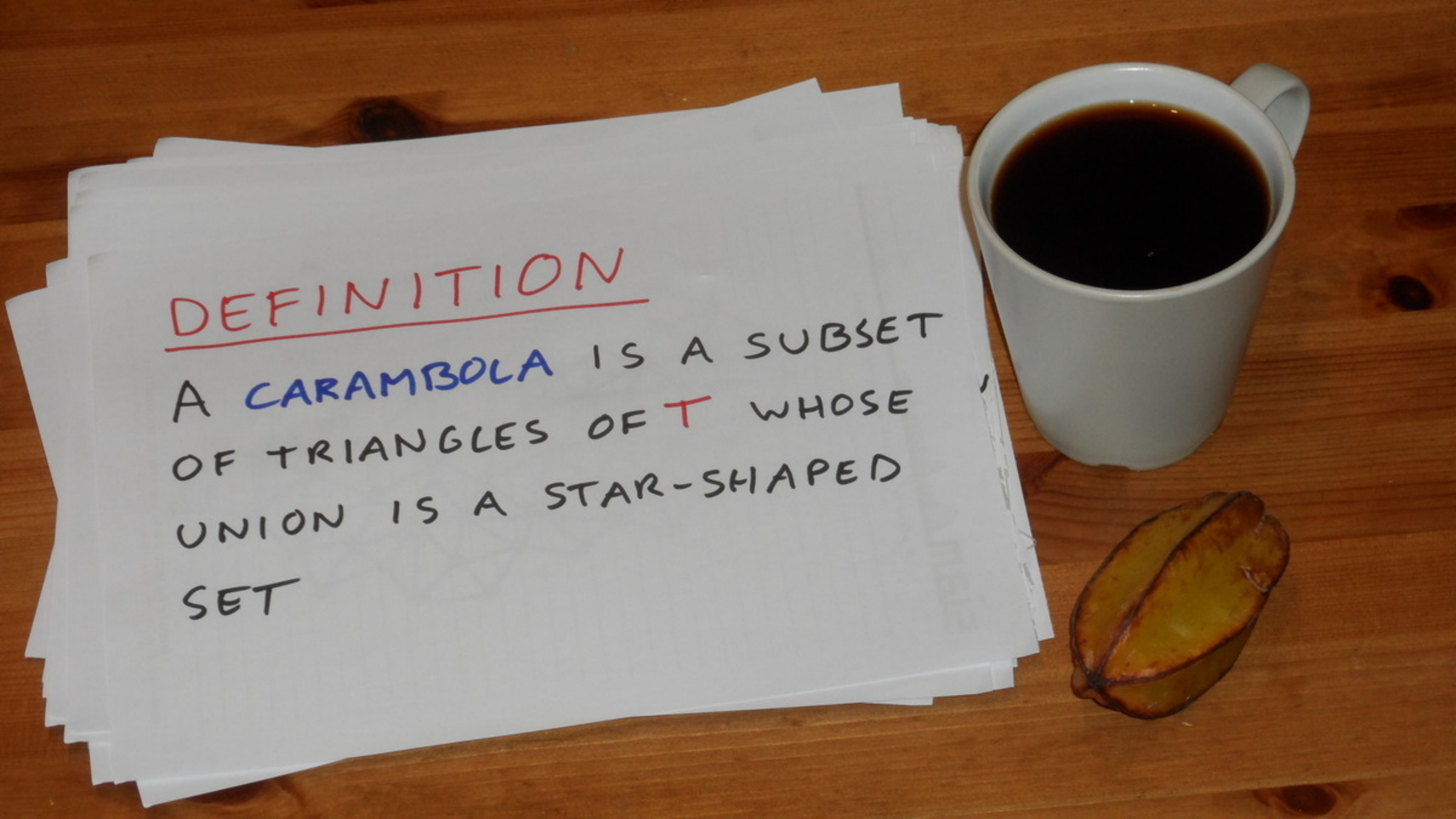
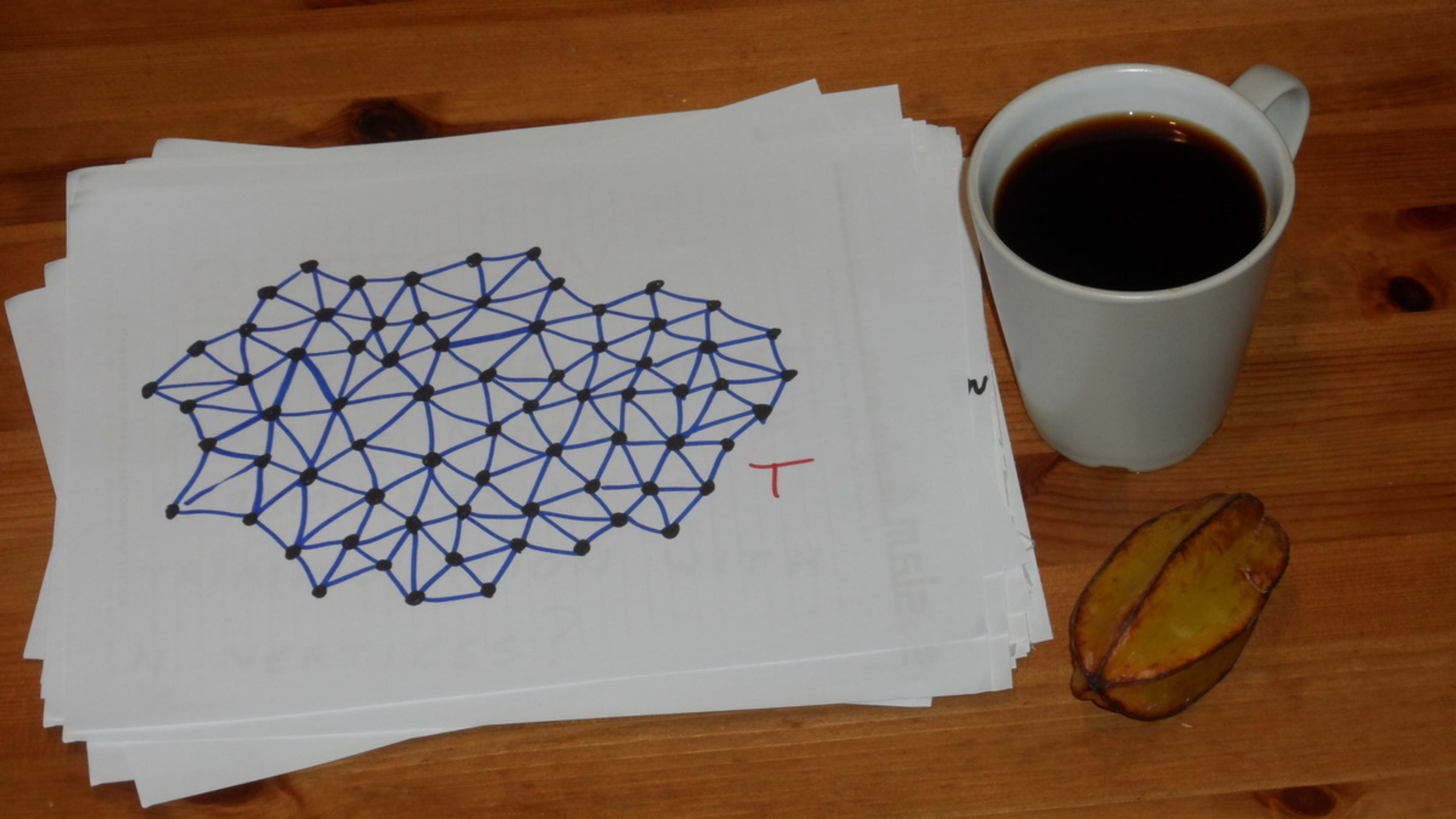

