

Boris
Aronov

Bettina
Speckmann

Maike
Buchin

Kevin
Buchin

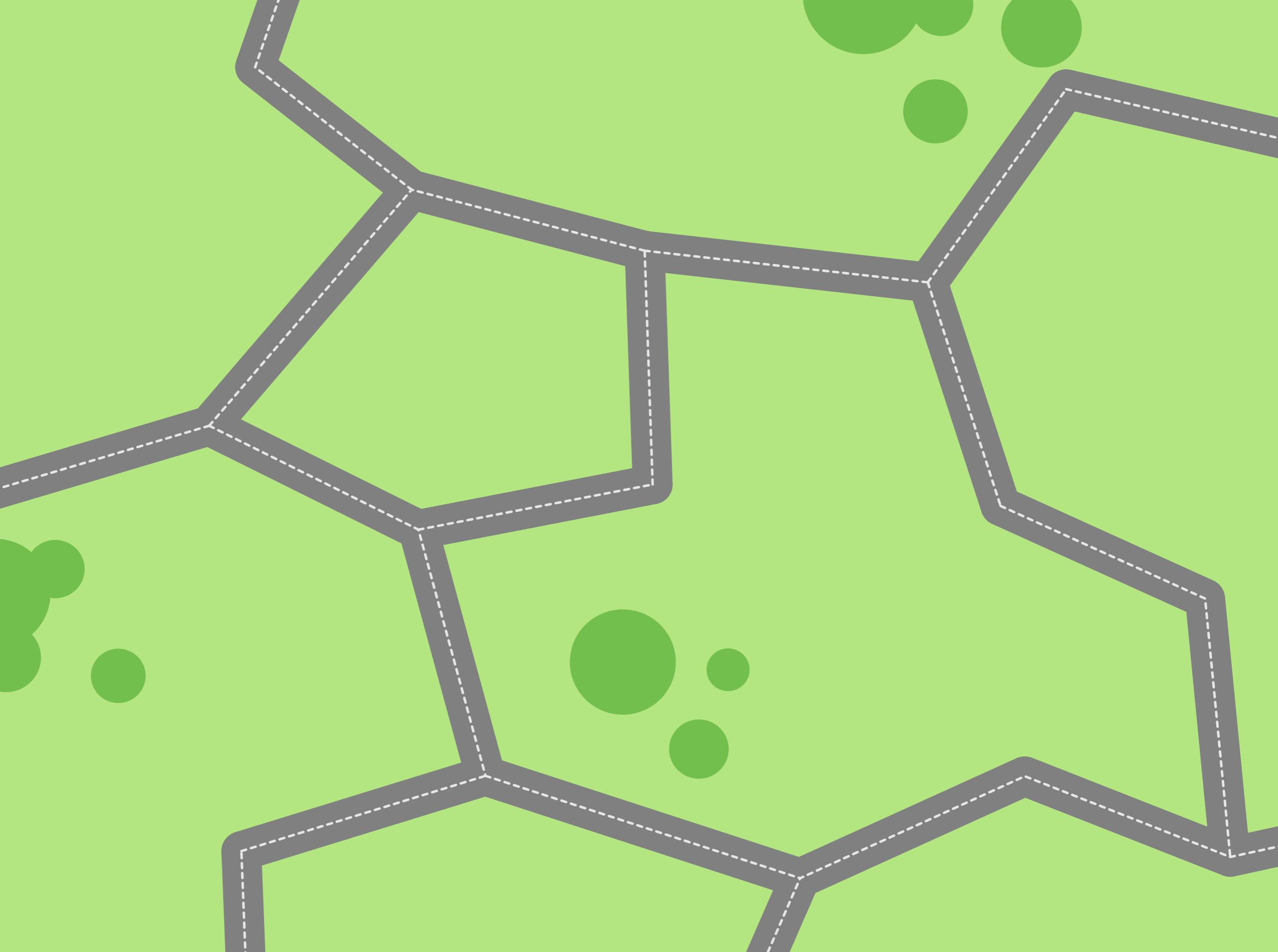
Connect The Dot
or
Computing Feed Links
with
Minimum Dilation

Jun
Luo

Rodrigo
Silveira

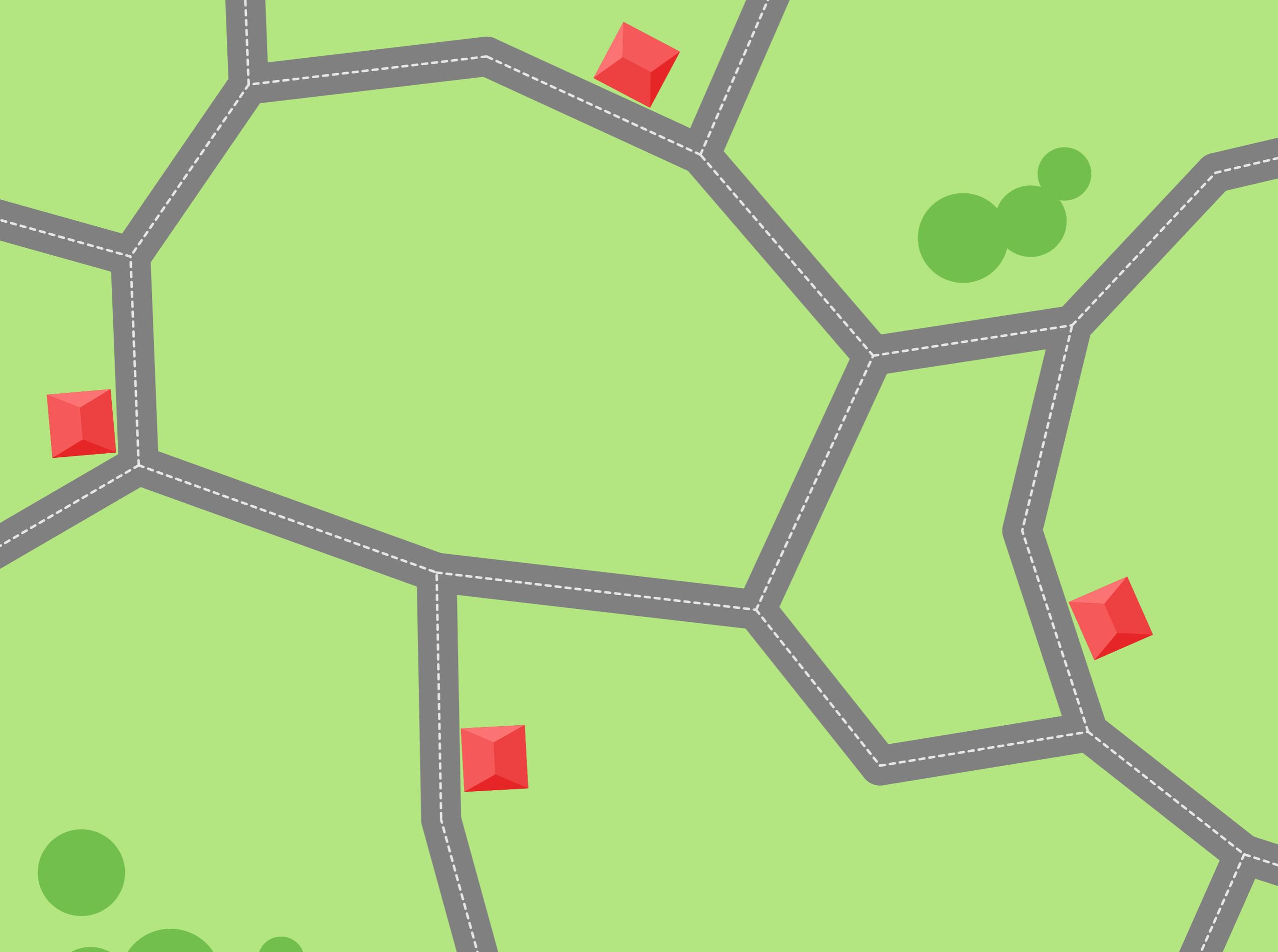
Marc
van Kreveld

Maarten
Löffler



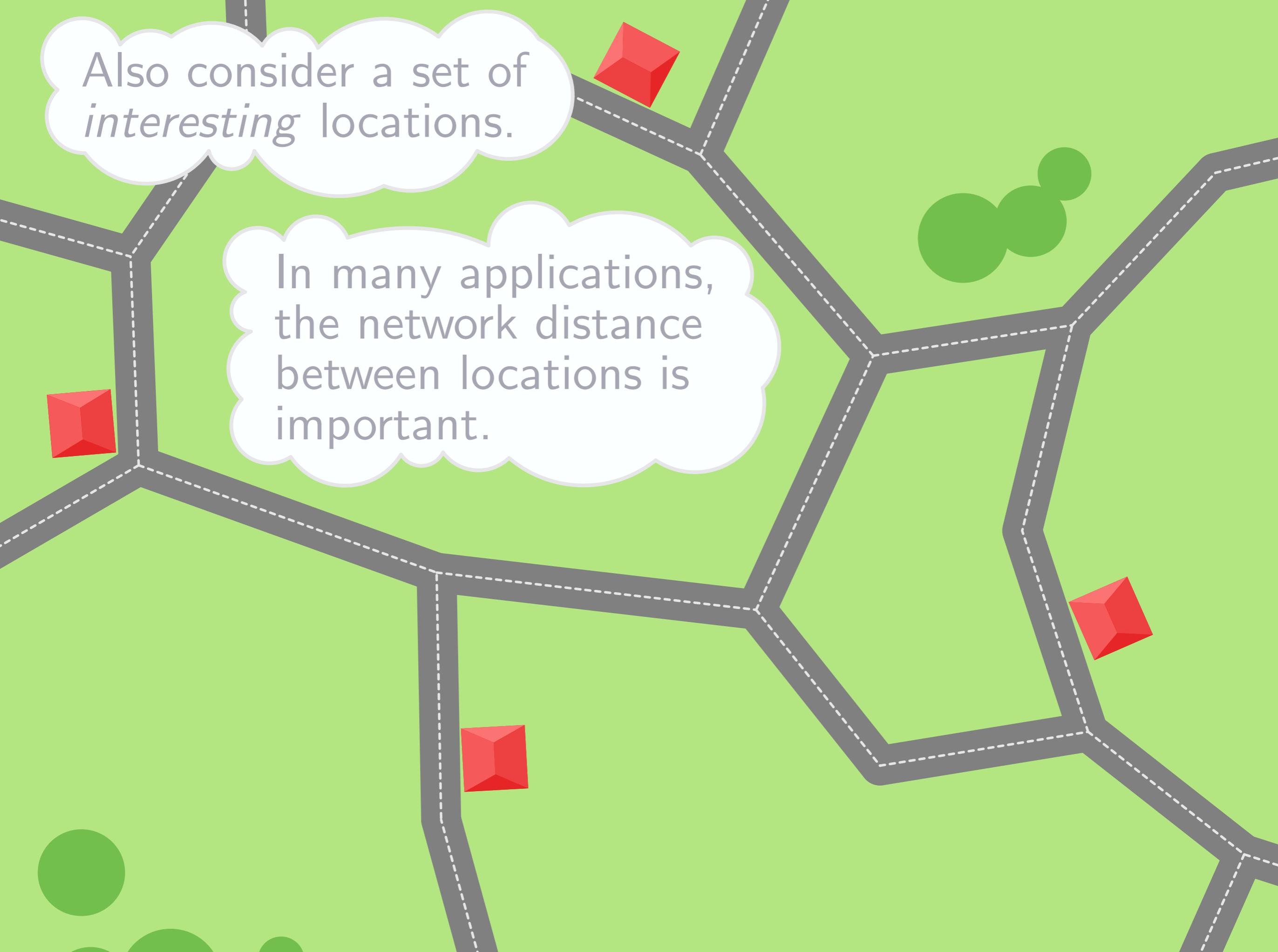
The image features a light green background with a network of dark grey roads. The roads are represented by solid lines with a dashed white line running parallel to them, creating a sense of depth. The network consists of several interconnected paths that form irregular, roughly hexagonal shapes. In the center of the network, there is a white, cloud-like speech bubble with a thin grey outline. Inside this bubble, the text "Consider a network of roads." is written in a simple, grey, sans-serif font. Scattered throughout the green background are several solid green circles of varying sizes, some of which are partially obscured by the road lines.

