Existence of Simple Tours through Imprecise Points

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Overview

• Imprecise points
• Imprecise simple polygons
• NP-hardness proof
  • Planar 3-SAT
  • Variables as scissors
  • Clauses
  • Further details
• Other results and consequences
Imprecise Points

- Unknown location
- Known region of possible locations
- Regions are simple geometric objects
  - Disc
  - Square
  - Rectangle
  - Convex polygon
  - Line segment
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Imprecise Simple Polygons

- Sequence of imprecise points
- Place one vertex in each region
- The result should be a simple polygon
- This problem is NP-hard
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Planar 3-SAT

- Boolean satisfiability problem
- At most three variables per clause
- Variable-clause graph must be planar
- Known to be NP-hard [Lichtenstein 1982]
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Scissors Gadget

- Configuration of four imprecise points
- Two distinct possible solutions
- Each variable will be represented by a number of scissors
  - Positive slope: true
  - Negative slope: false
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Scissors Chain
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Clause Gadget

- Configuration of three imprecise points
- Two fixed parts of the tour
- Three distinct possible solutions
- Each solution will be connected to one of the clause’s variables
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Variables and Clauses
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Further Details

- **Splitting variables**
  - Variables will occur in many clauses
  - Chains need to move vertically

- **Connecting tour parts**
  - Many small pieces of tour need to become one big tour
  - Bridges to cross chains
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Conclusions

• Finding a simple polygon with imprecise vertices as vertical line segments is NP-hard

• Other results
  • Imprecision model: any other connected region
  • Finding the shortest simple tour through \( n \) regions
Questions?