COUNTING
CARAMBOLAS
MAARTEN LÖFFLER
ANDRÉ SCHULZ
CSABA TÓTH


INTRODUCTION


DEFINITION
A CARAMBOLA IS A SUBSET
OF TRIANGLES OF T WHOSE
UNION IS A STAR-SHAPED
SET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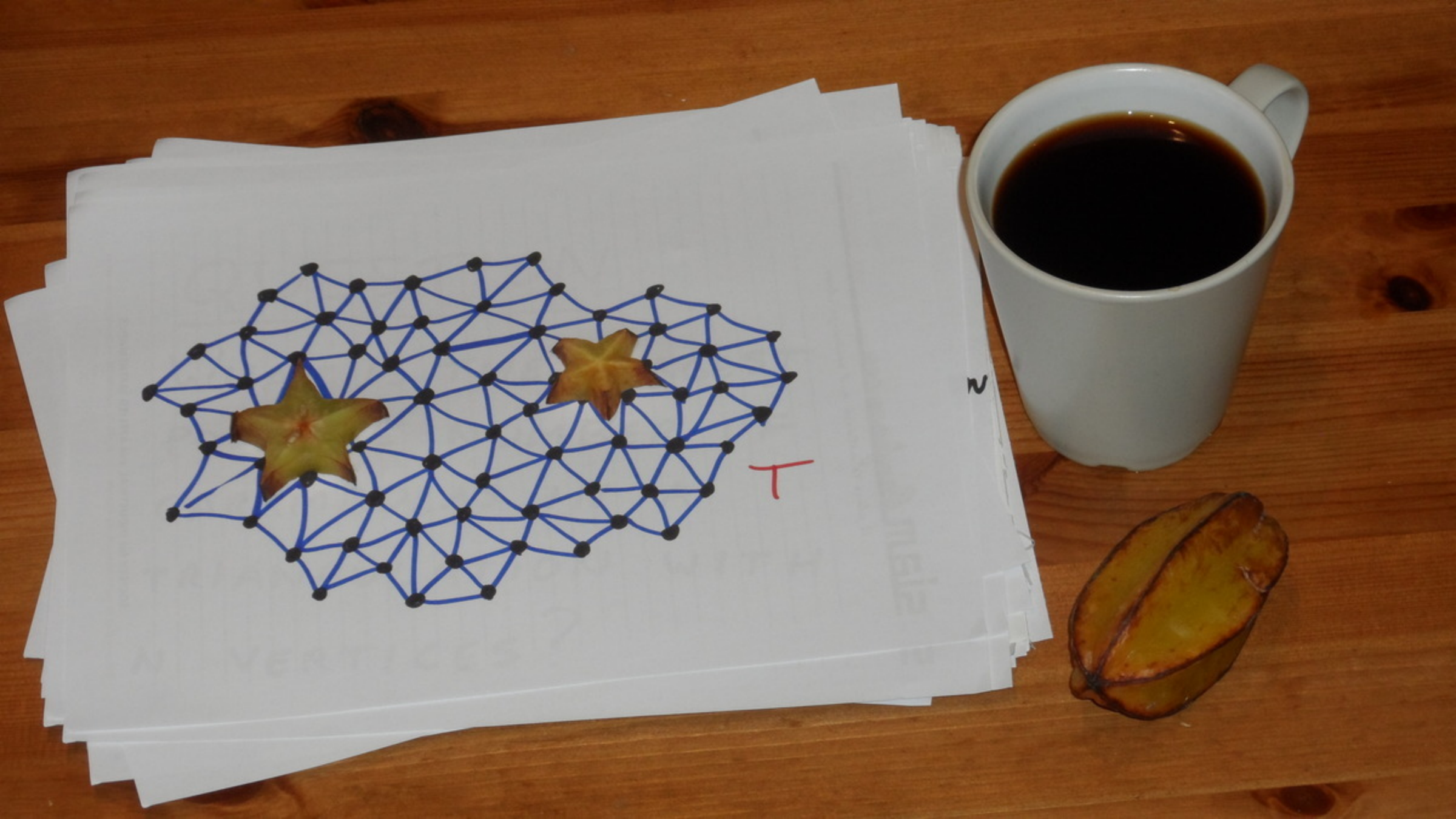
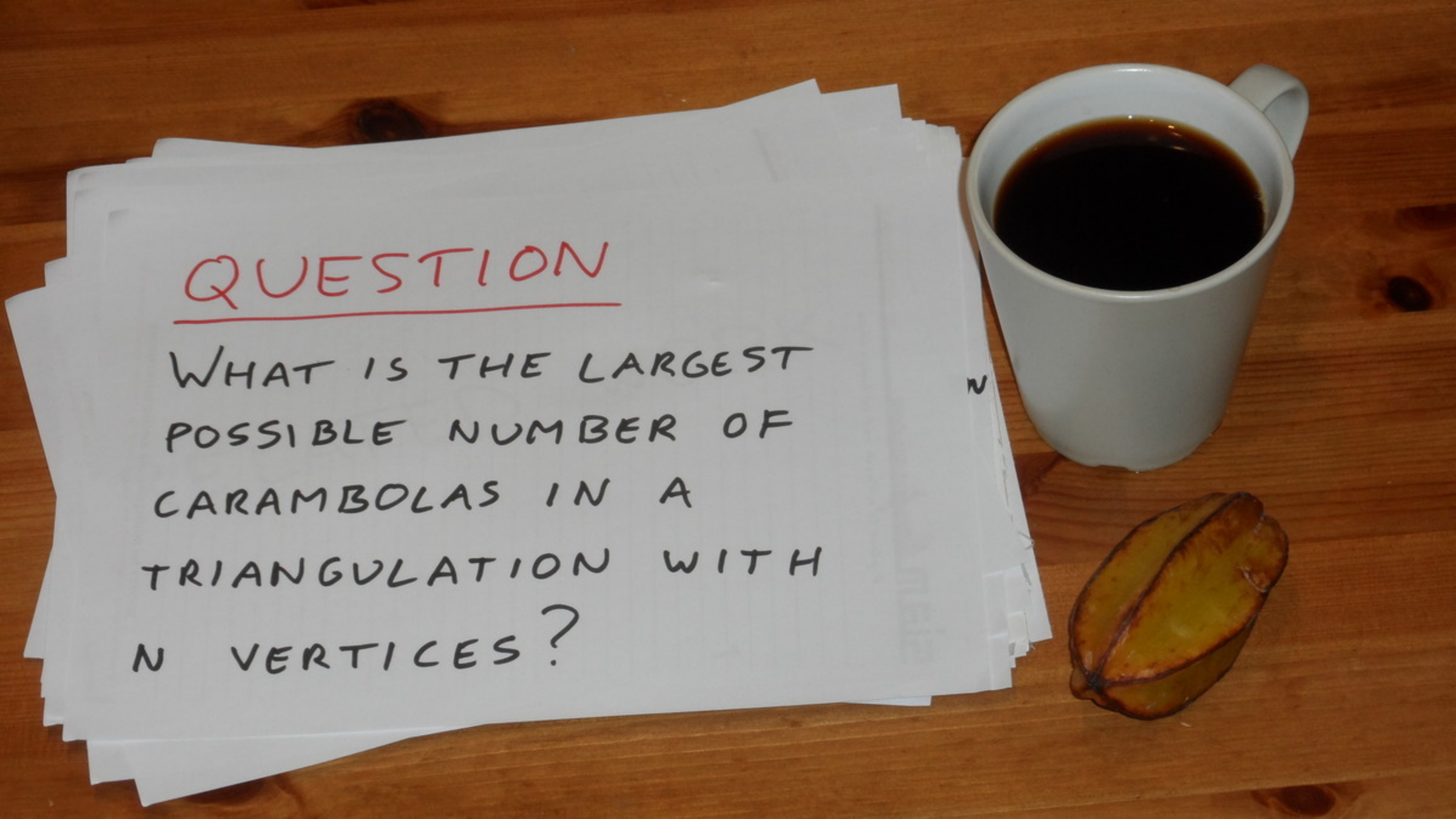
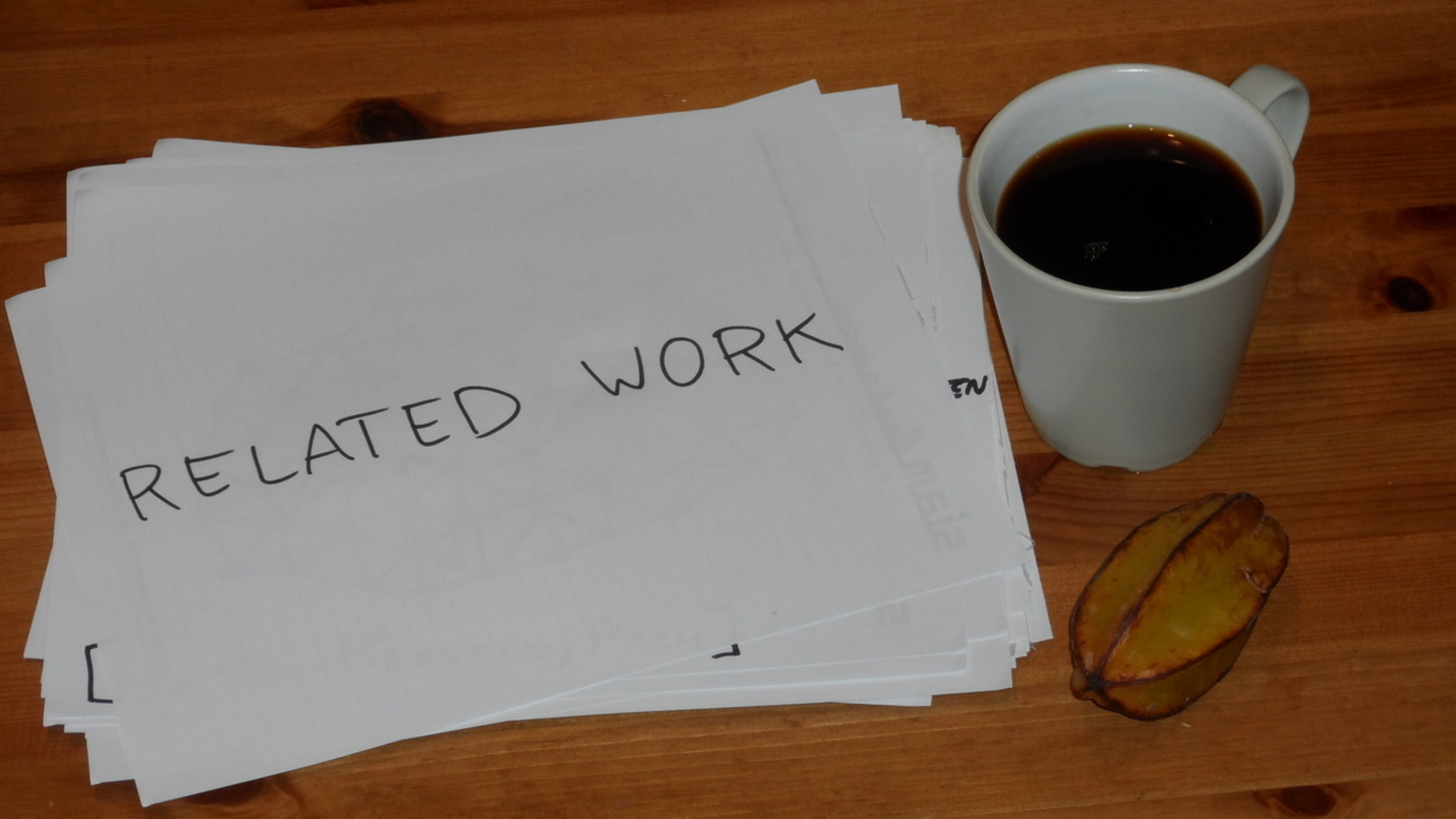
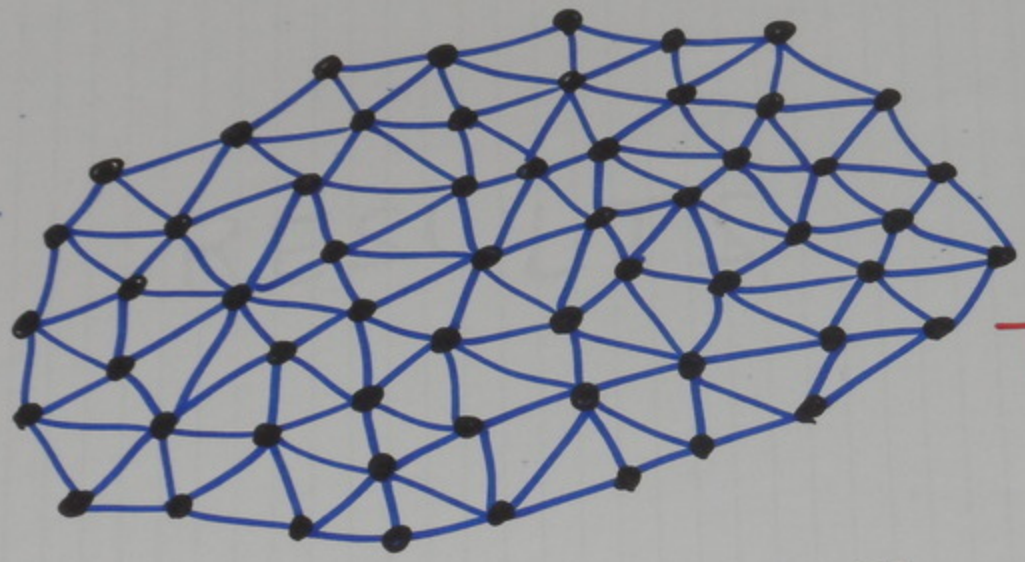



QUESTION
WHAT IS THE LARGEST POSSIBLE NUMBER OF CARAMBOLAS IN A TRIANGULATION WITH N VERTICES?
N


RELATED WORK



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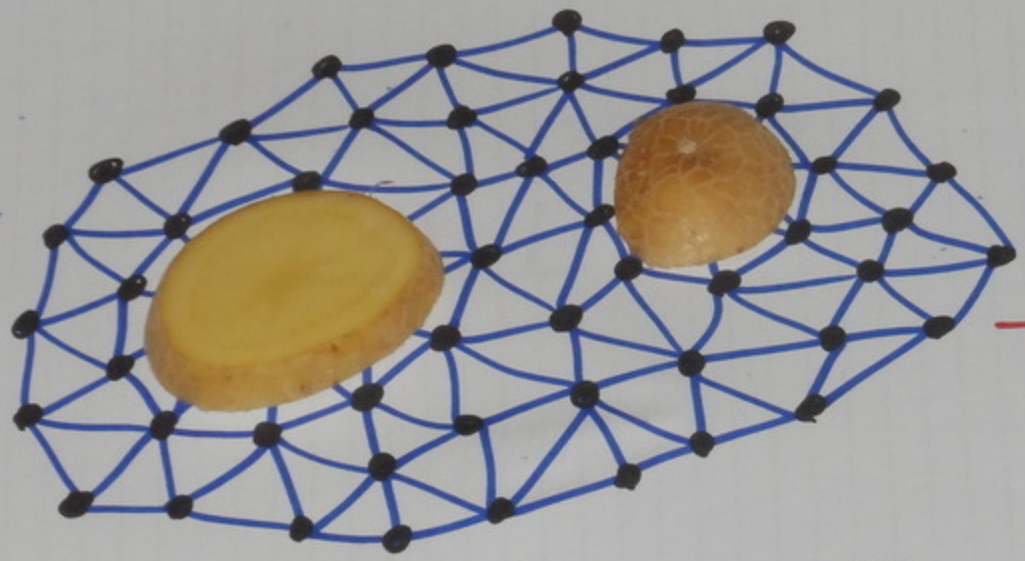
[L., VAN KREVELD, PACH]