Consider a network of roads.



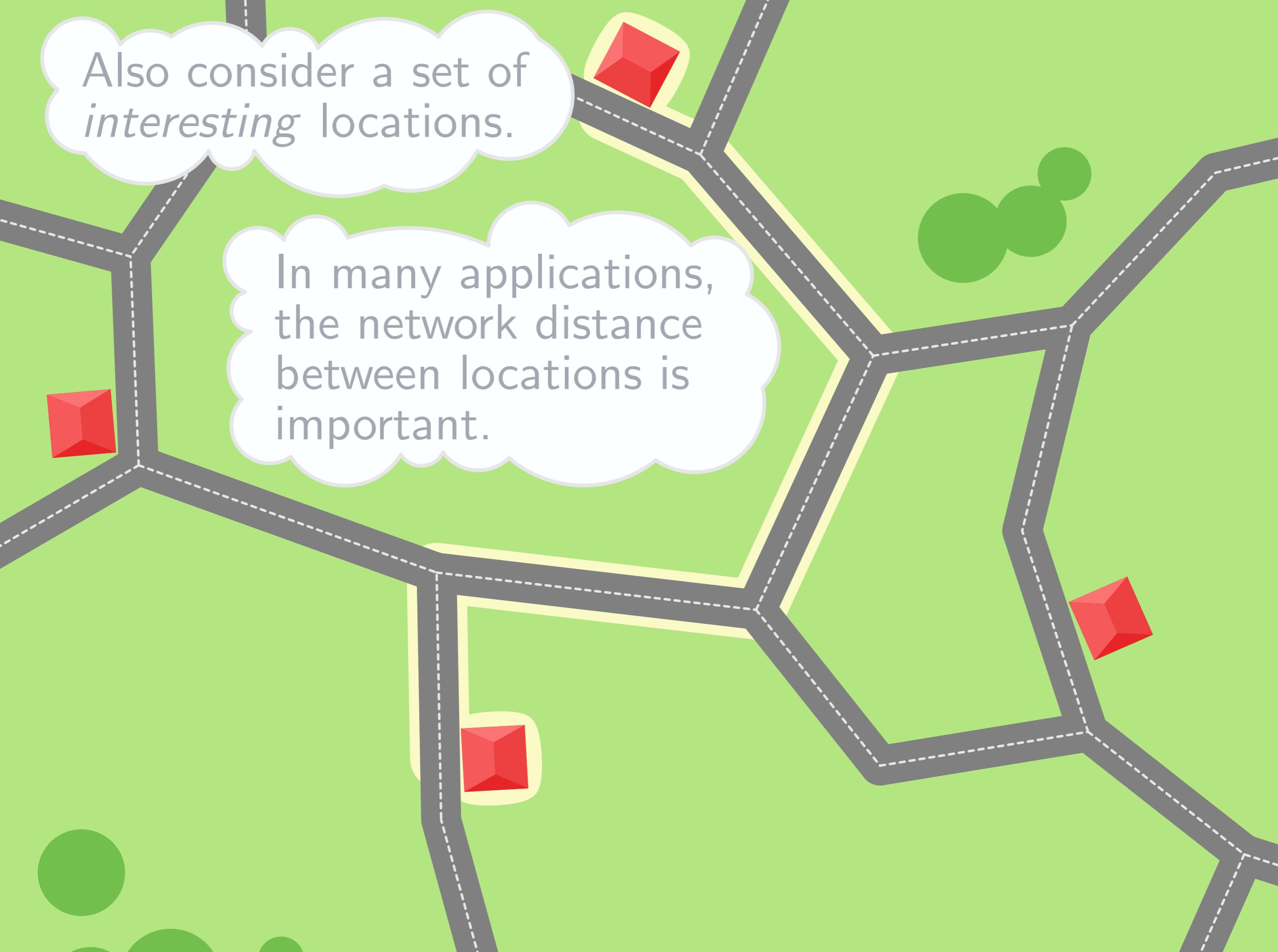
Also consider a set of *interesting* locations.



A stylized map with a network of roads and interesting locations. The background is light green. A network of dark grey roads with dashed white lines runs across the map. There are four red 3D cube-like shapes representing interesting locations. In the top right, there are three green circles of varying sizes representing trees. In the bottom left, there are also three green circles of varying sizes.

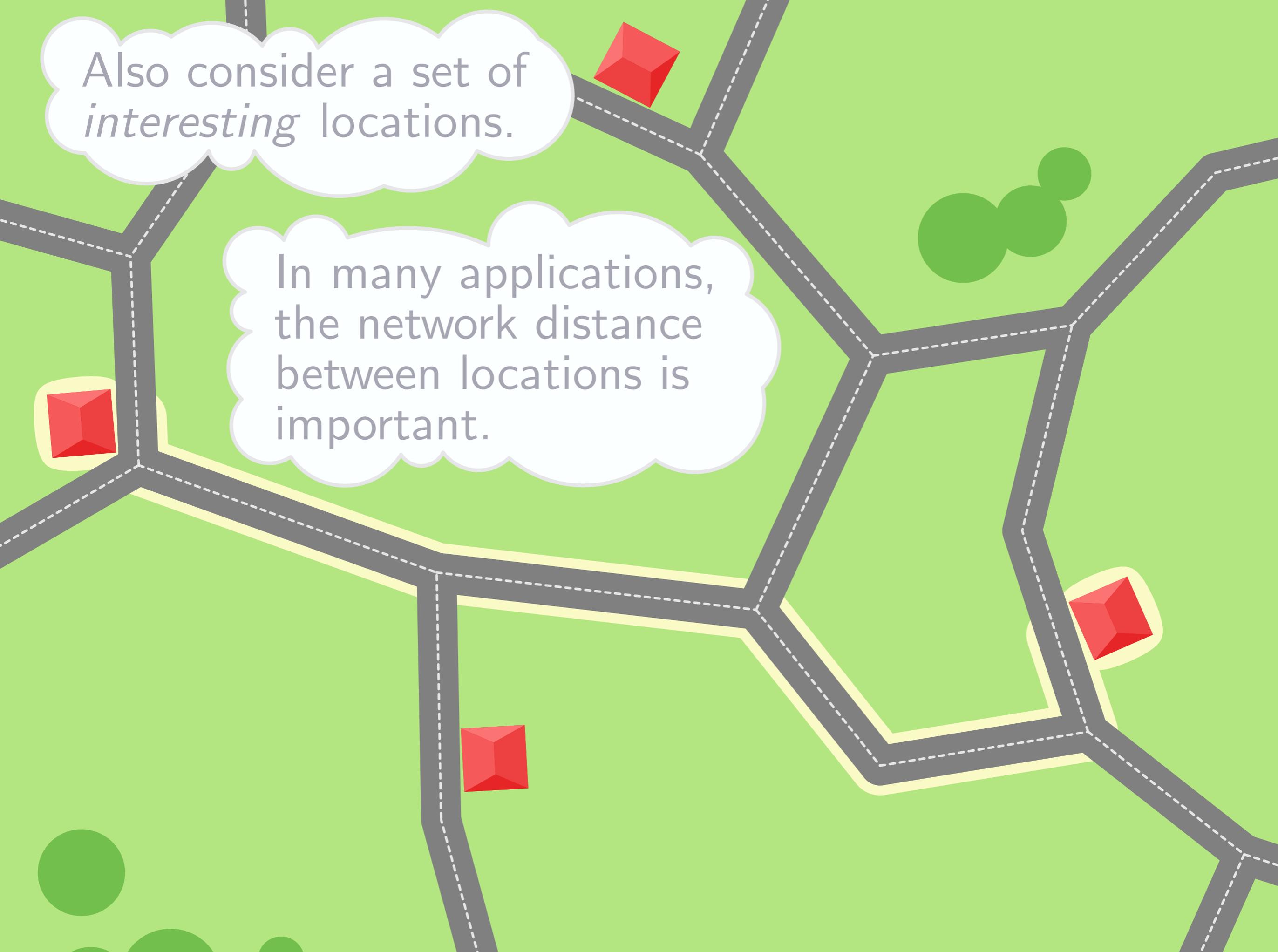
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In many applications,
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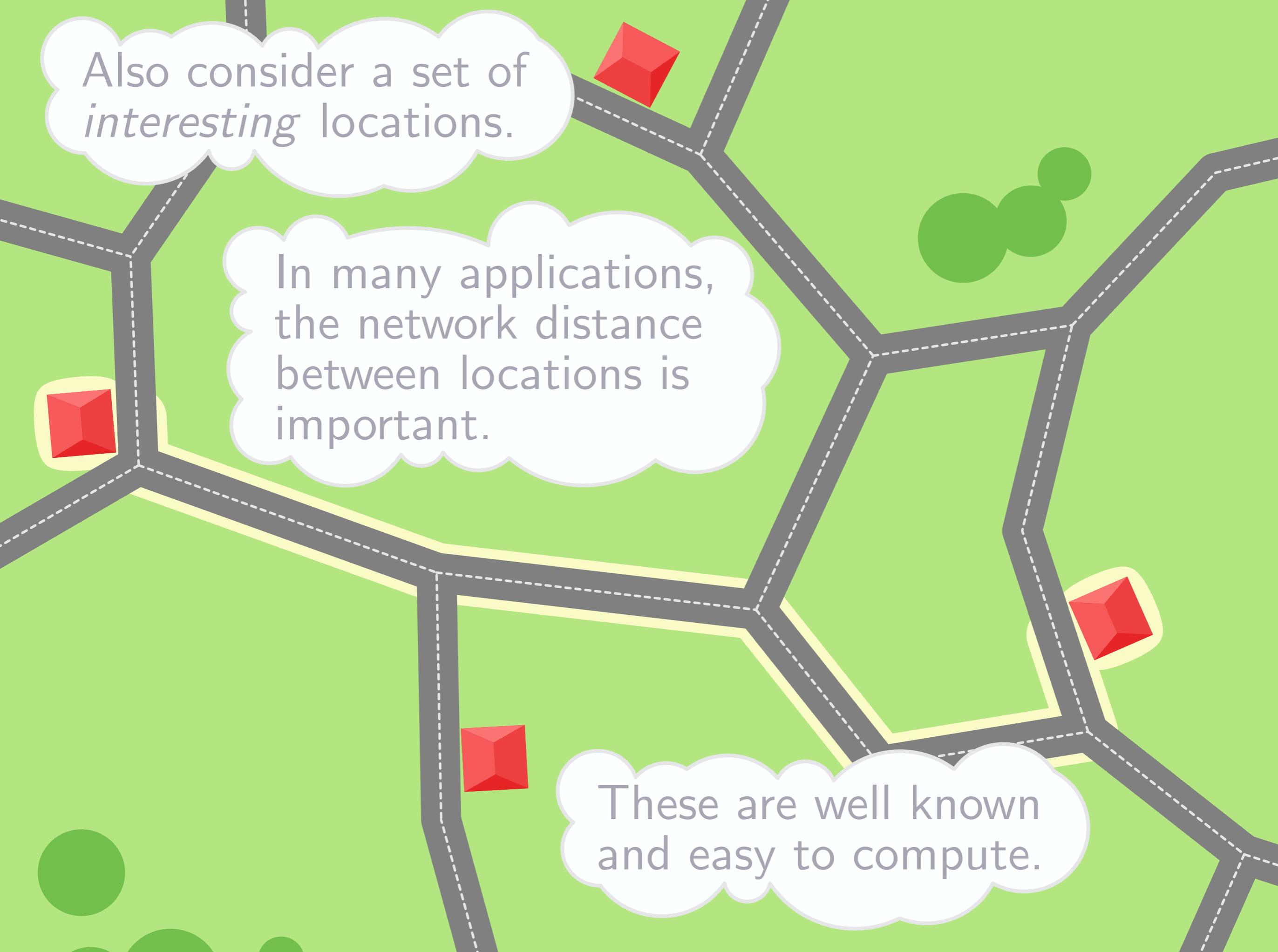
Also consider a set of *interesting* locations.

In many applications, the network distance between locations is important.

A network diagram on a green background. A grey road network is shown with dashed lines indicating the underlying structure. A yellow path highlights a specific route through the network. Four red cube markers are placed at various points along the network. Two callout boxes with white backgrounds and grey borders contain text. The background is decorated with green circles of varying sizes, representing trees or bushes.

Also consider a set of *interesting* locations.

In many applications, the network distance between locations is important.

A network diagram on a light green background. The network consists of several interconnected nodes and edges. A path of nodes and edges is highlighted in yellow. There are four red cube markers placed at various nodes: one at the top, one at the left, one at the bottom, and one at the right. There are also several green circles of varying sizes scattered around the network.

Also consider a set of *interesting* locations.

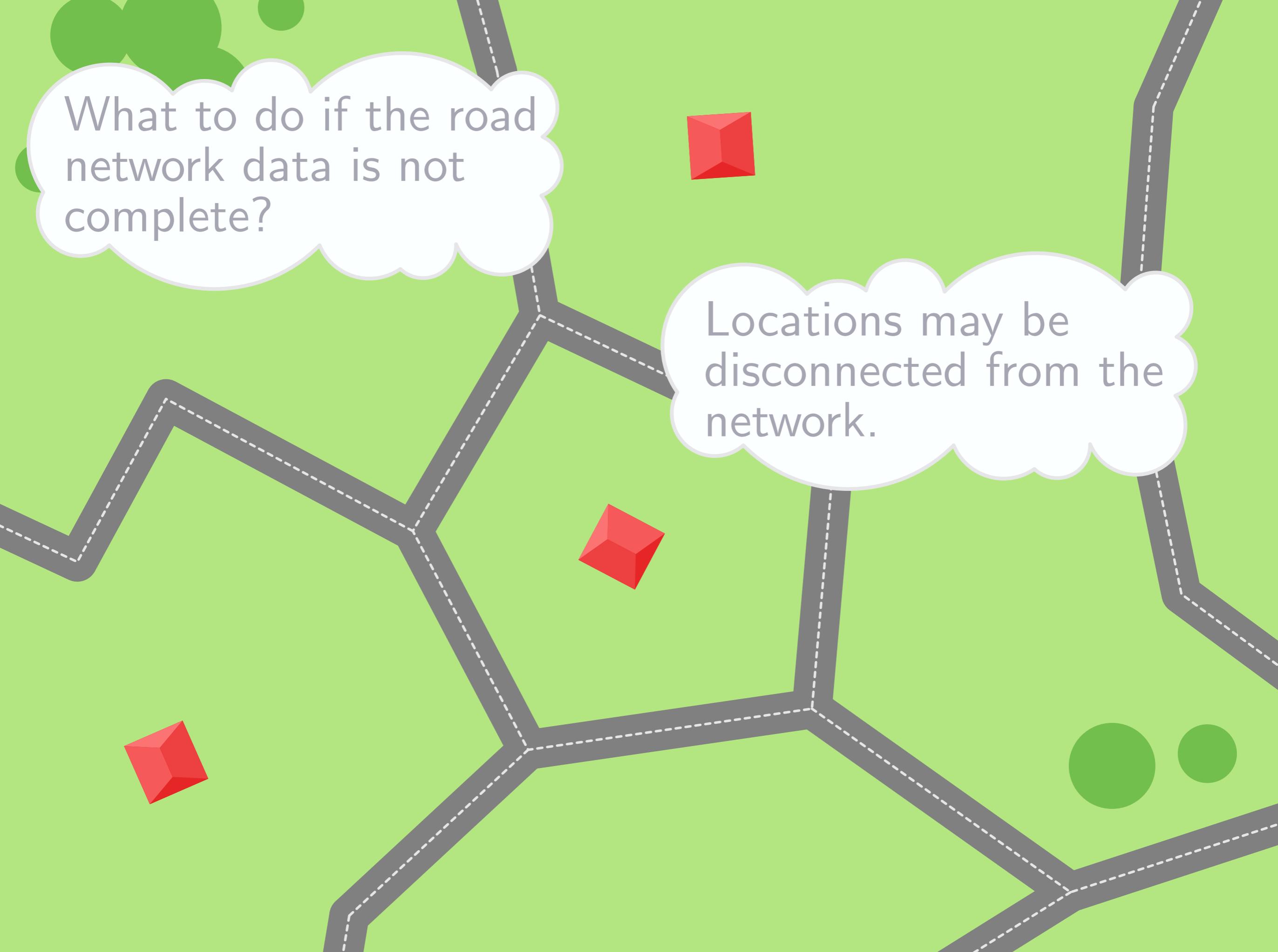
In many applications, the network distance between locations is important.

These are well known and easy to compute.



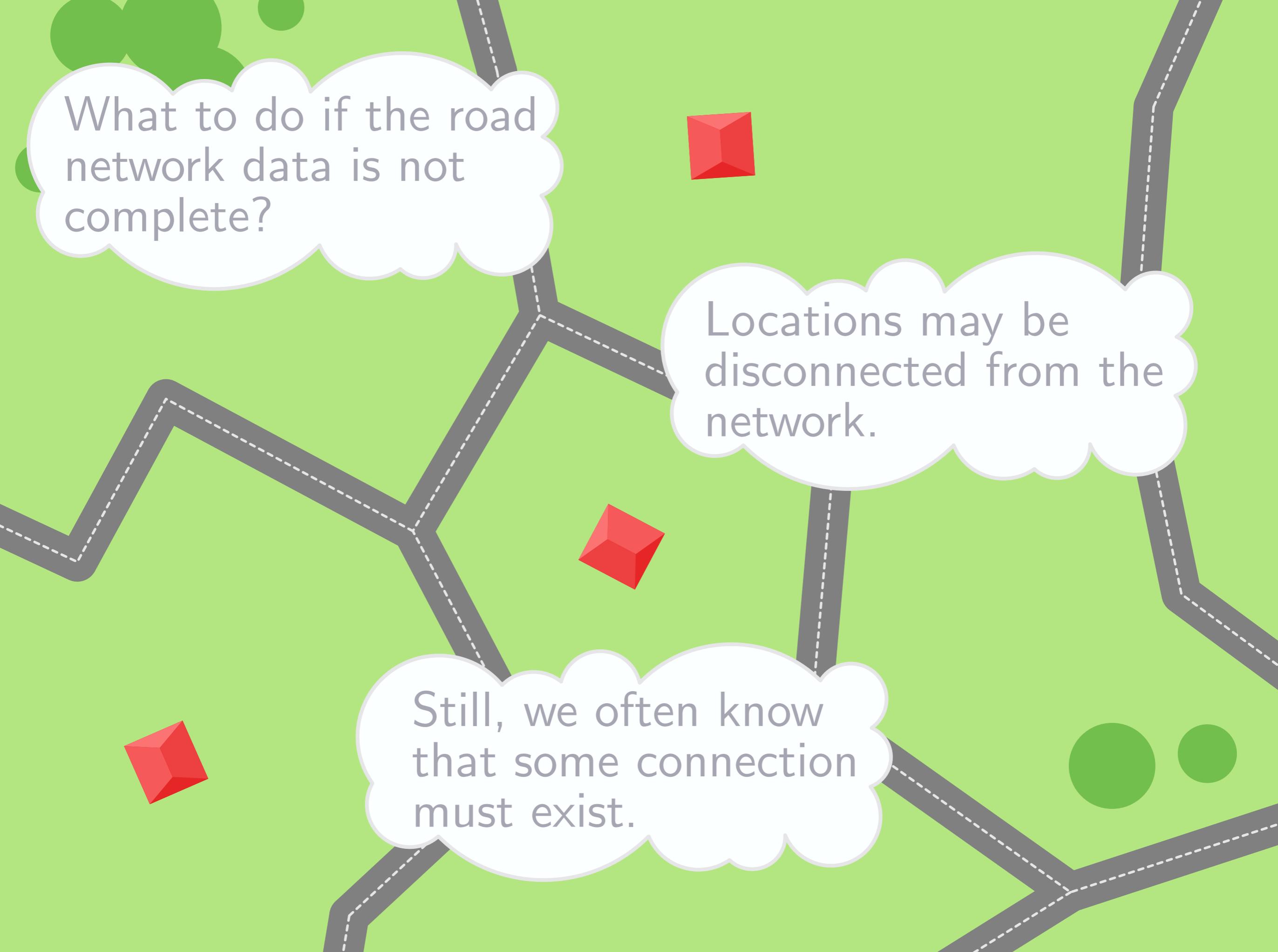
What to do if the road network data is not complete?





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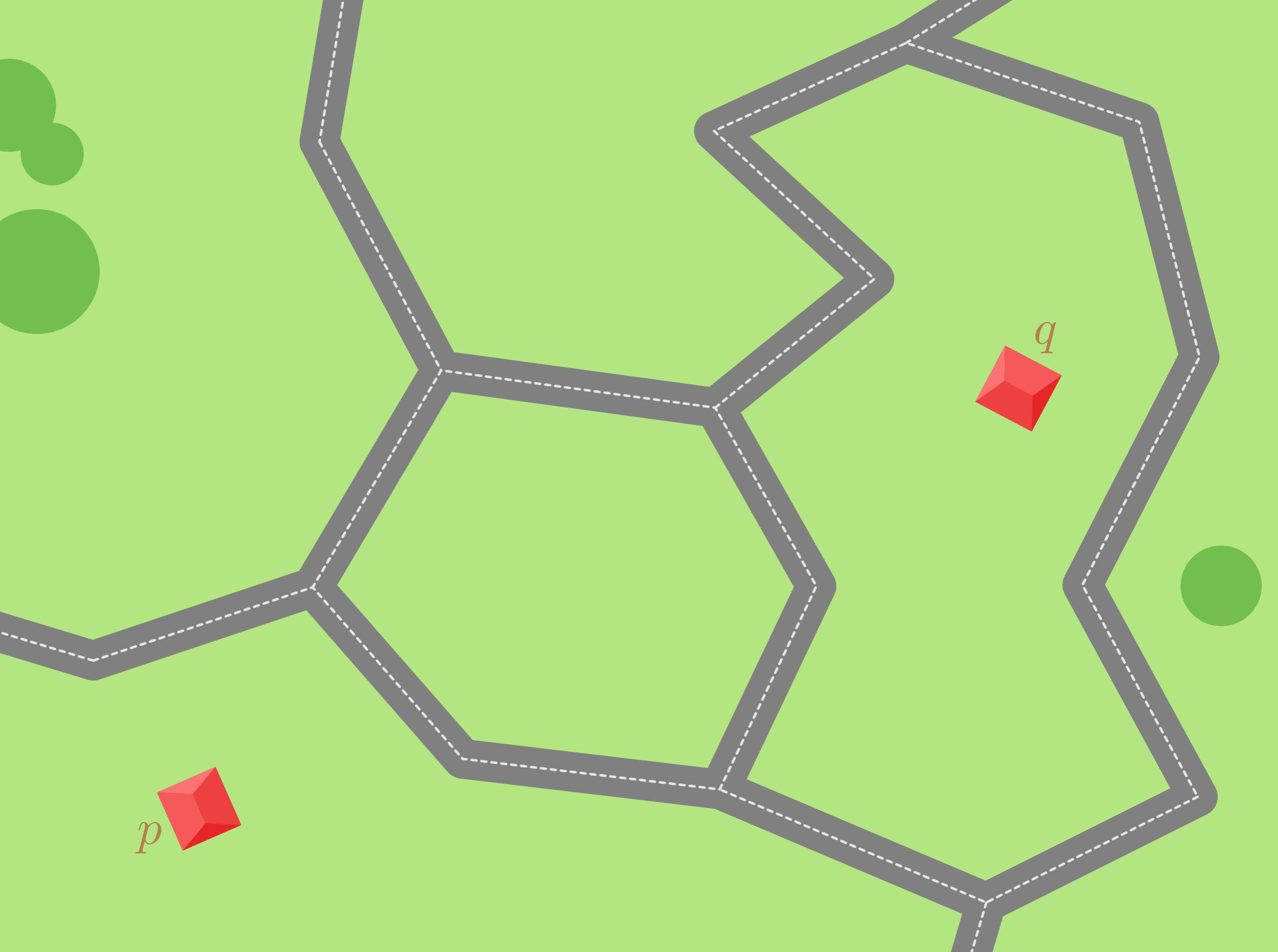
Locations may be disconnected from the network.