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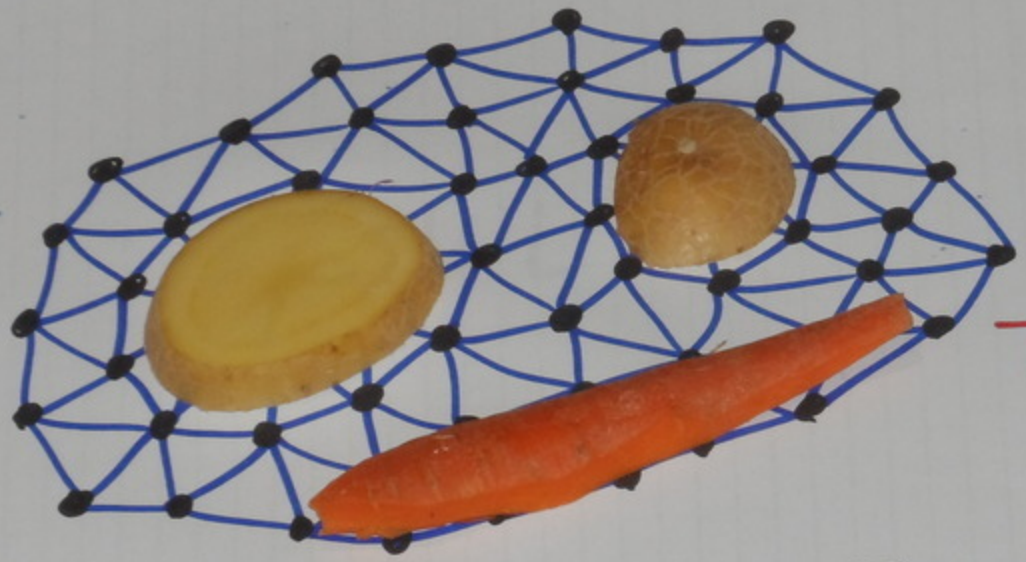
[L., VAN KREVELD, PACH]



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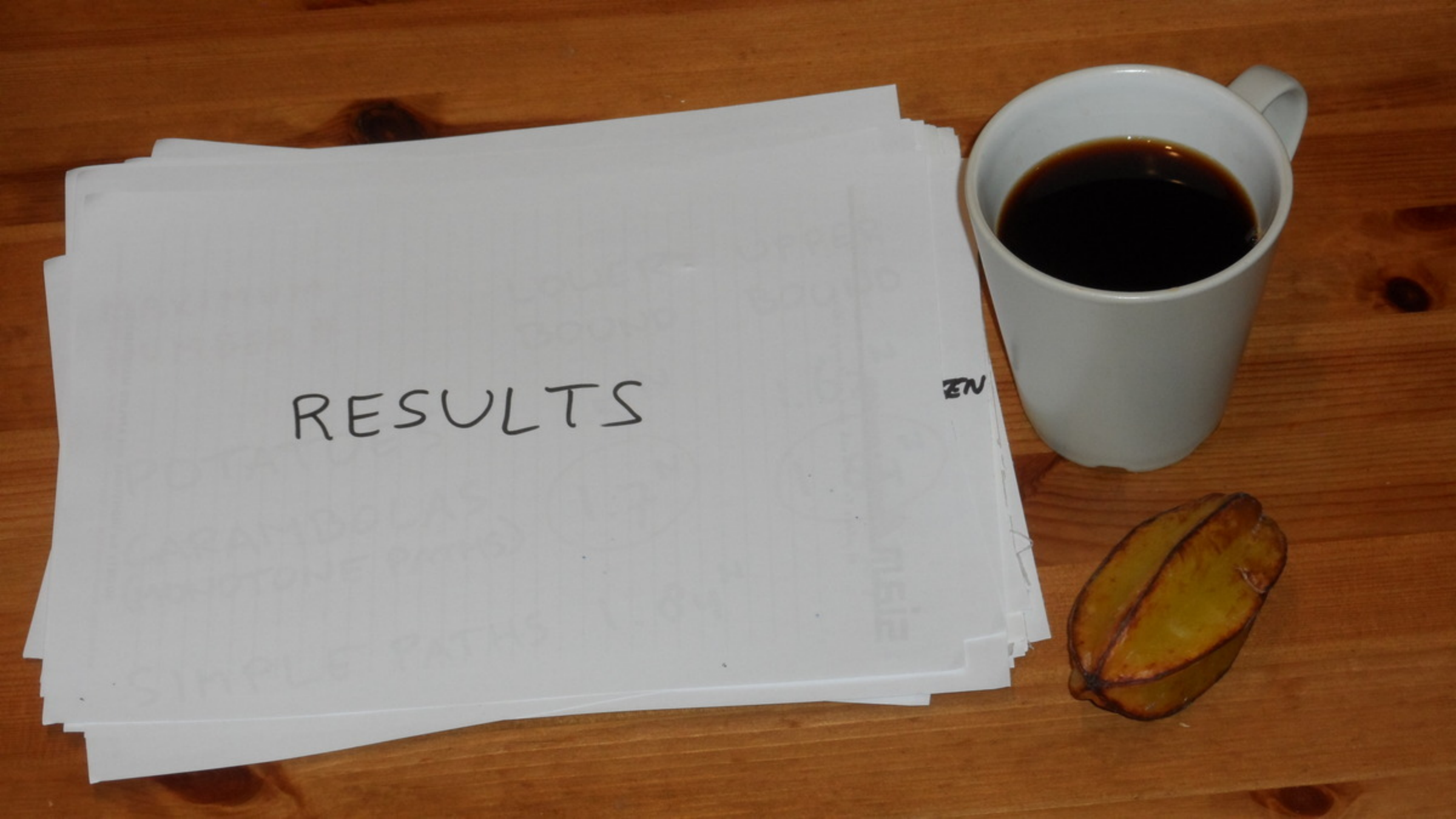
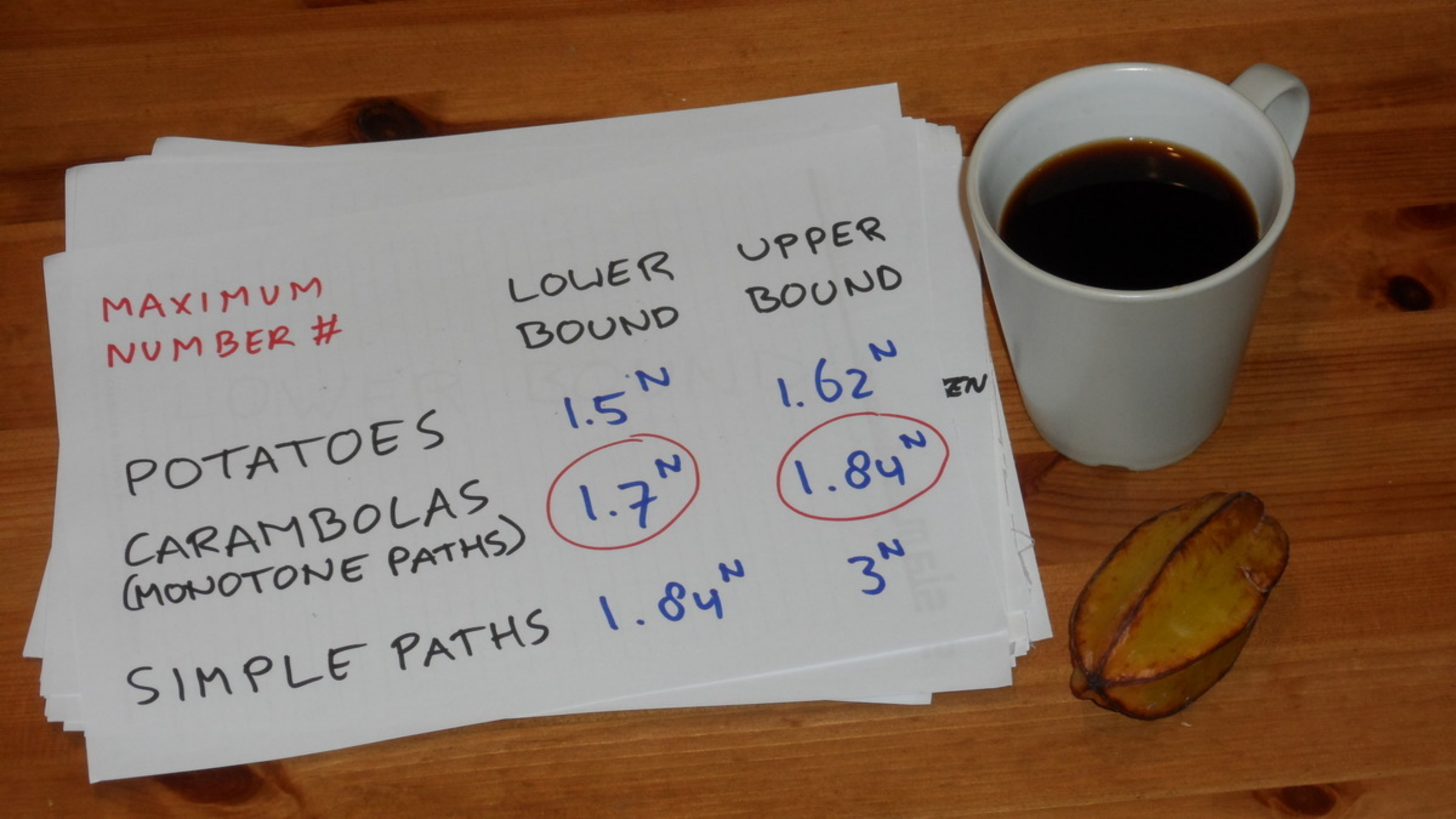
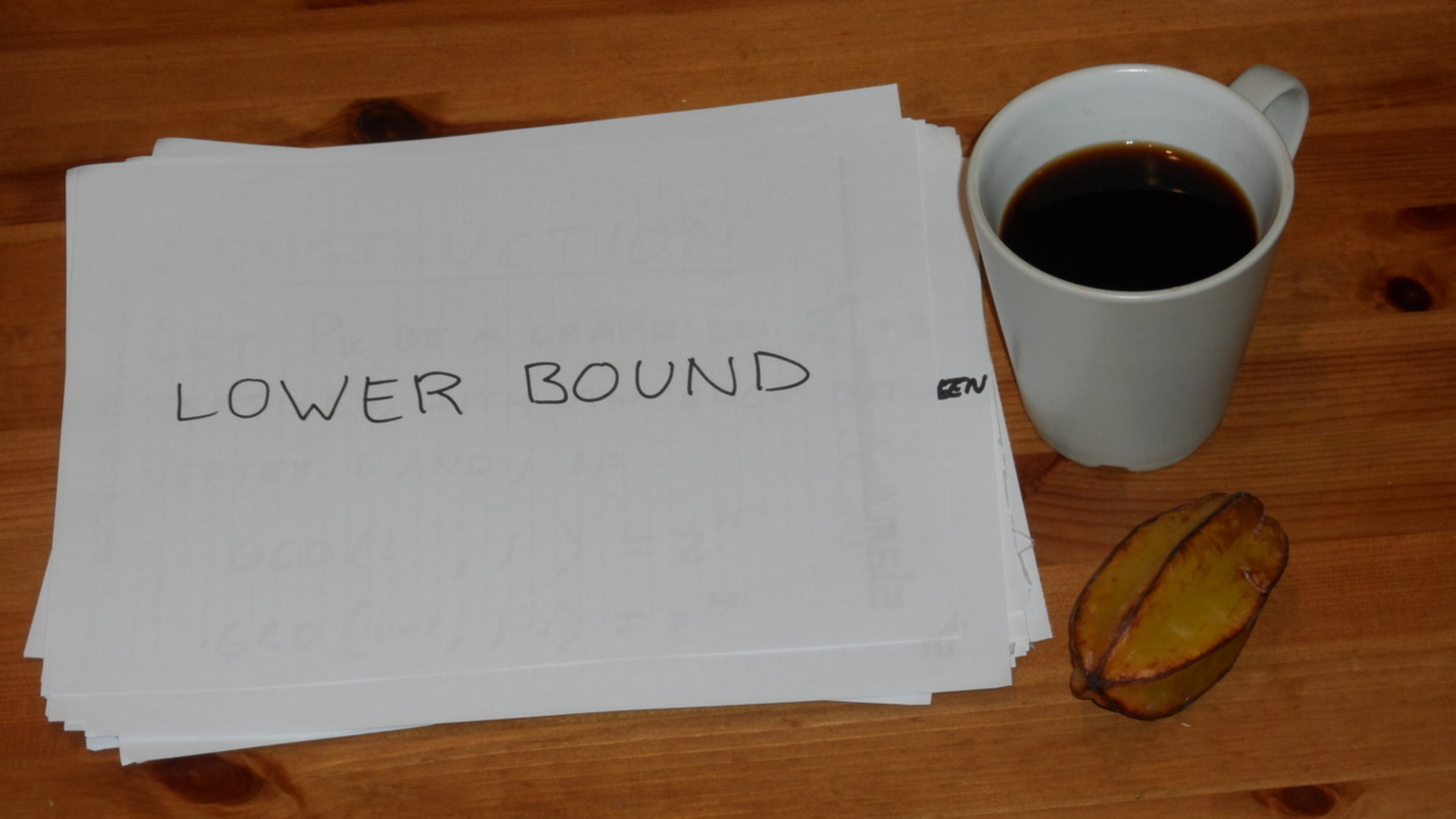
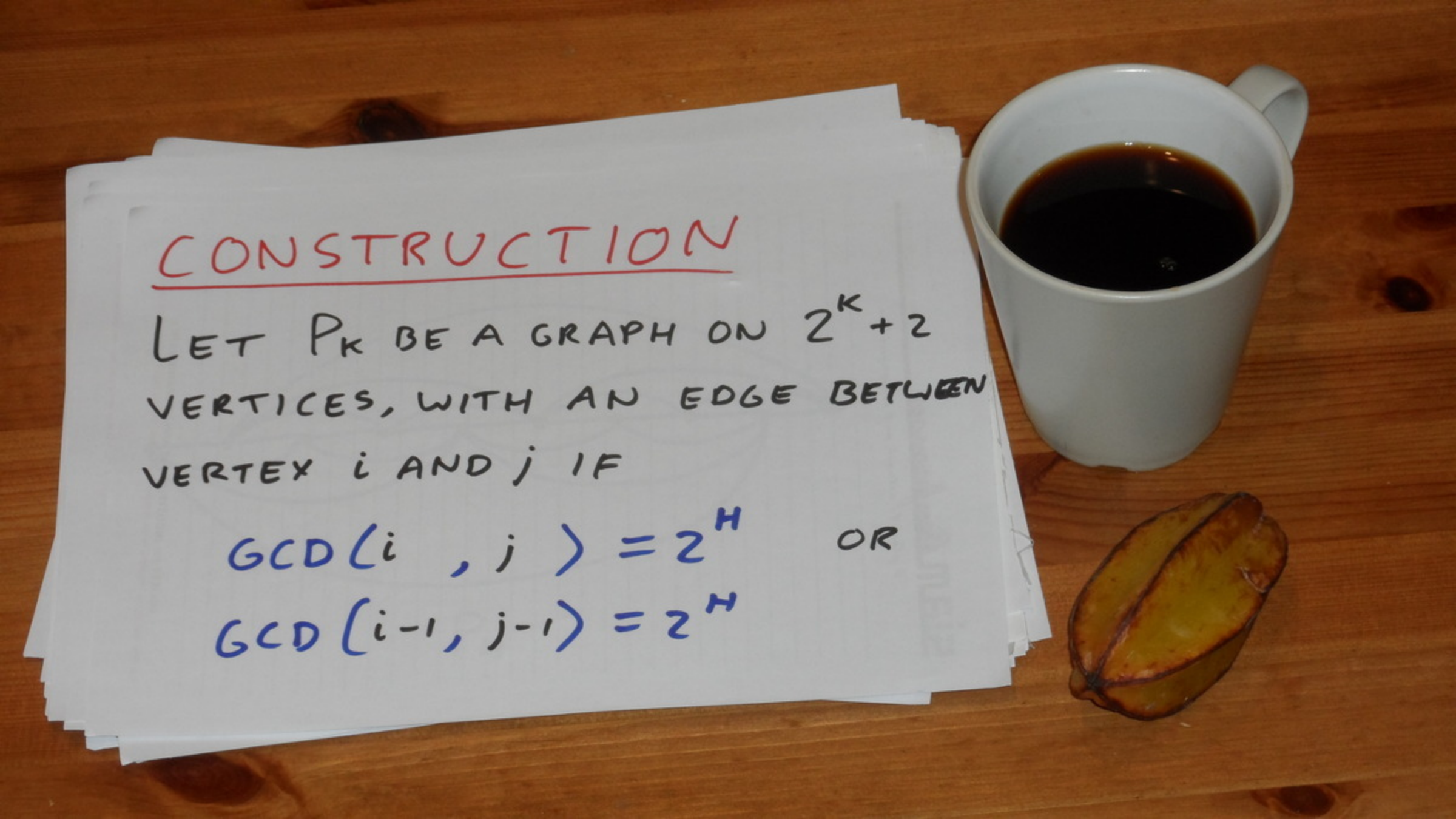
EN