What to do if the road network data is not complete?

Locations may be disconnected from the network.

Still, we often know that some connection must exist.



p

q

There must be some path from p to q .



There must be some path from p to q .

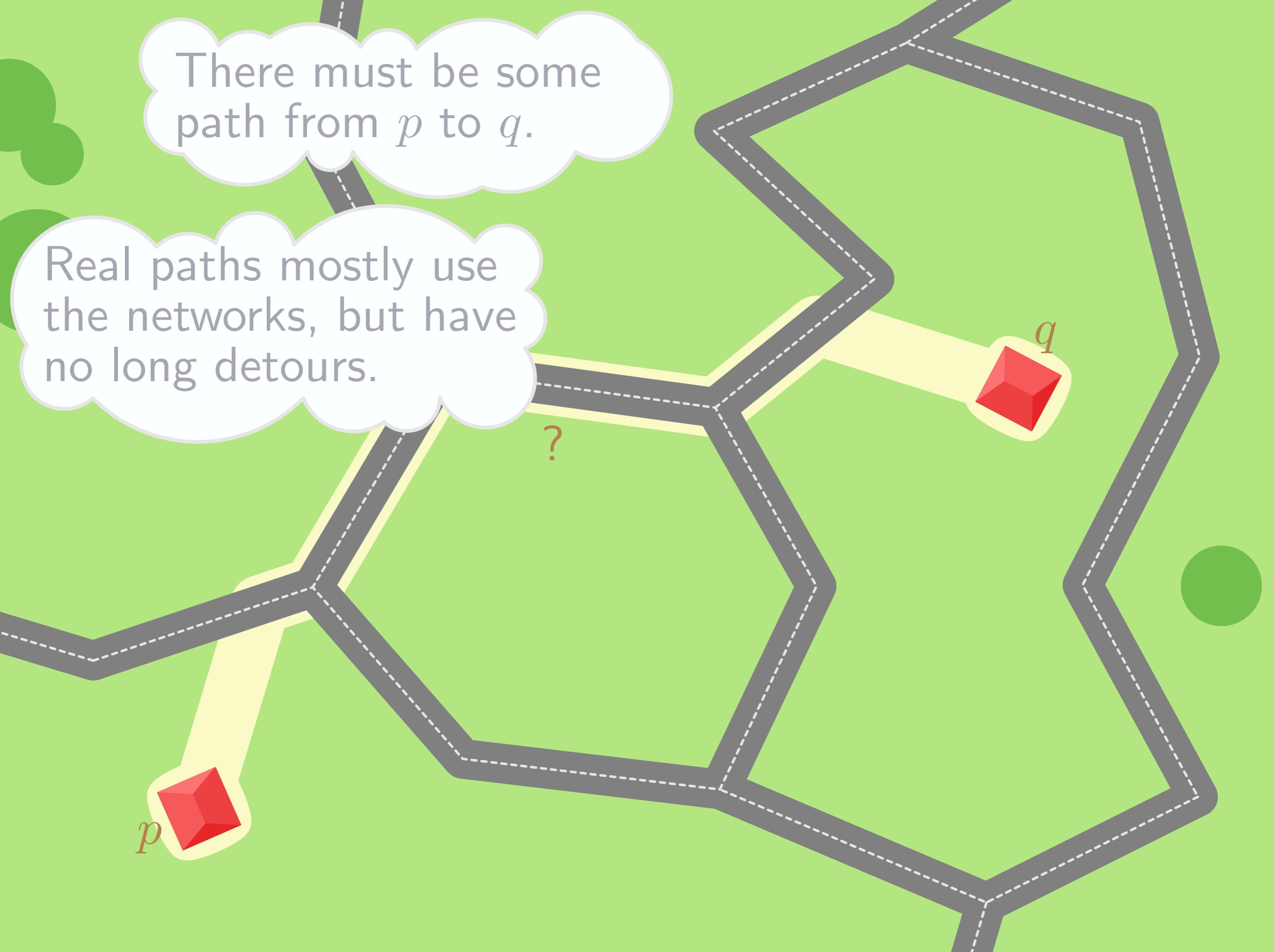


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Real paths mostly use the networks, but have no long detours.



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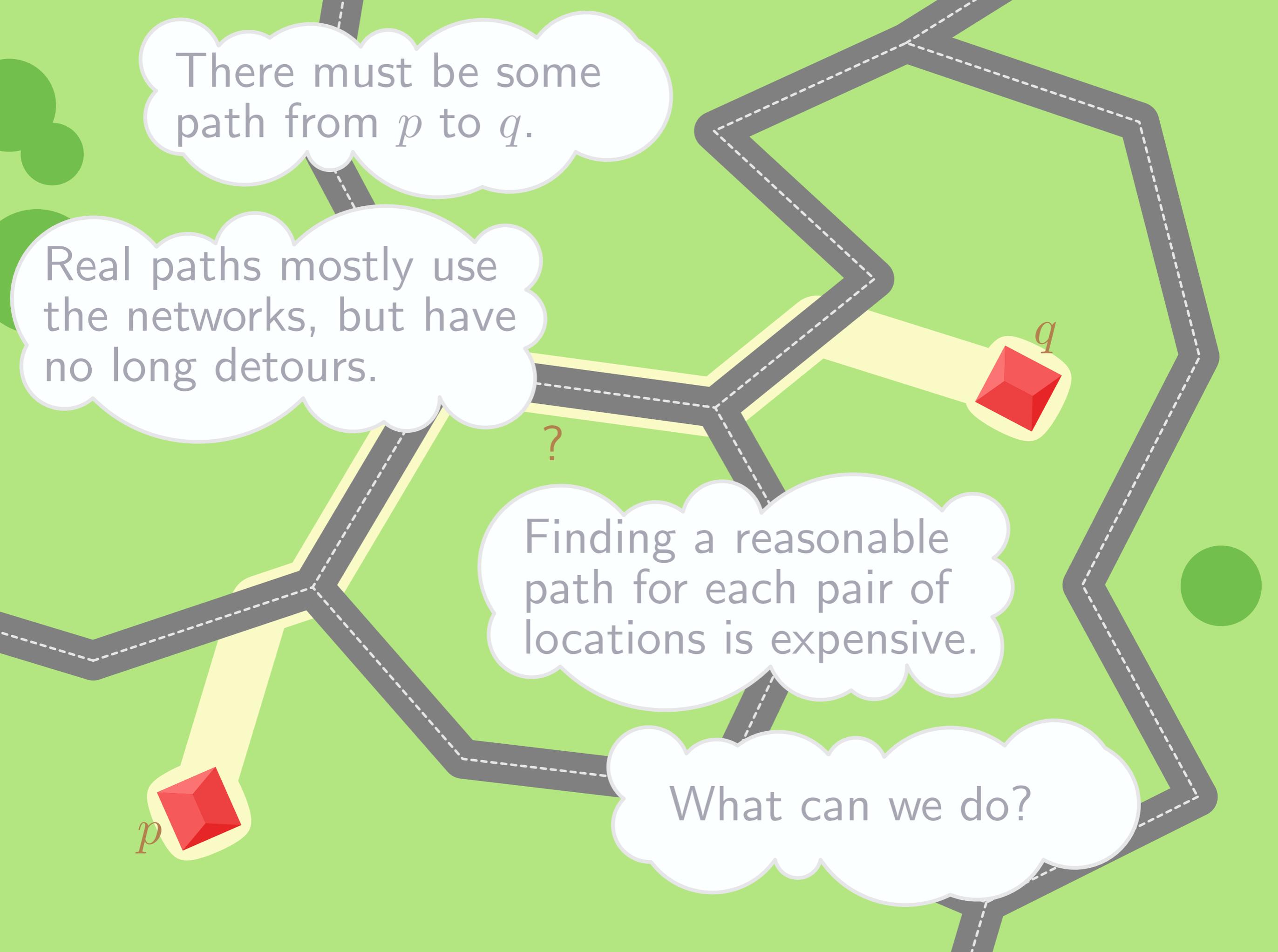
Real paths mostly use the networks, but have no long detours.

Finding a reasonable path for each pair of locations is expensive.

p

q

?

A network diagram on a green background. A dark grey path with a dashed white line inside starts at a red gem labeled 'p' at the bottom left and ends at a red gem labeled 'q' at the top right. A yellow highlight follows the path from 'p' through several nodes to 'q'. A red question mark is placed at a junction where the path branches. Three white thought bubbles with grey text are overlaid on the diagram.

There must be some path from p to q .

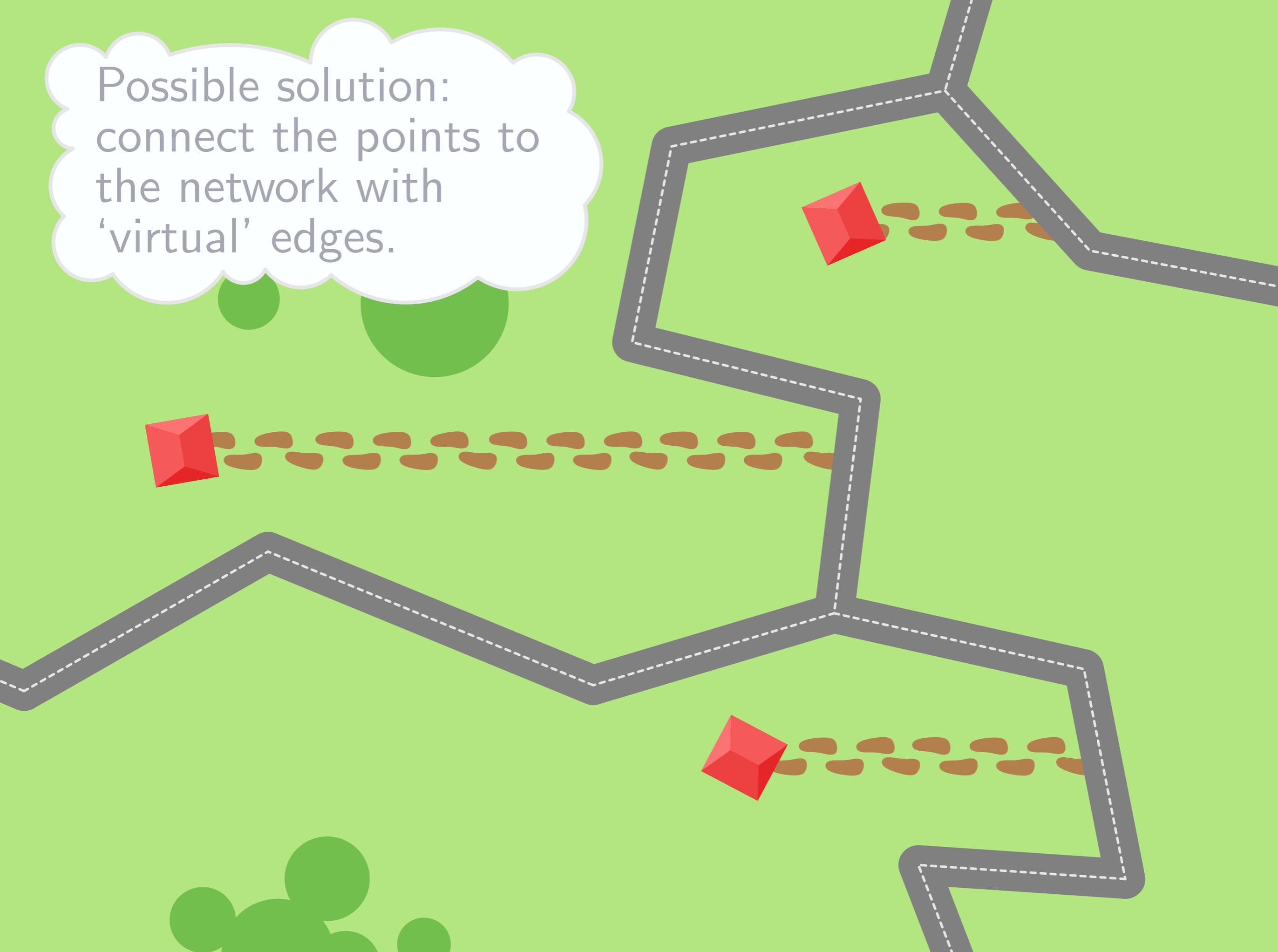
Real paths mostly use the networks, but have no long detours.

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What can we do?

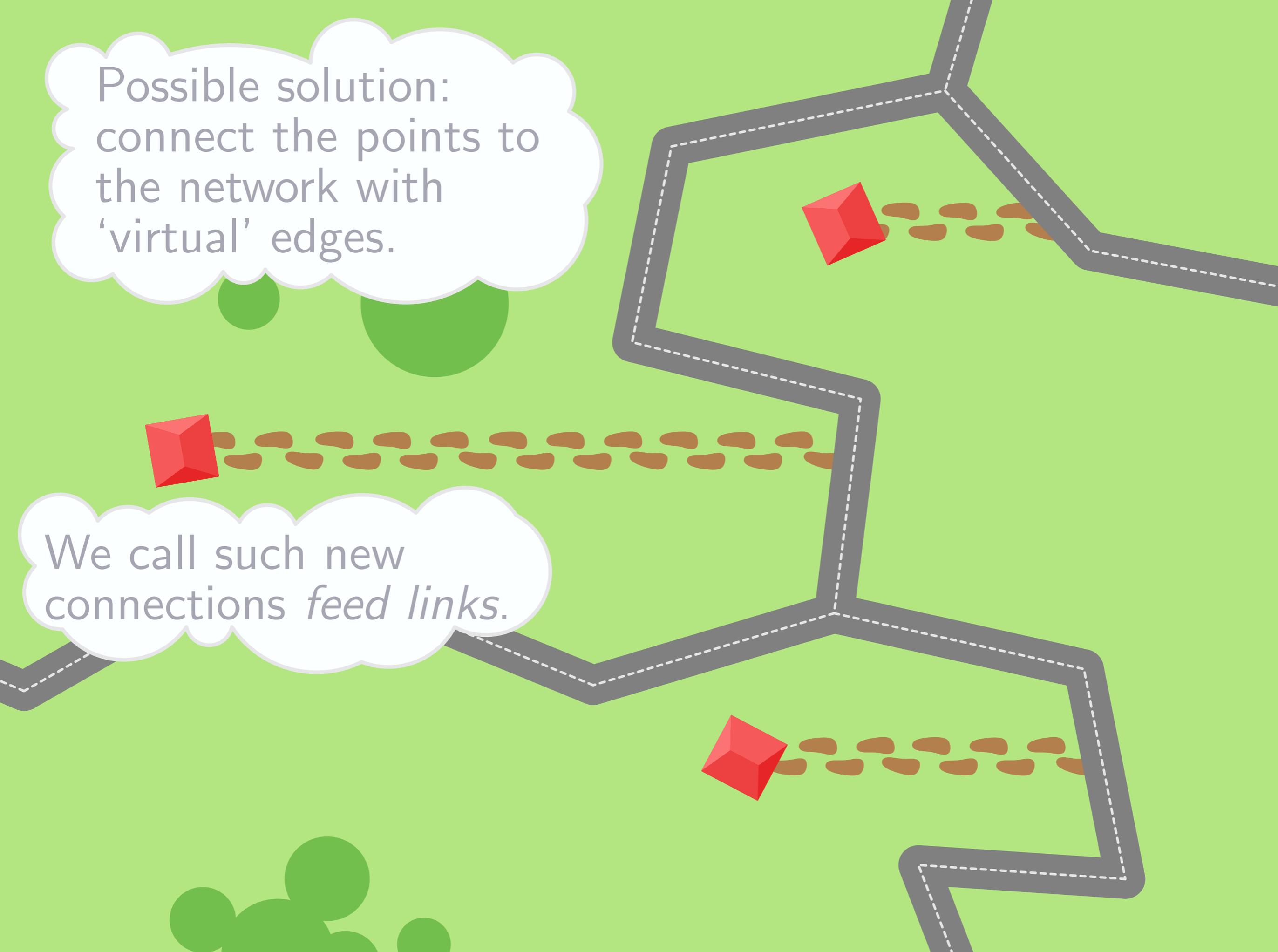


Possible solution:
connect the points to
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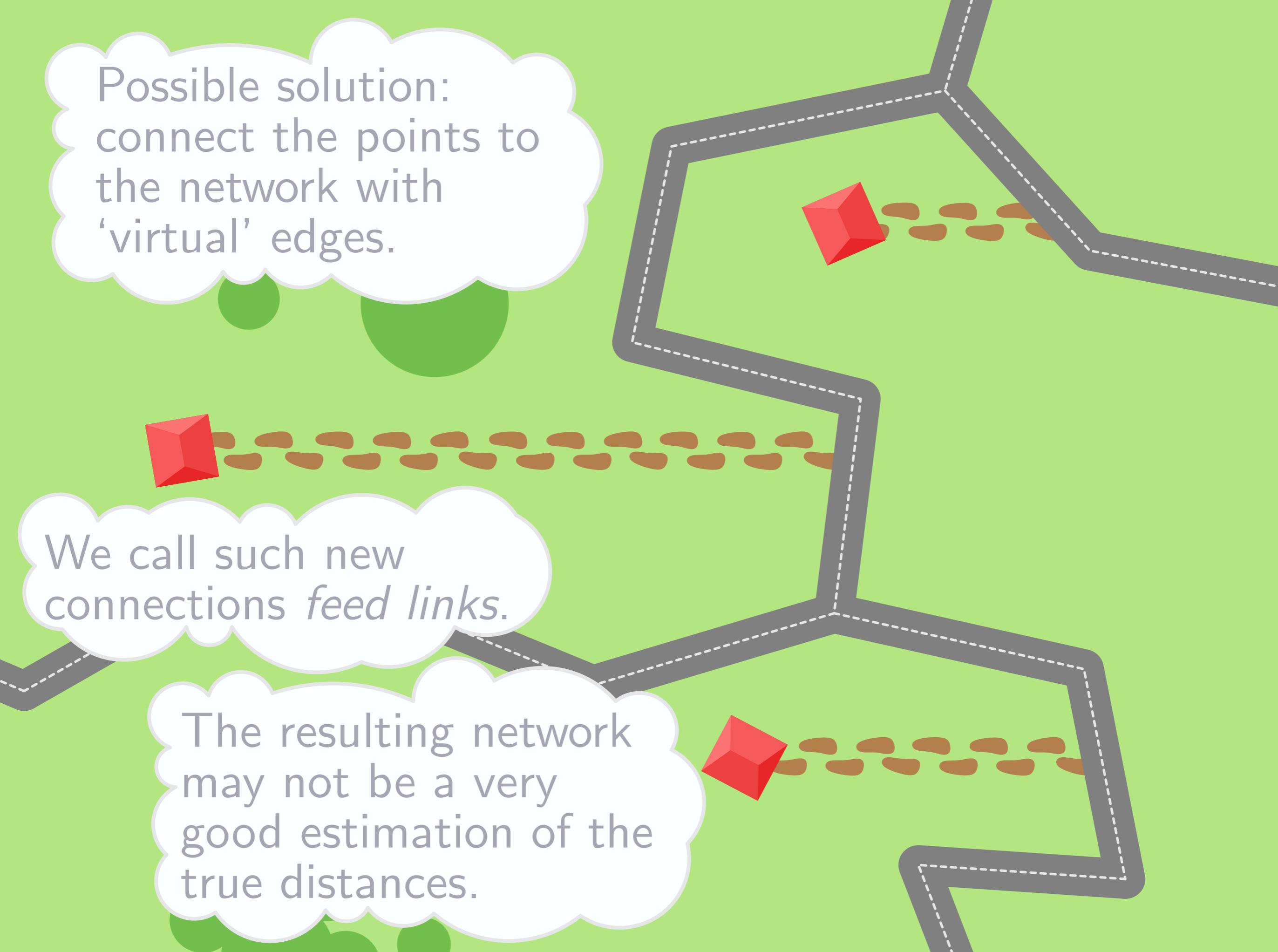
We call such new
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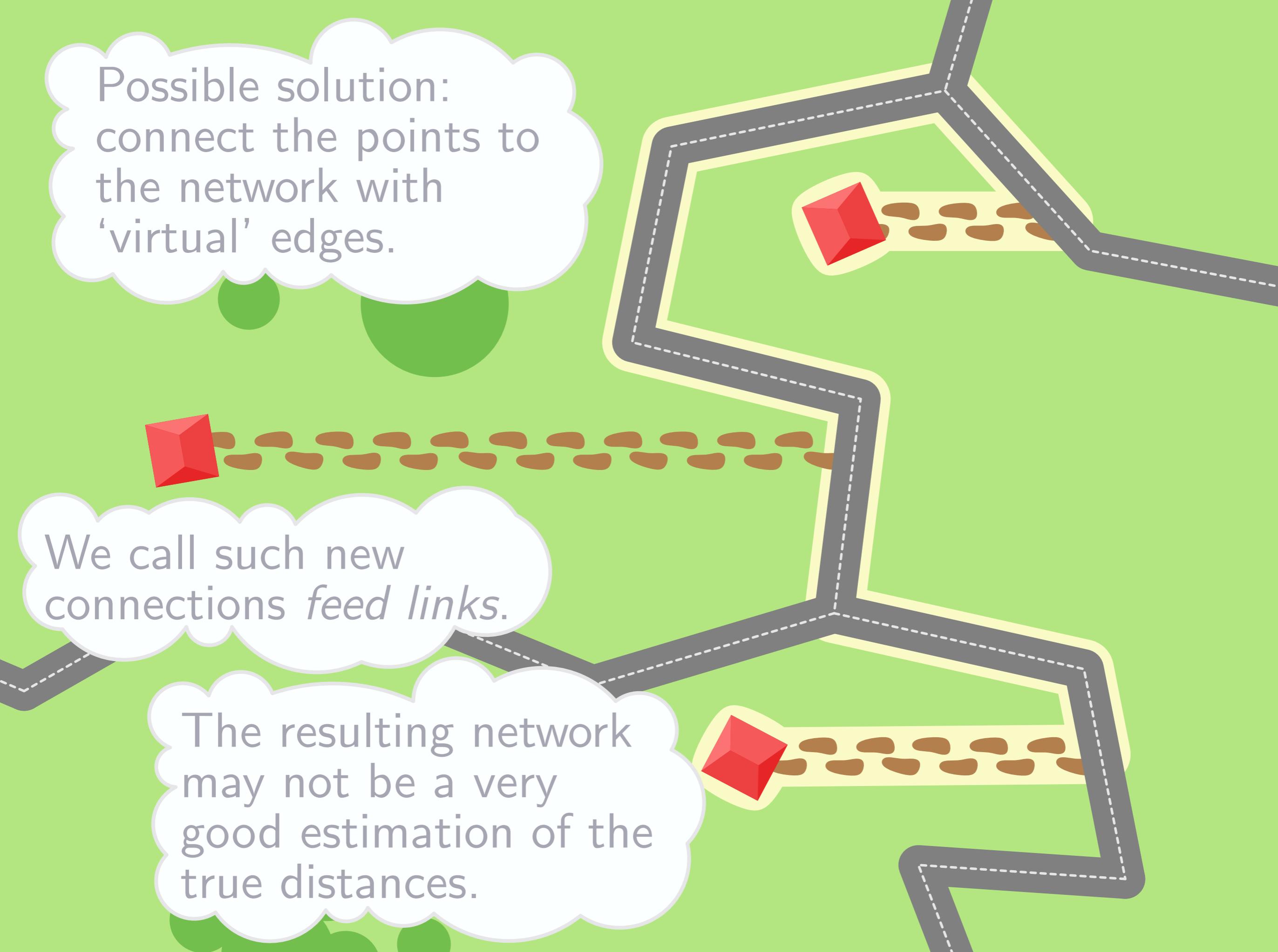
The resulting network
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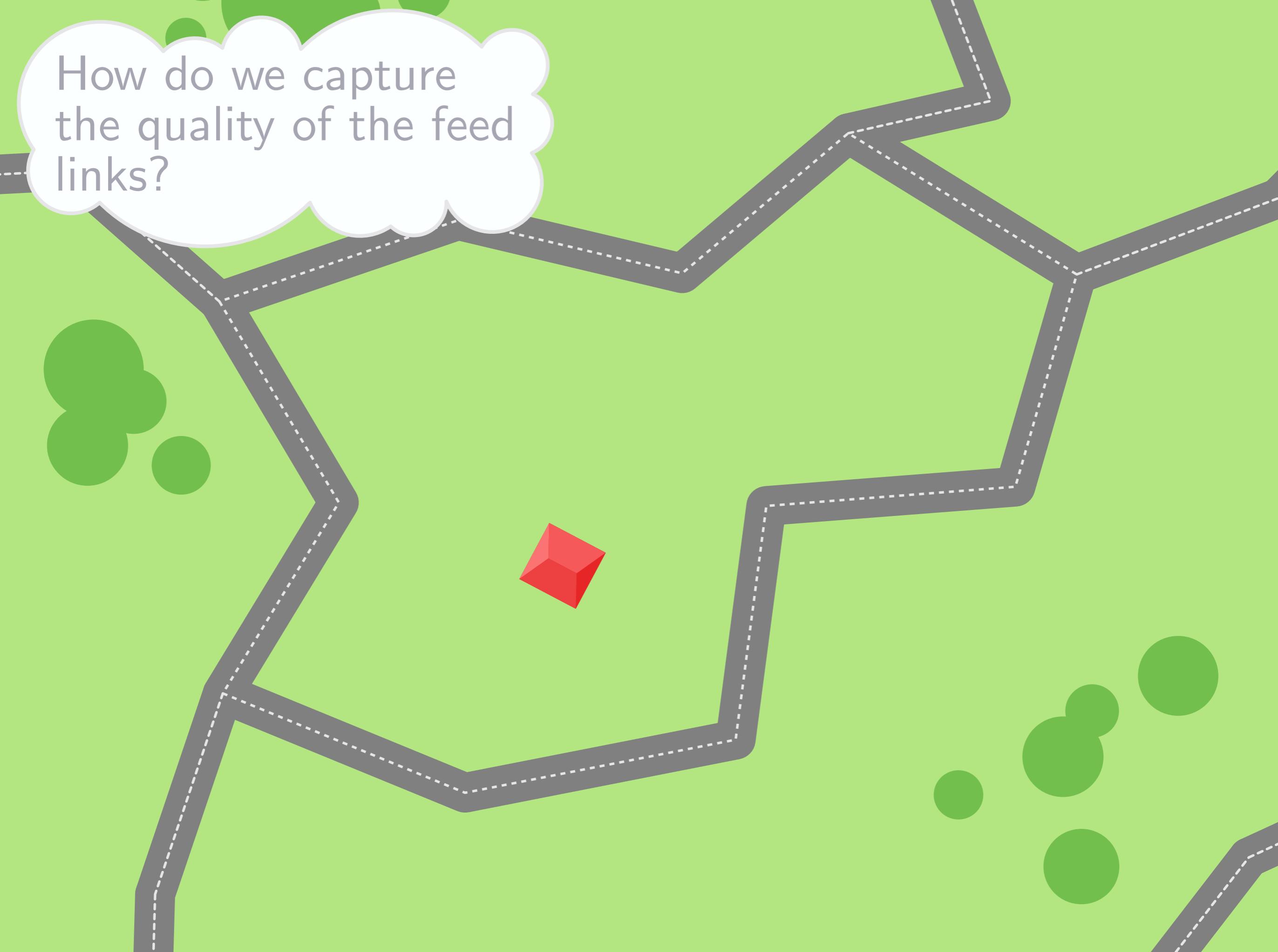
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How do we capture the quality of the feed links?