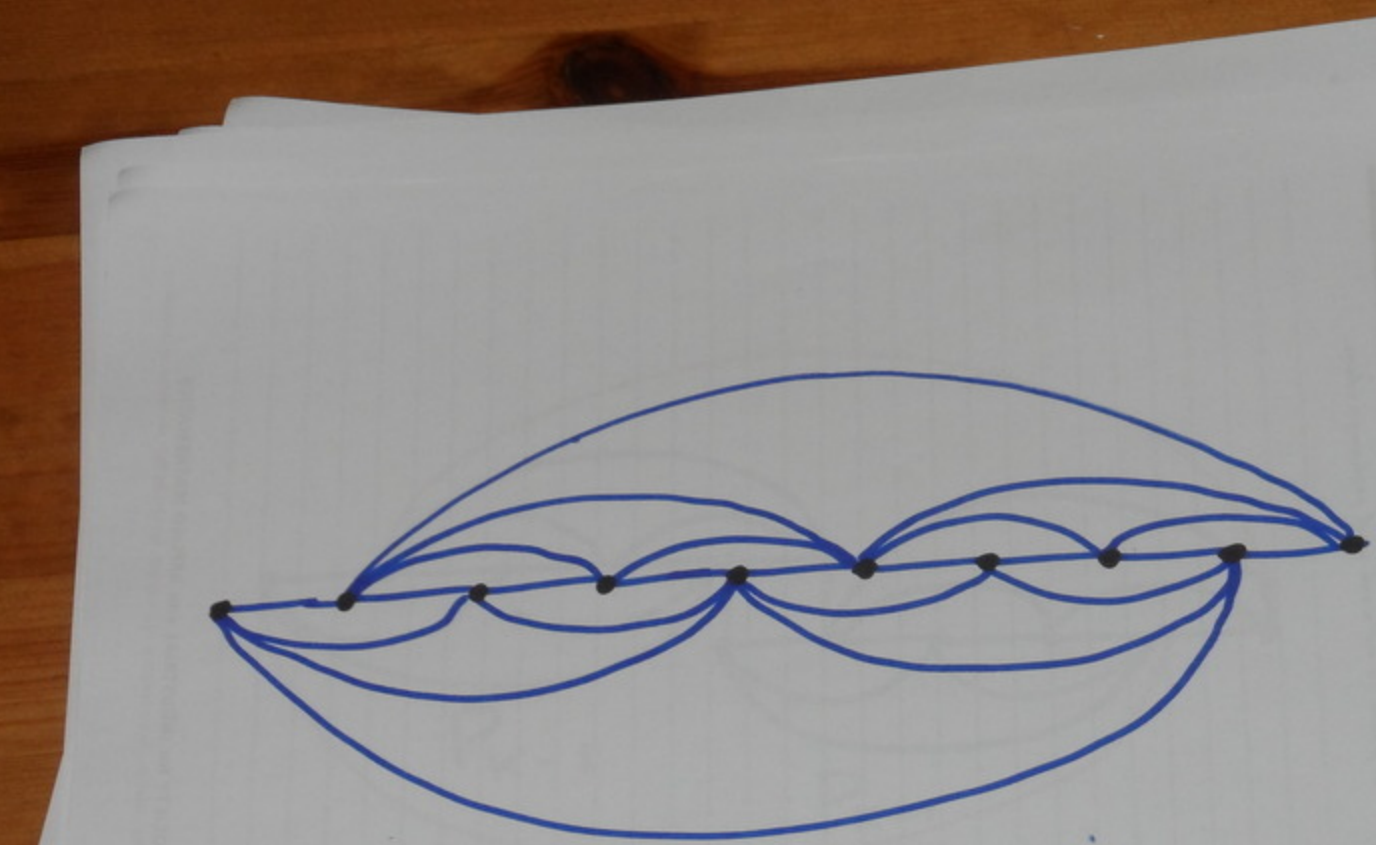
[L., VAN KREVELD, PACH]