A stylized map on a light green background. A grey path with a dashed white line runs through the scene. A red 3D cube is positioned in the center. There are several green circles of various sizes scattered around. Two white thought bubbles with grey outlines contain text.

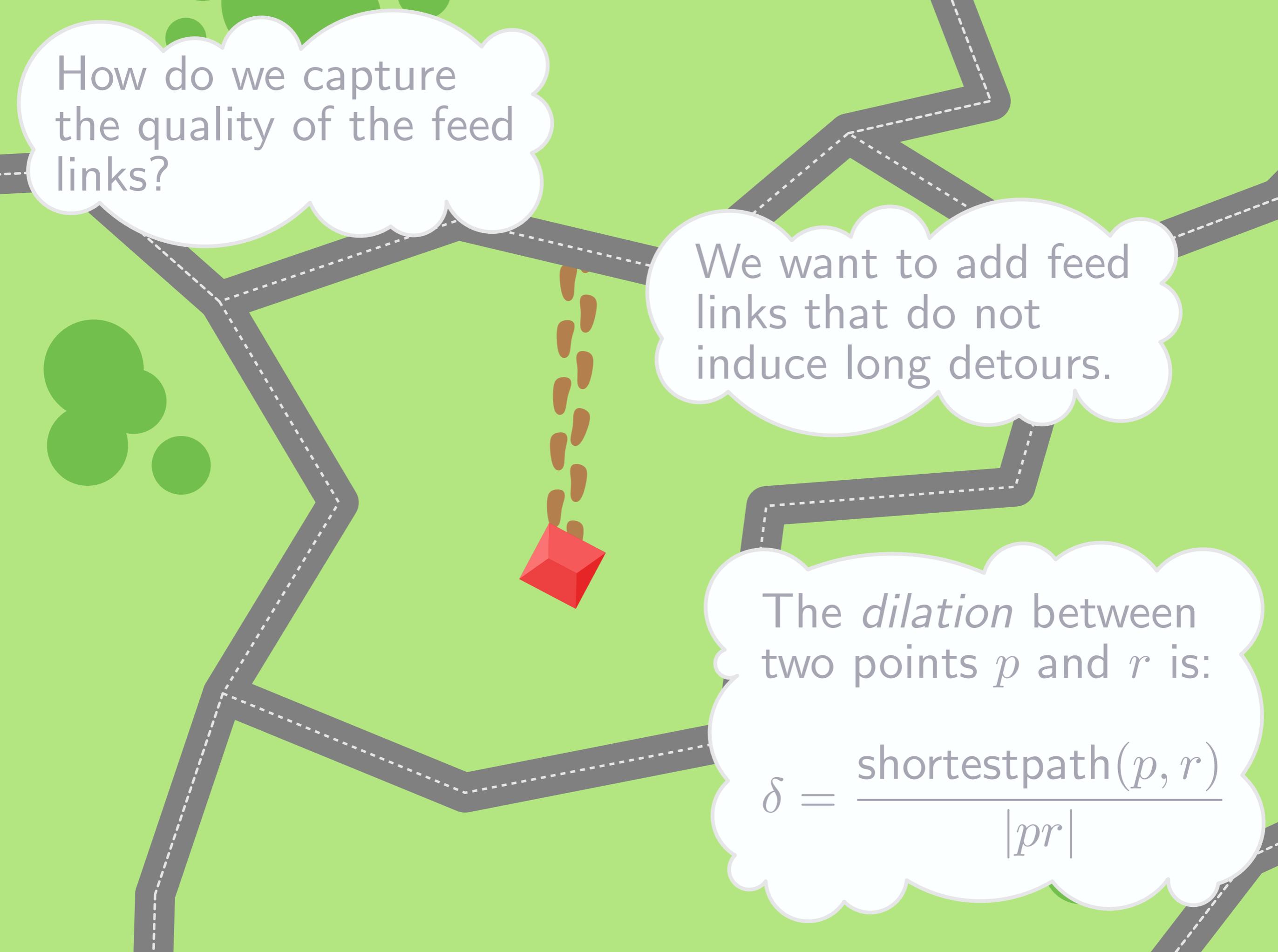
How do we capture the quality of the feed links?

We want to add feed links that do not induce long detours.

A stylized map with a light green background. A network of grey roads is shown, with a dashed white line indicating a path. A red cube is positioned in the center of the map, representing a destination. A vertical trail of brown footprints leads from the top of the map down to the red cube. Two white thought bubbles with grey outlines are present. The first bubble is in the top left, containing the text 'How do we capture the quality of the feed links?'. The second bubble is in the top right, containing the text 'We want to add feed links that do not induce long detours.' There are several green circles of various sizes scattered across the map, representing trees or bushes.

How do we capture the quality of the feed links?

We want to add feed links that do not induce long detours.

A network diagram on a green background. It features a main network of dark grey lines with dashed white outlines, branching out from the left. A red 3D cube is positioned in the lower-middle area. A brown, dashed path leads from the cube upwards to a junction in the network. Three white thought bubbles with grey outlines contain text. The background also includes some green circular shapes representing trees or bushes.

How do we capture the quality of the feed links?

We want to add feed links that do not induce long detours.

The *dilation* between two points p and r is:

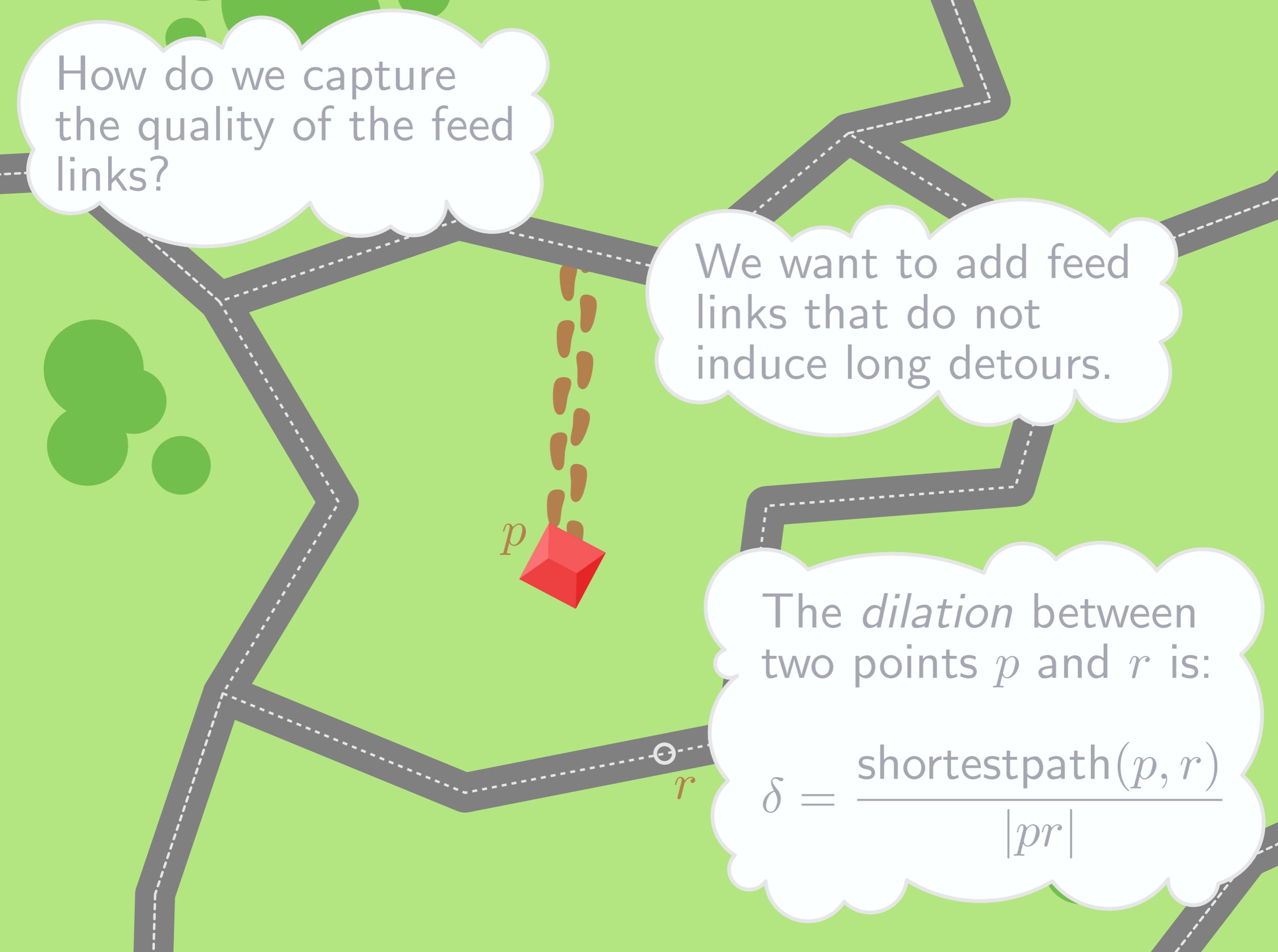
$$\delta = \frac{\text{shortestpath}(p, r)}{|pr|}$$

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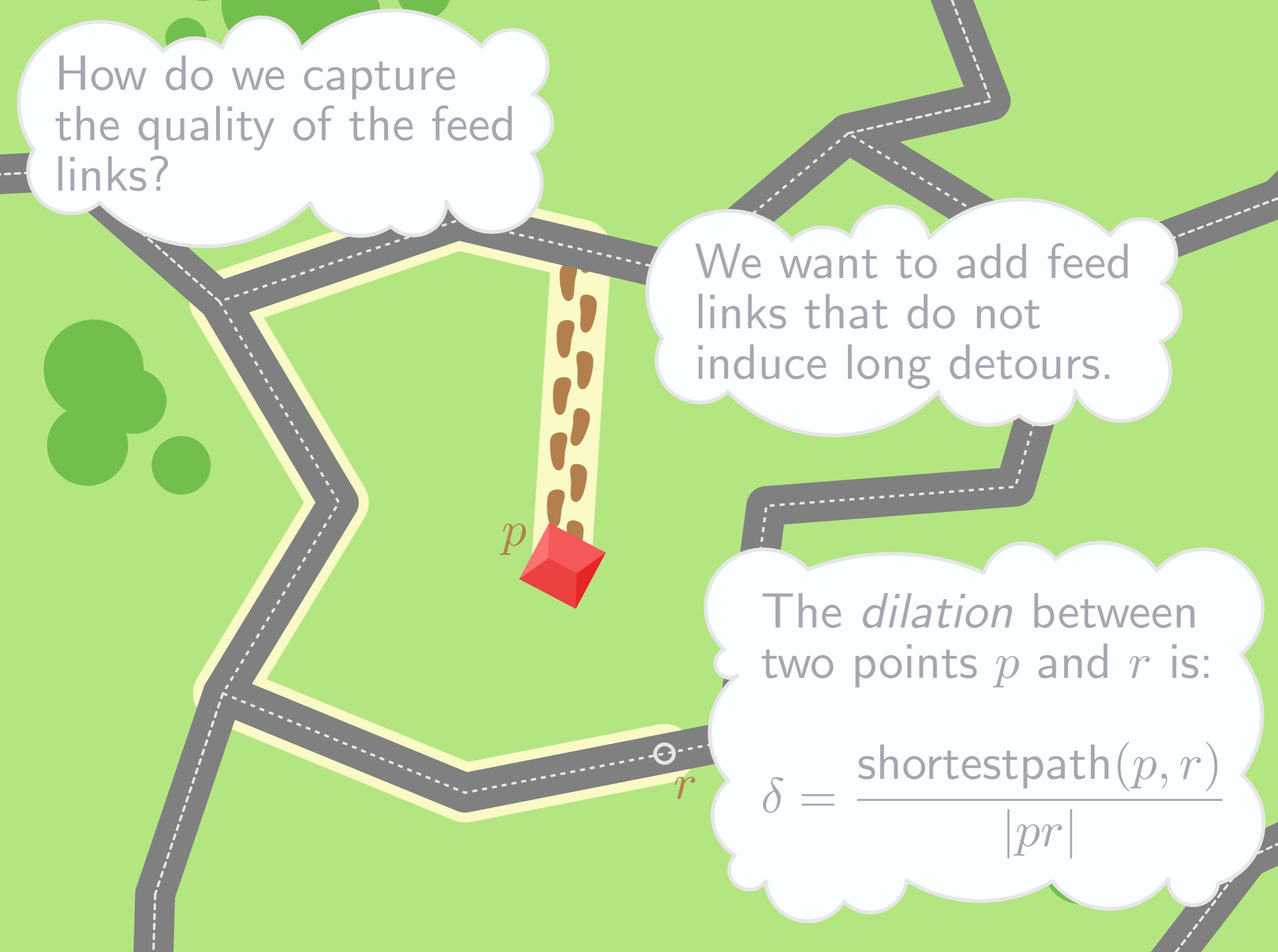


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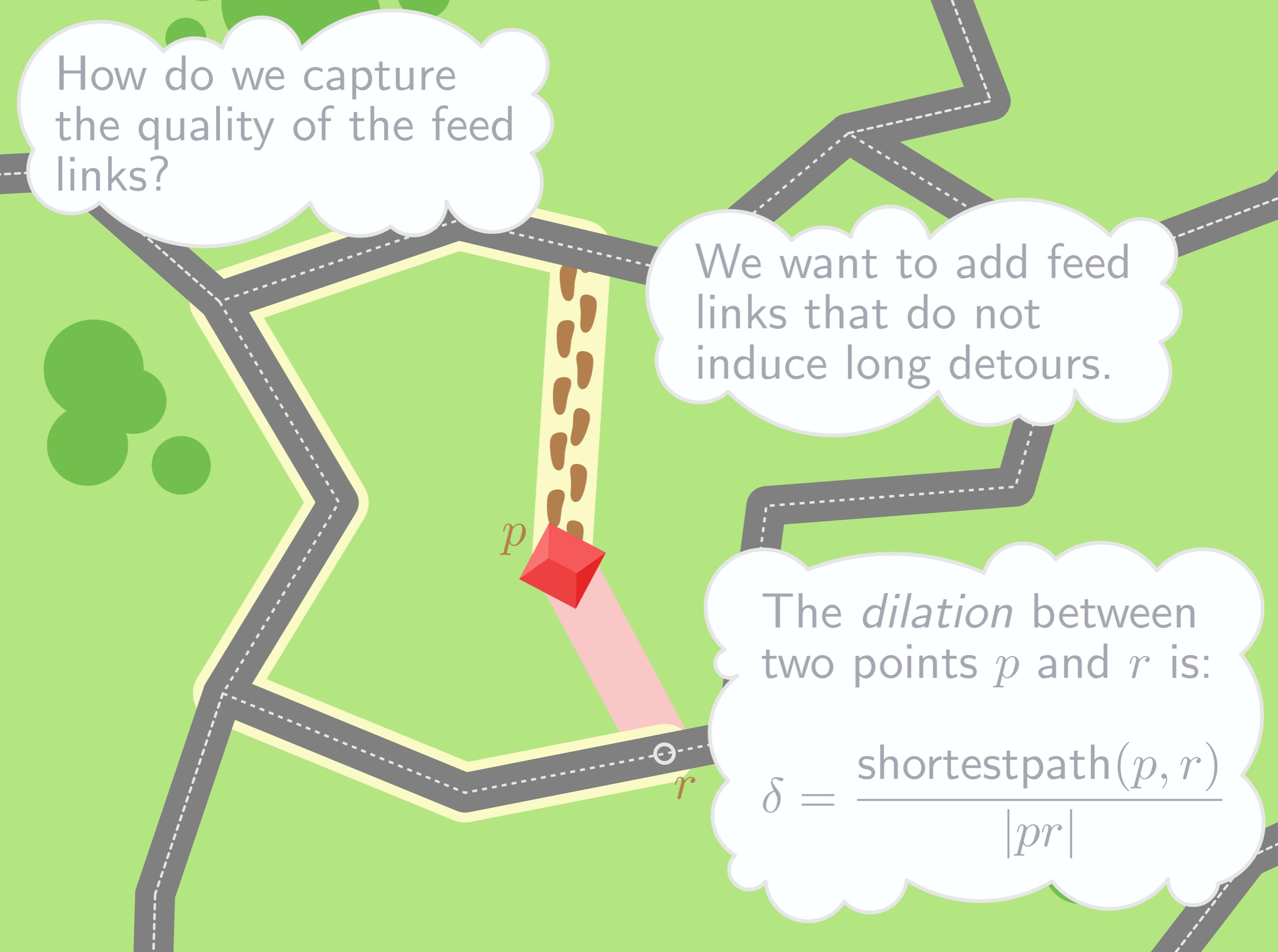


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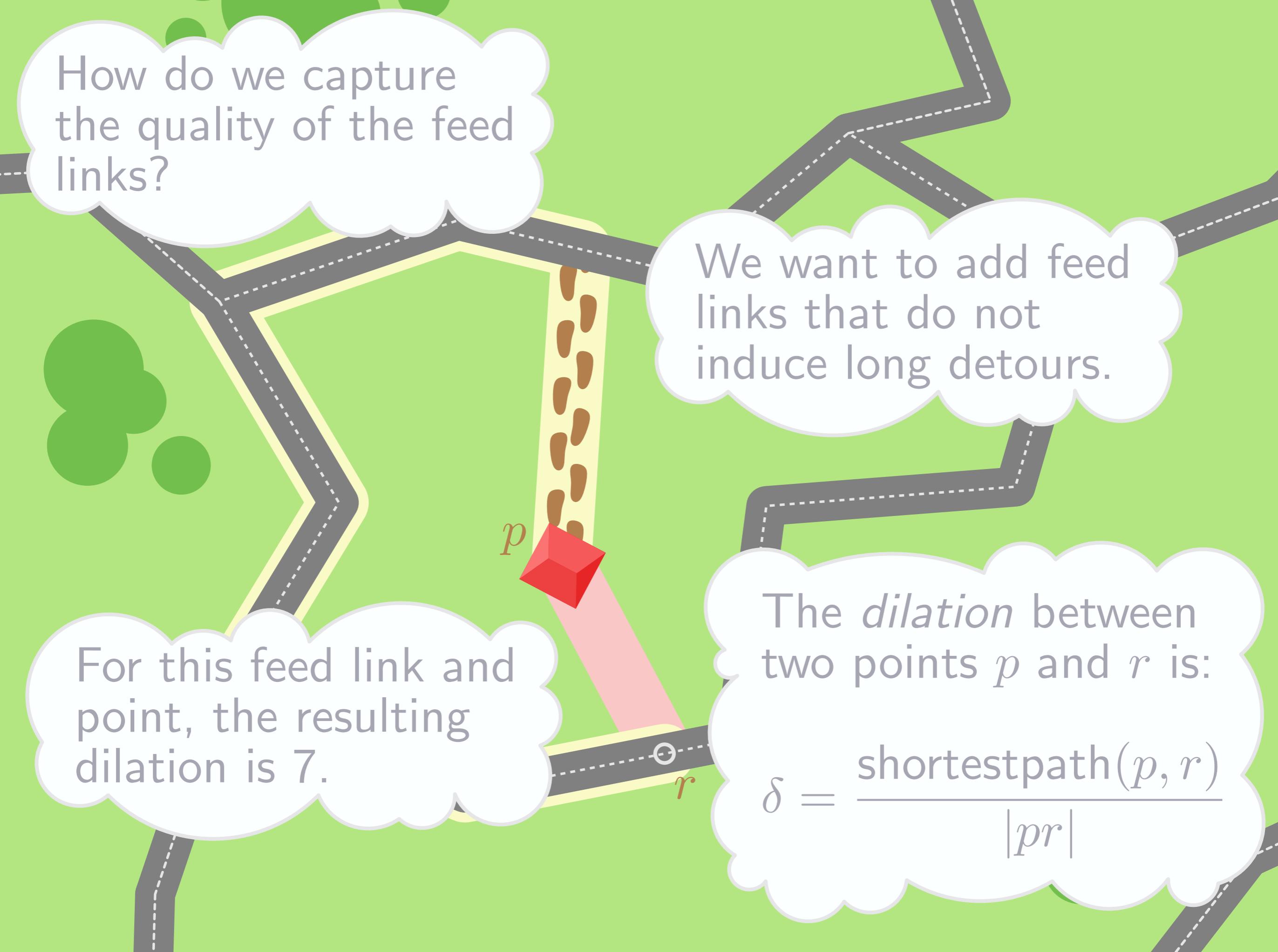
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For this feed link and point, the resulting dilation is 7.