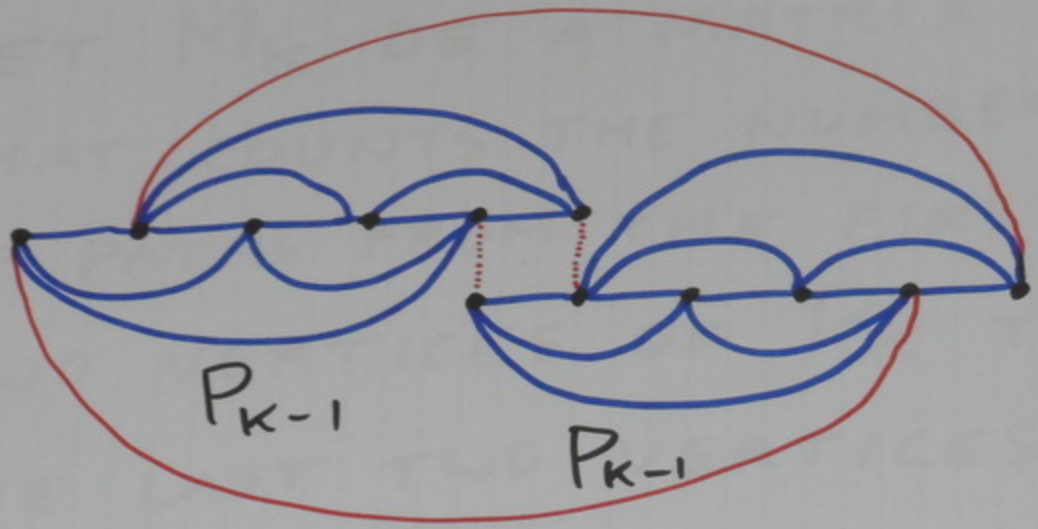

RESULTS

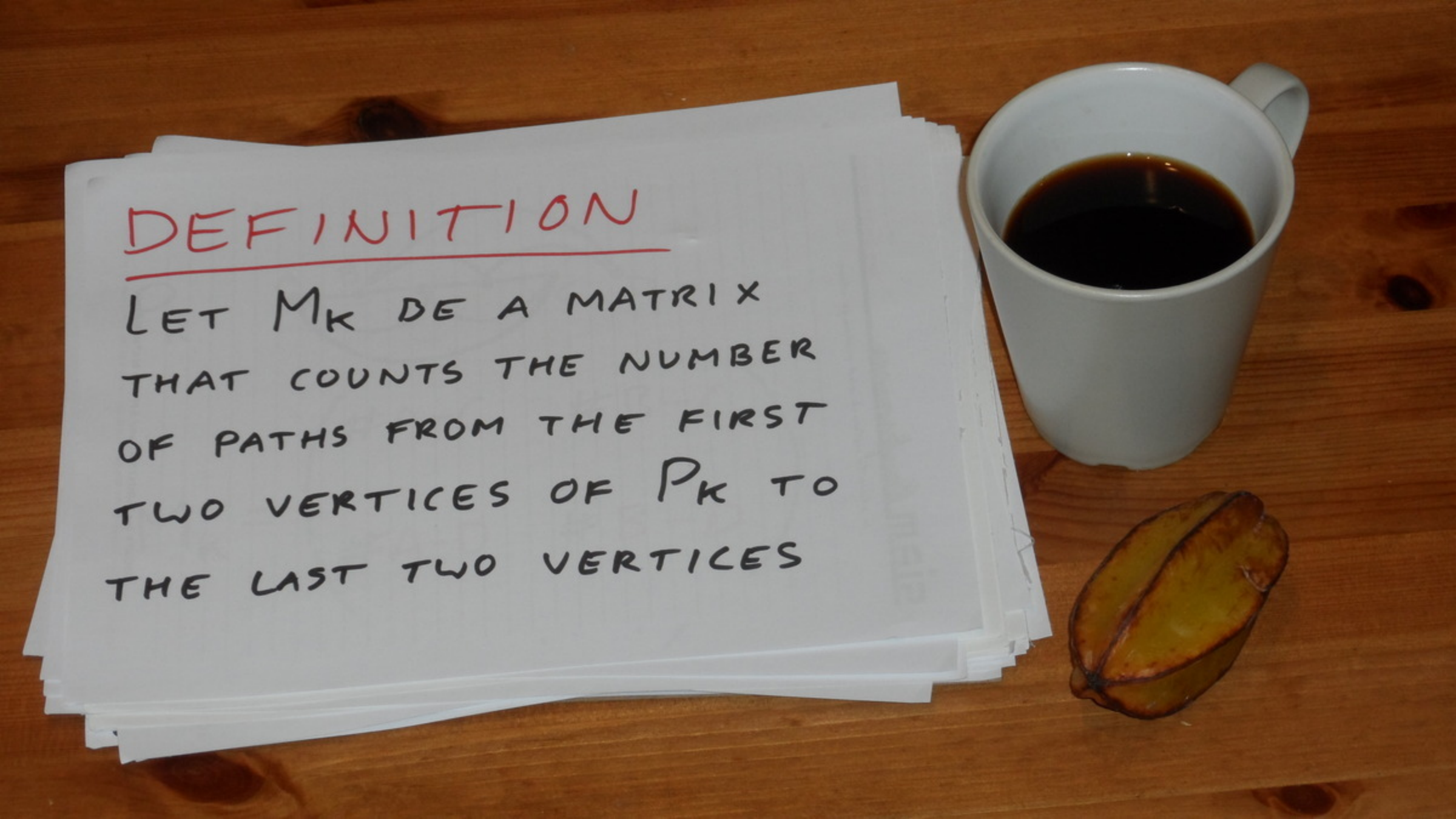

MAXIMUM NUMBER #
LOWER BOUND
UPPER BOUND
POTATOES
1.5^N
1.62^N
EN
CARAMBOLAS (MONOTONE PATHS)
1.7^N
1.84^N
SIMPLE PATHS
1.84^N
3^N


LOWER BOUND
KEN

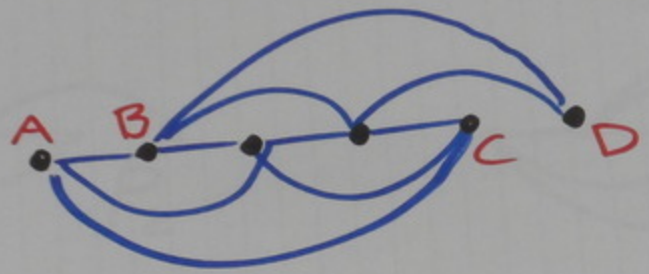

CONSTRUCTION
LET P_K BE A GRAPH ON 2^K+2 VERTICES, WITH AN EDGE BETWEEN VERTEX i AND j IF
$GCD(i, j) = 2^H$ OR
$GCD(i-1, j-1) = 2^H$


P_4

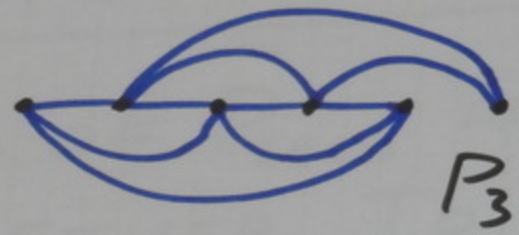

P_{K-1}
P_{K-1}
P_K


DEFINITION
LET M_k BE A MATRIX THAT COUNTS THE NUMBER OF PATHS FROM THE FIRST TWO VERTICES OF P_k TO THE LAST TWO VERTICES

P_K



$$M_K = \begin{pmatrix} \#A\text{-}C & \#B\text{-}C \\ \#A\text{-}D & \#B\text{-}D \end{pmatrix}$$

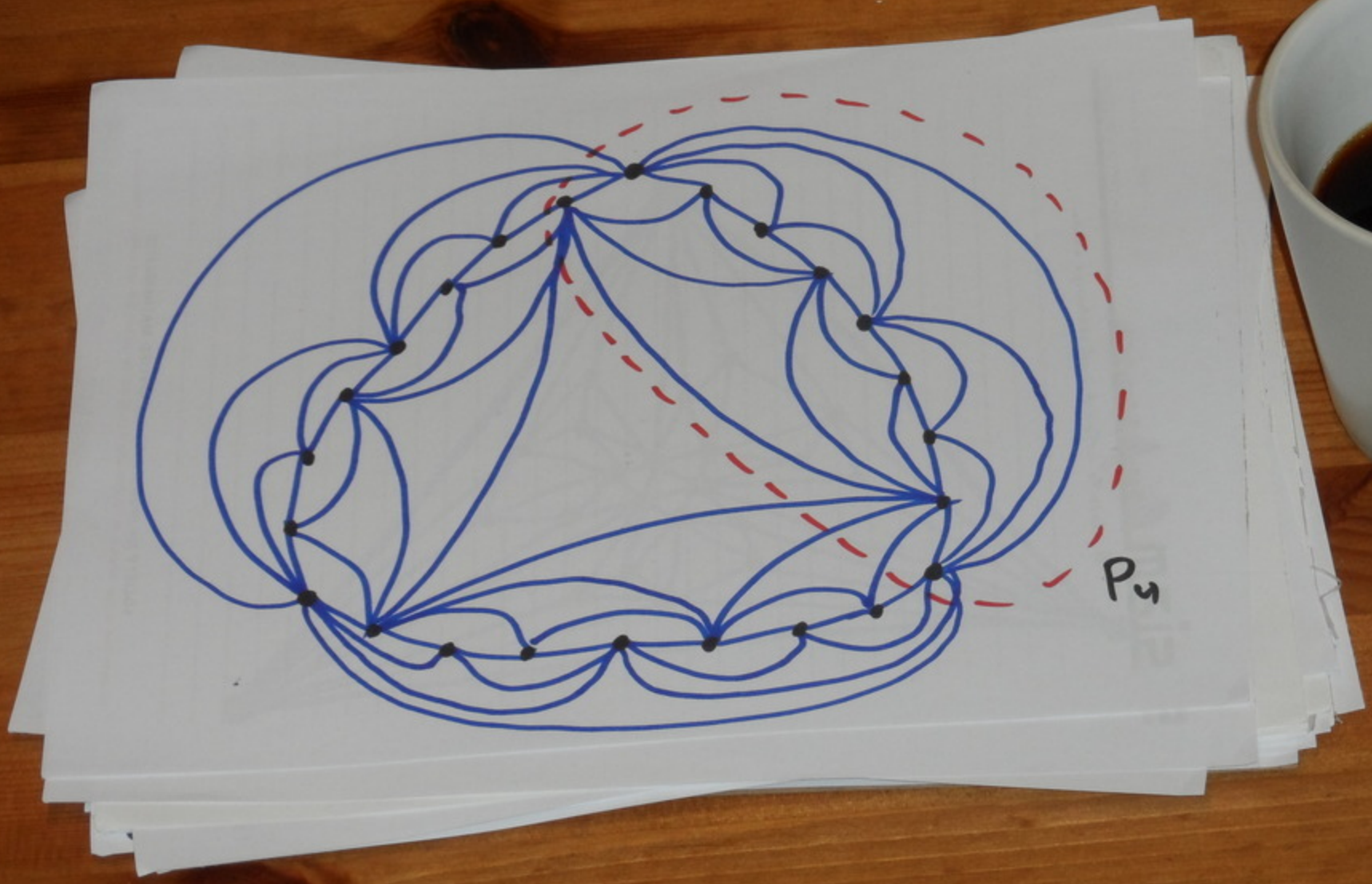


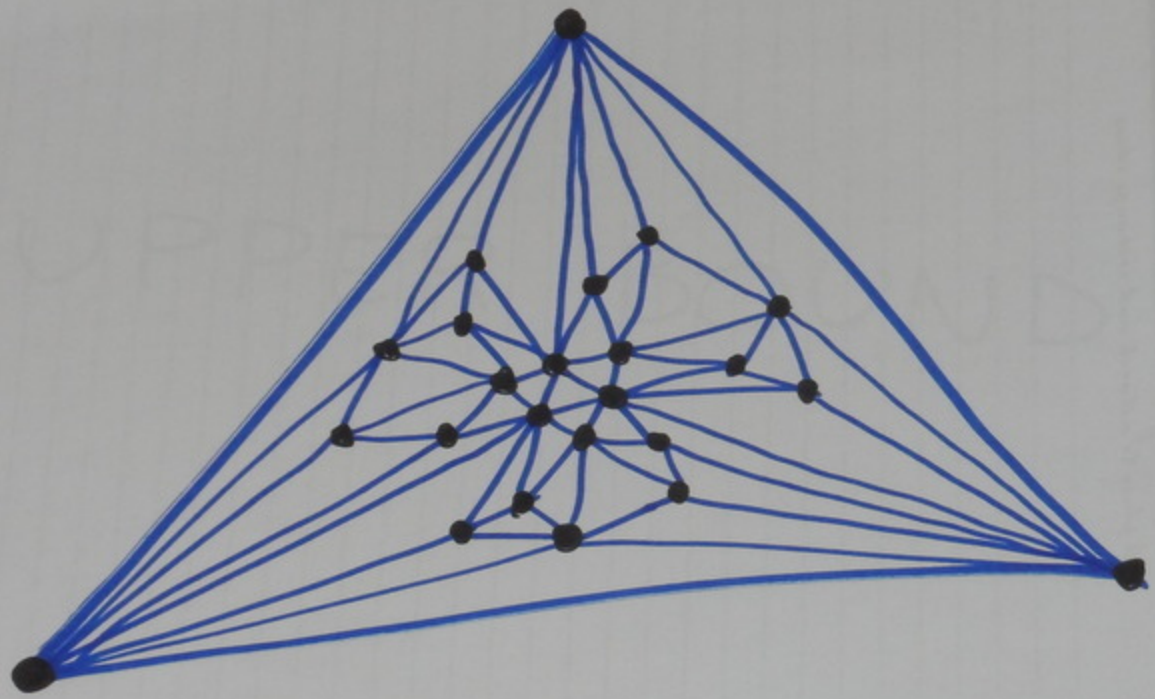
$$M_2 = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \qquad M_3 = \begin{pmatrix} 6 & 3 \\ 4 & 3 \end{pmatrix}$$

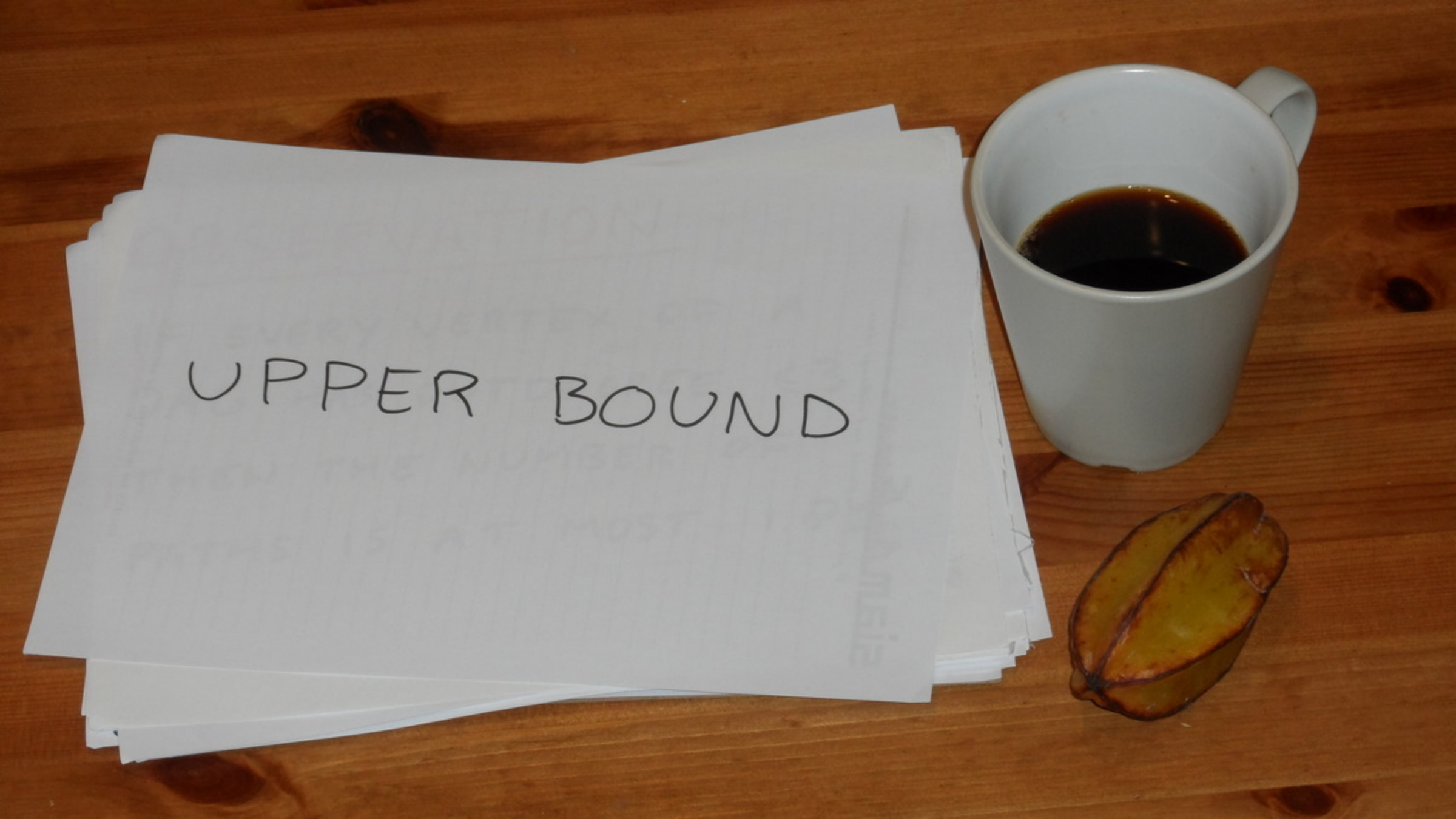
$$M_k = M_{k-1}^2 + \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

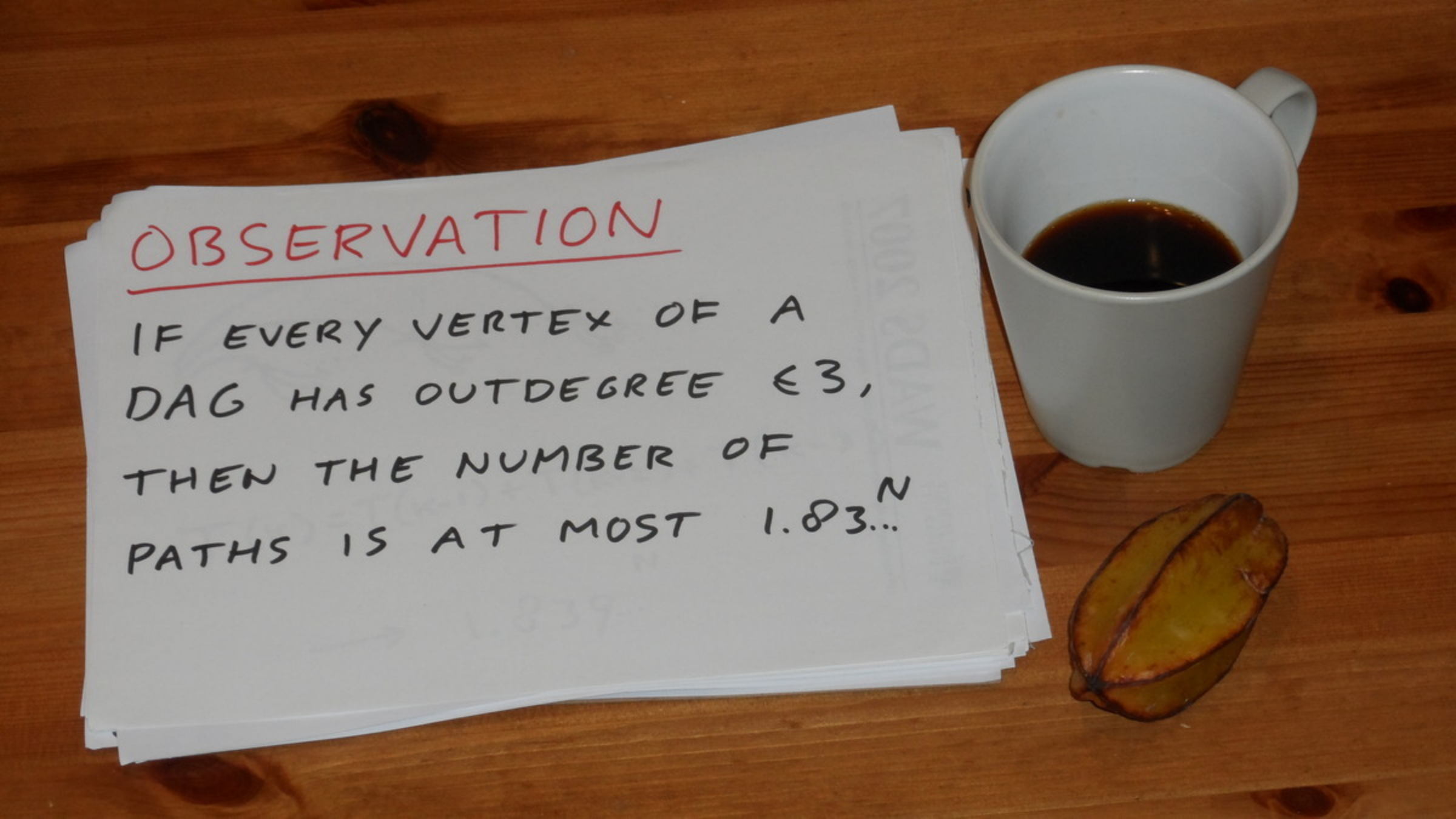
BACK TO CARAMBOLAS

CREATE 3 COPIES OF P_{LOGN} AND GLUE THEM TOGETHER. TO GET A STRAIGHT EMBEDDING, ALTERNATE VERTICES INSIDE AND OUTSIDE. MAINTAIN CYCLICAL ORDER TO ENSURE STAR-SHAPES.


P_4

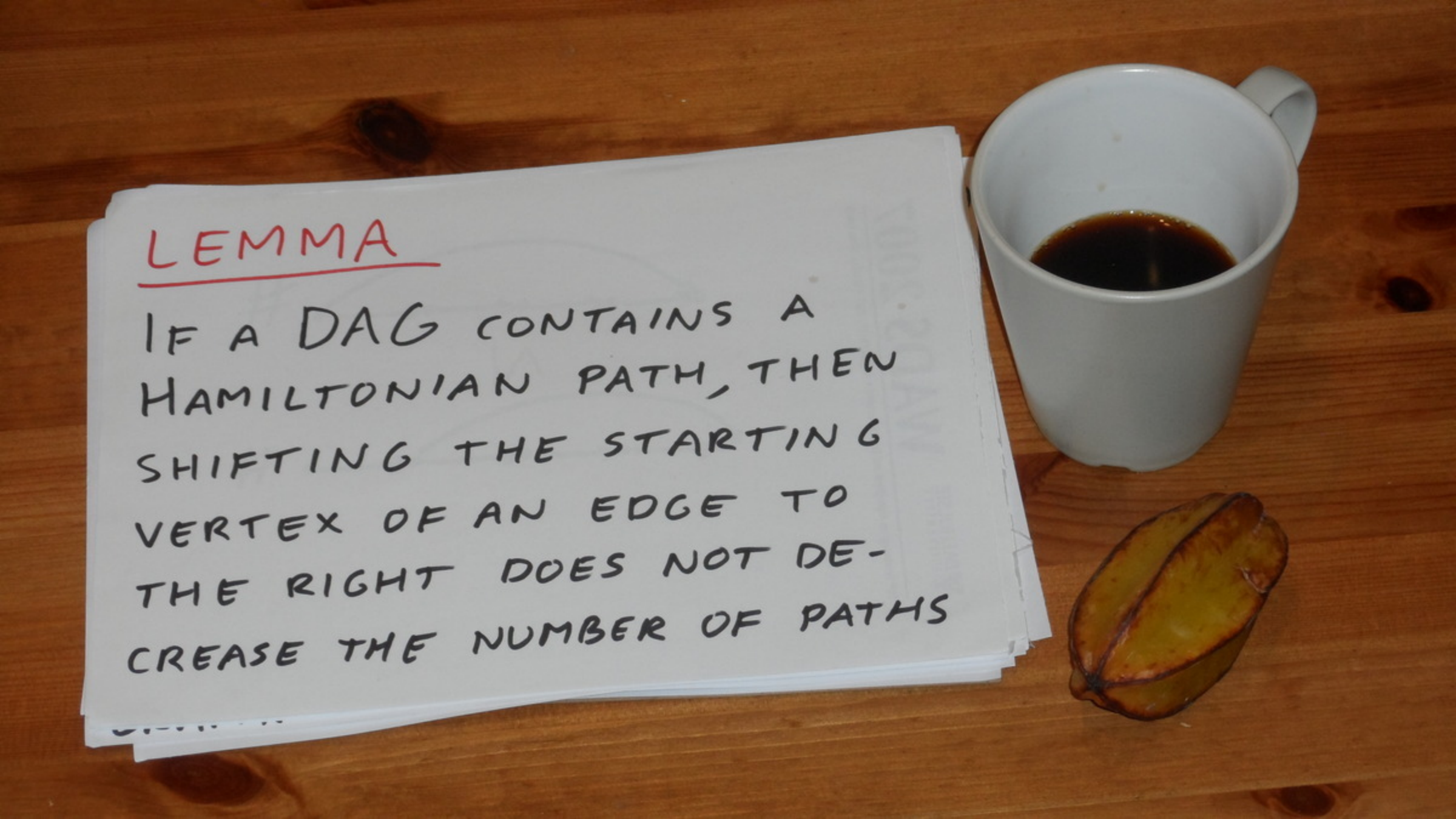


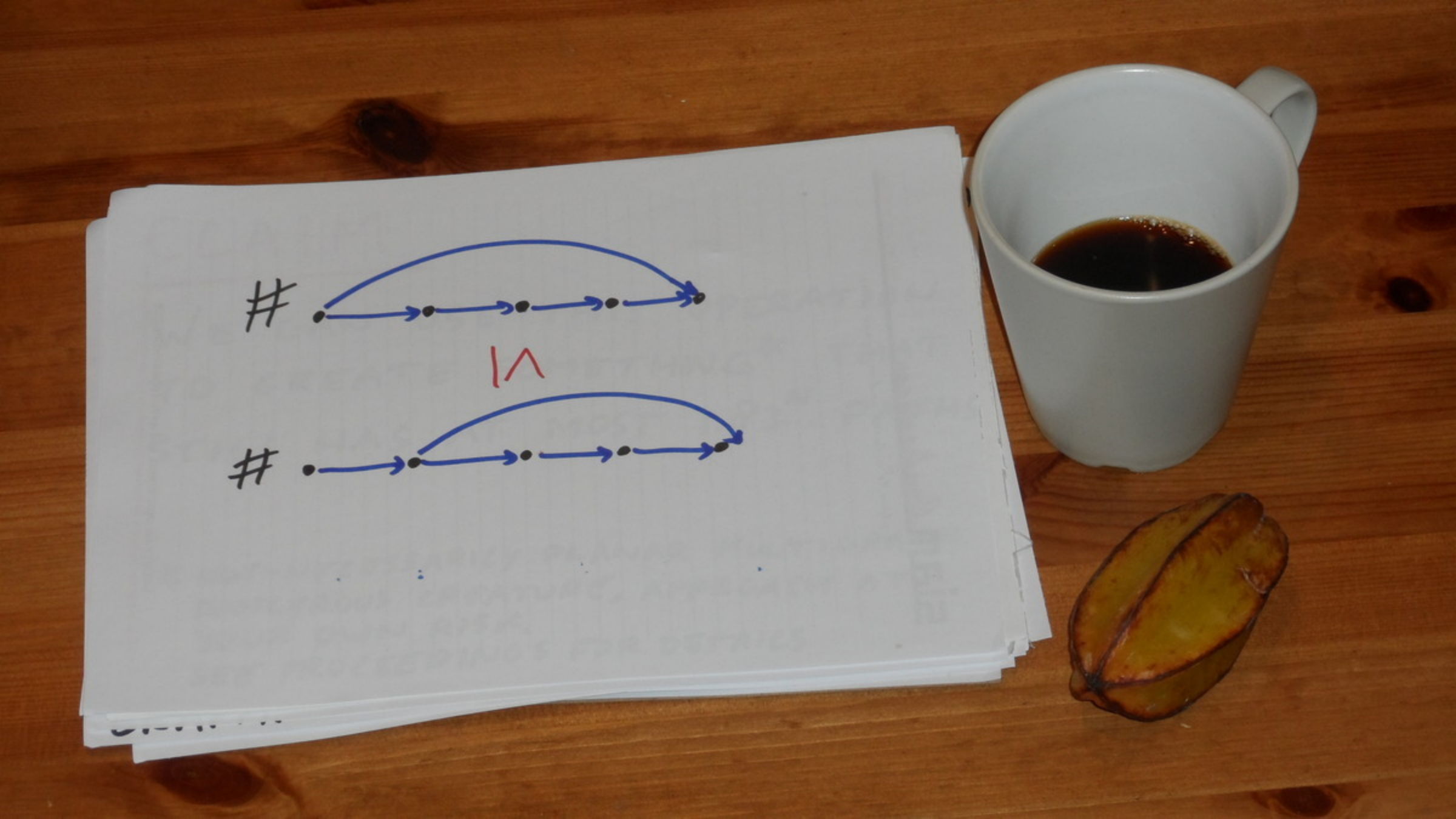

UPPER BOUND

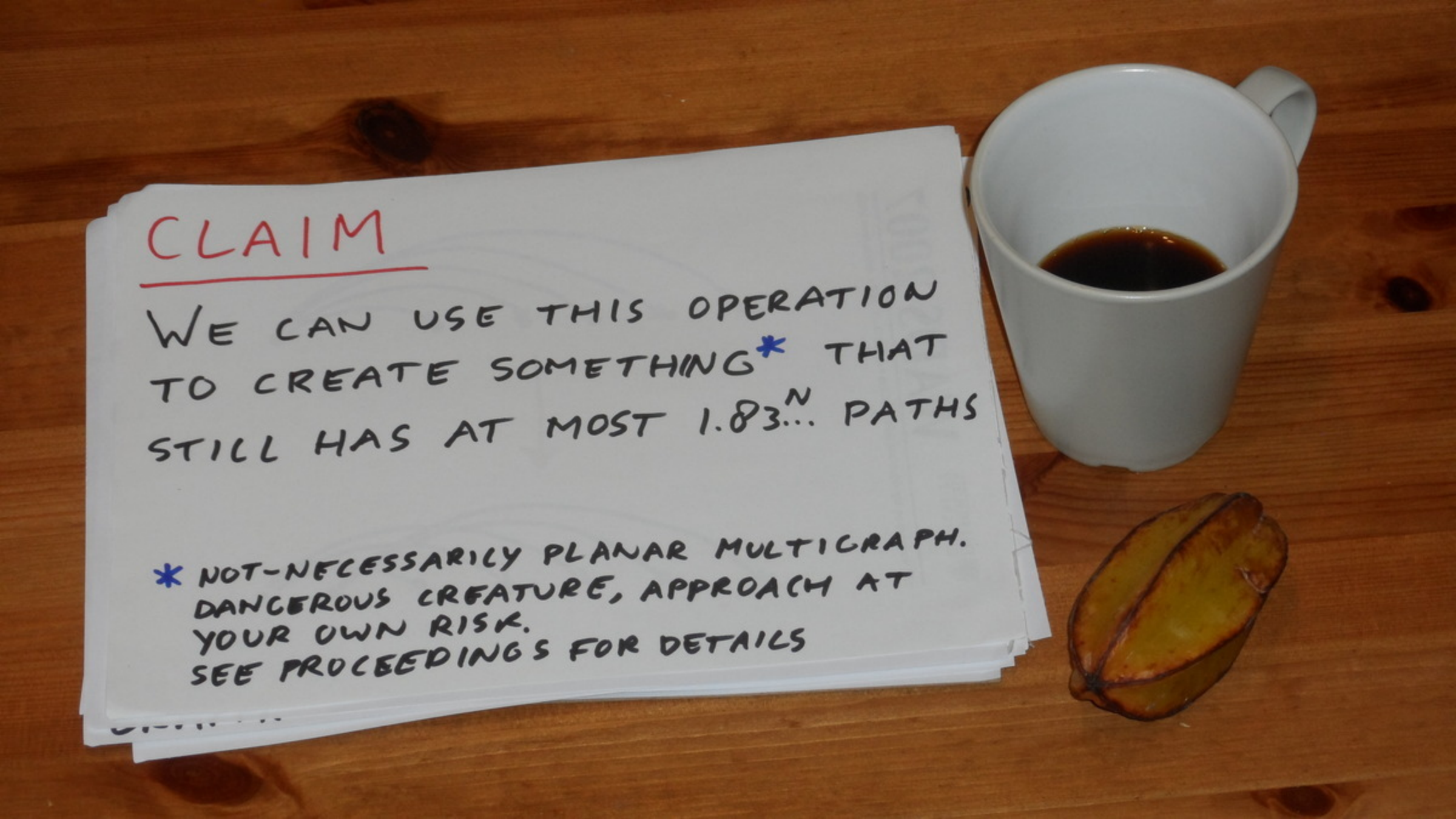

OBSERVATION
IF EVERY VERTEX OF A DAG HAS OUTDEGREE ≥3, THEN THE NUMBER OF PATHS IS AT MOST 1.83^N

$$T(k) = T(k-1) + T(k-2) + T(k-3)$$

$$\rightarrow 1.839\ldots^{N}$$

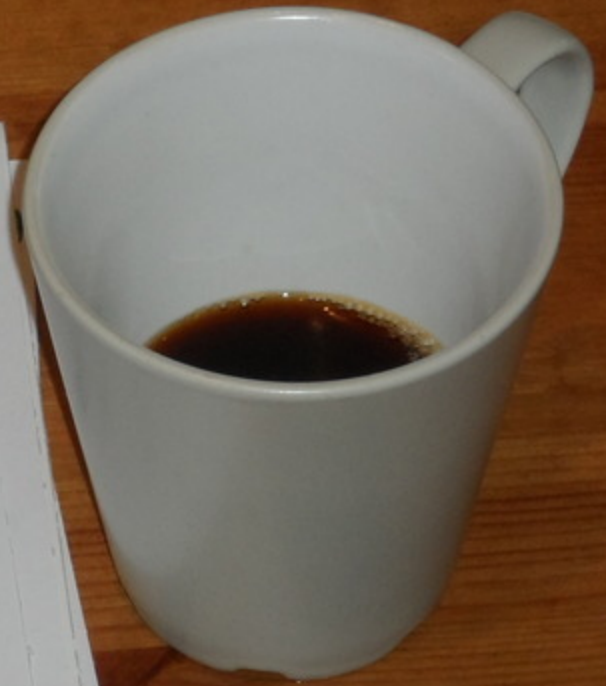

LEMMA
IF A DAG CONTAINS A HAMILTONIAN PATH, THEN SHIFTING THE STARTING VERTEX OF AN EDGE TO THE RIGHT DOES NOT DECREASE THE NUMBER OF PATHS

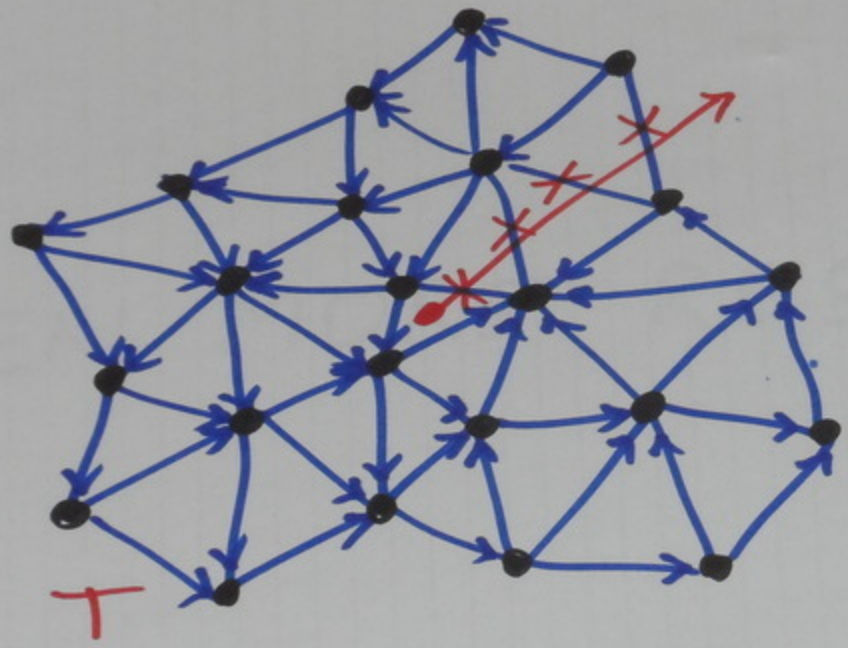


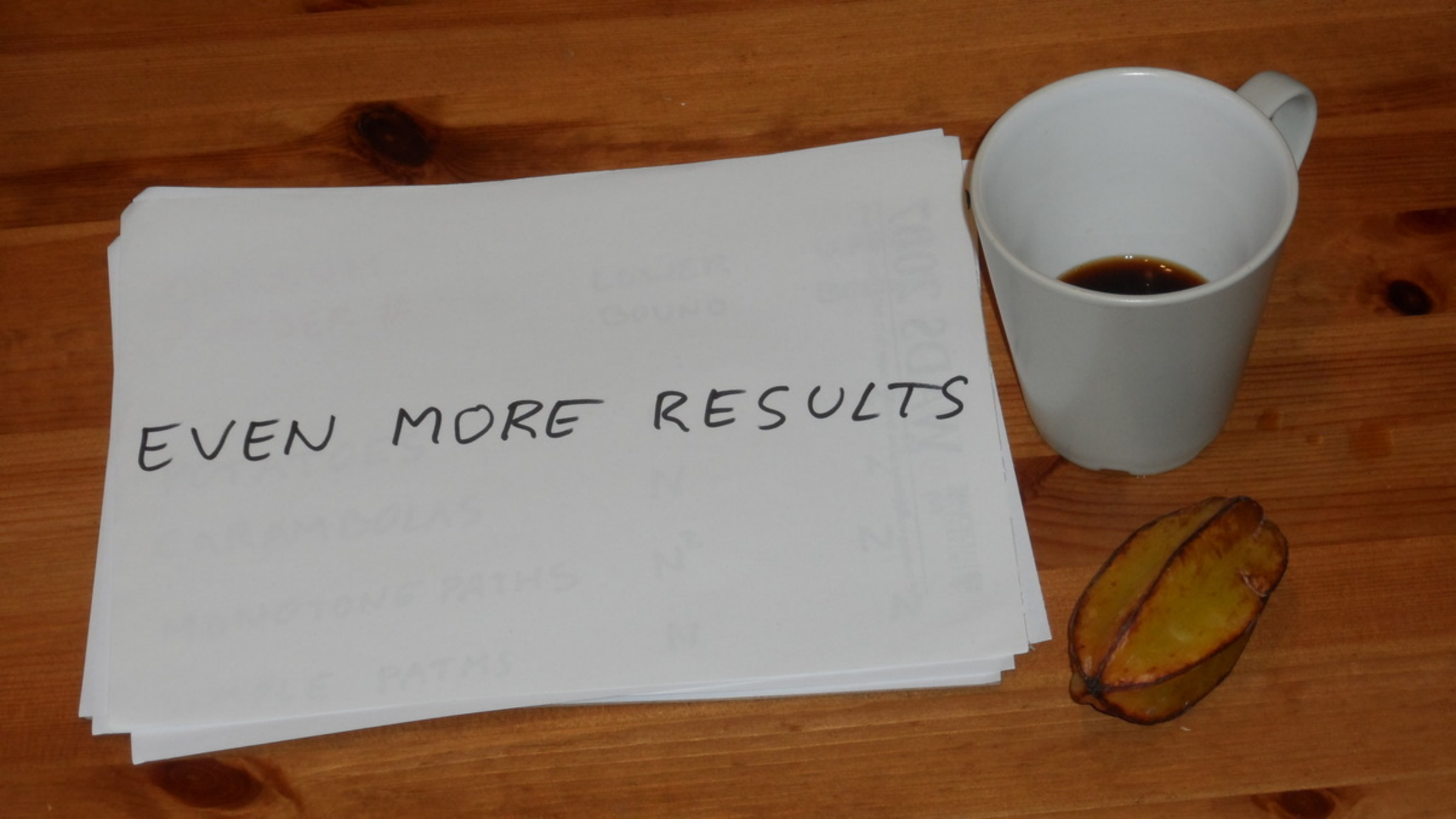

CLAIM
WE CAN USE THIS OPERATION TO CREATE SOMETHING* THAT STILL HAS AT MOST 1.83^N... PATHS
* NOT-NECESSARILY PLANAR MULTIGRAPH. DANGEROUS CREATURE, APPROACH AT YOUR OWN RISK. SEE PROCEEDINGS FOR DETAILS

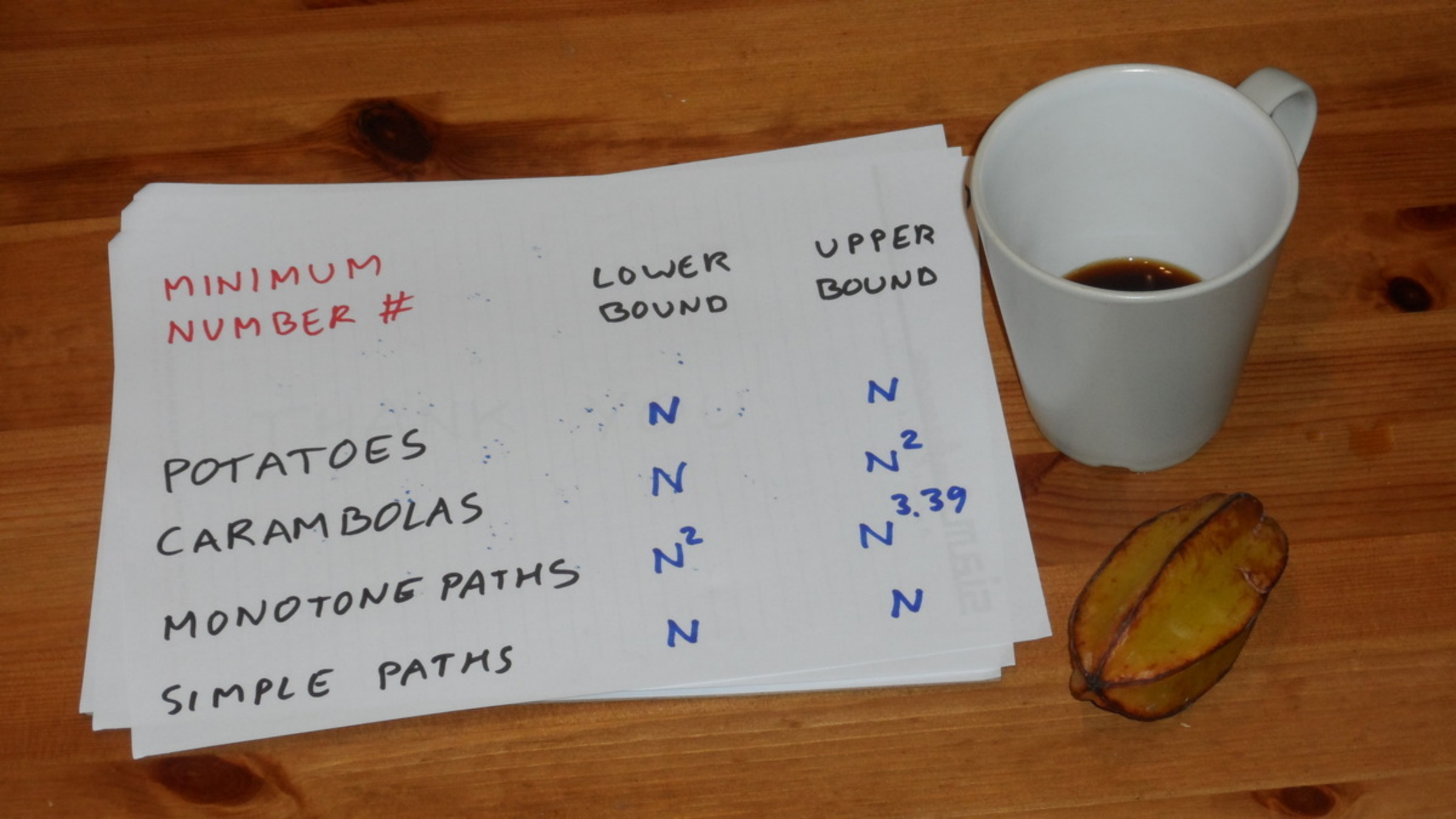
BACK TO CARAMBOLAS

GIVEN A TRIANGULATION T, GUESS THE CENTER POINT AND ORIENT ALL EDGES AROUND IT. SHOOT A RAY AND REMOVE ALL EDGES IT INTERSECTS. APPLY PREVIOUS BOUND TO REMAINING GRAPH.






EVEN MORE RESULTS


MINIMUM NUMBER #
LOWER BOUND
UPPER BOUND
POTATOES
N
N
CARAMBOLAS
N
N^2
MONOTONE PATHS
N^2
N^3.39
SIMPLE PATHS
N
N


THANK YOU!


DID YOU ENJOY THIS TALK?
WADS, AUGUST 12, 16:30