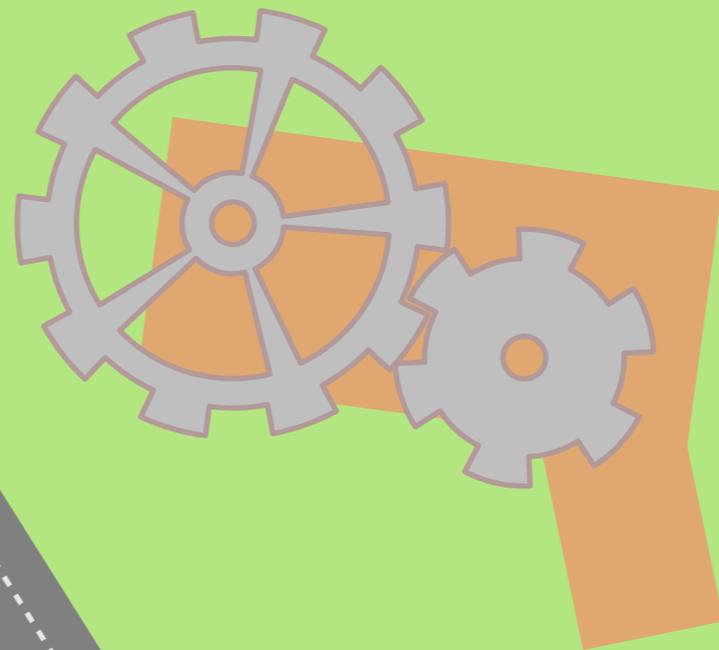
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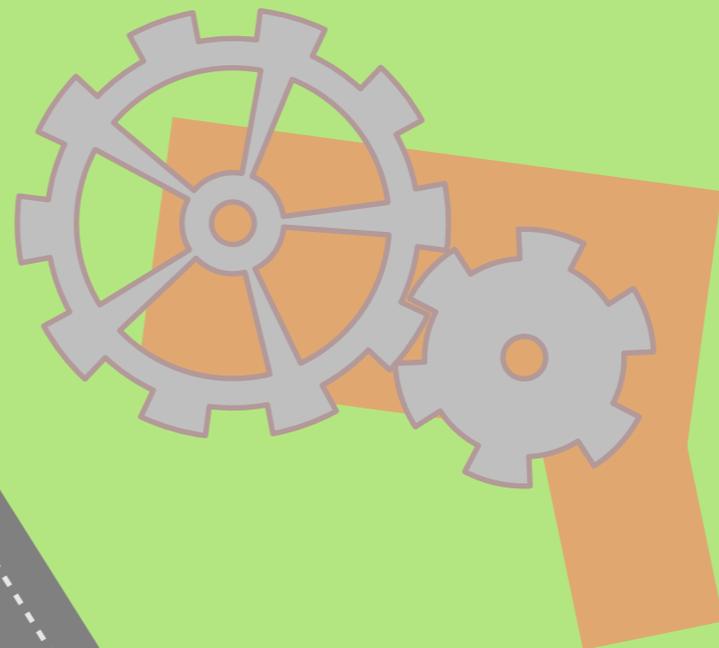
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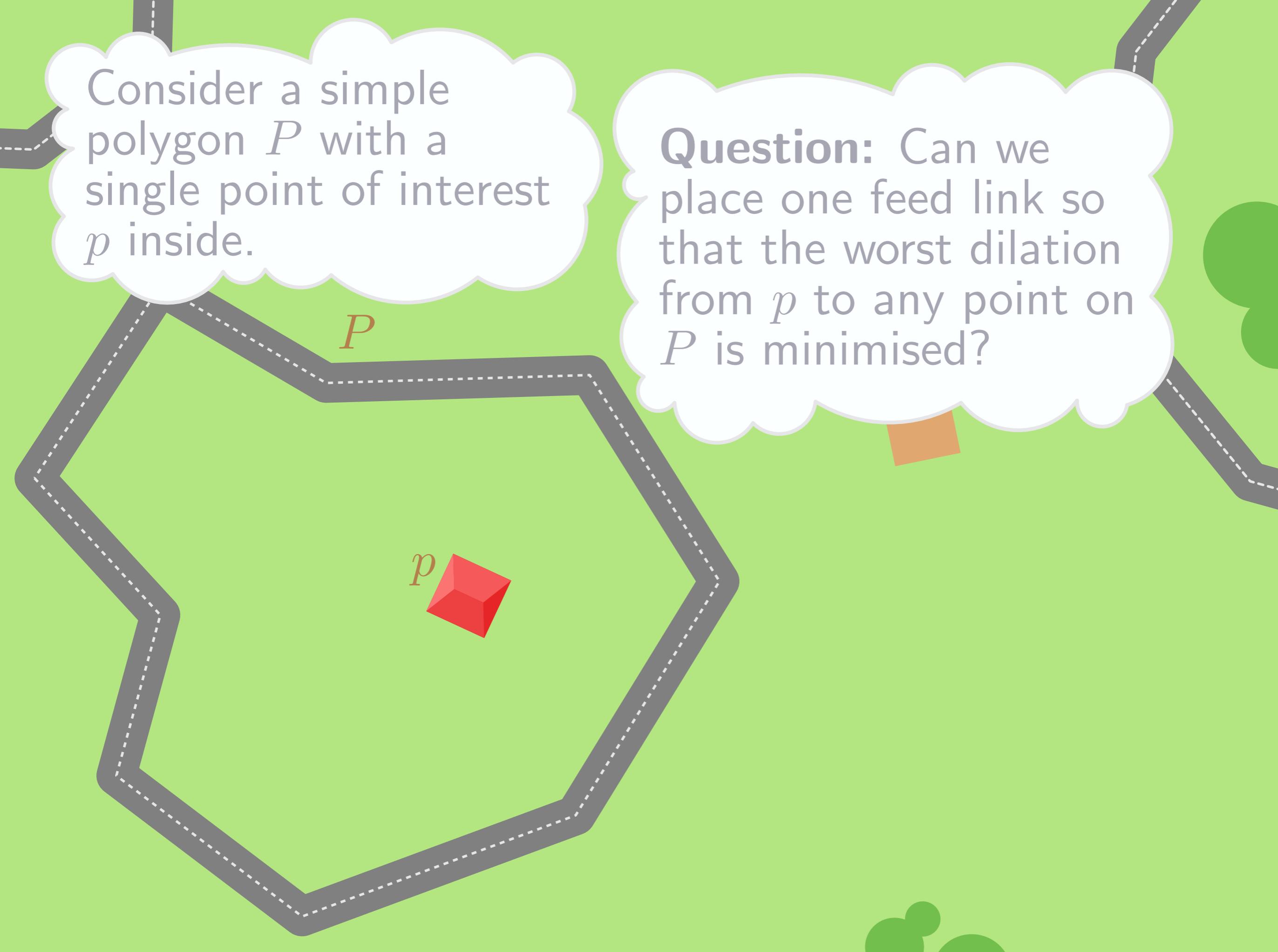
P

p



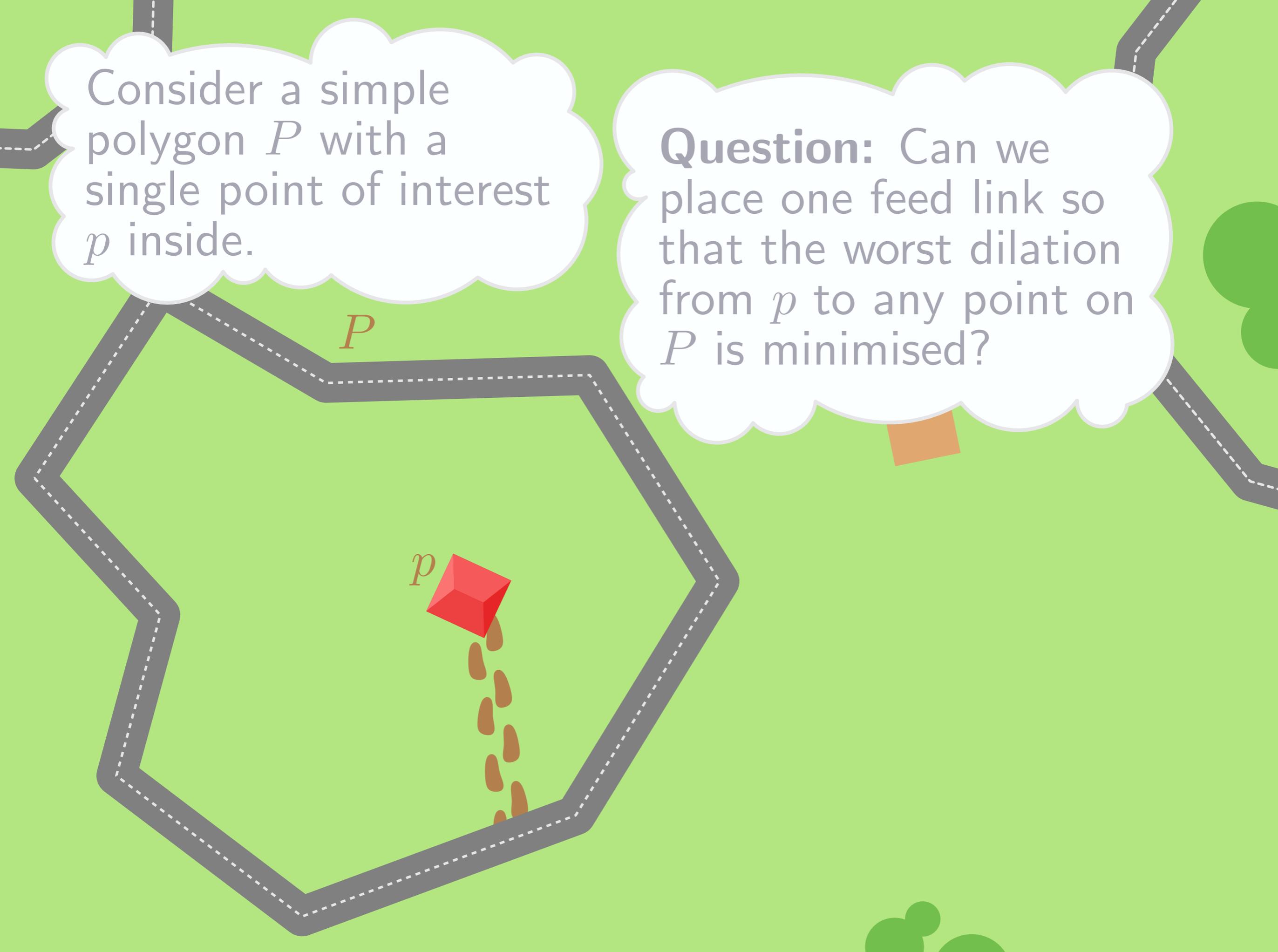
Consider a simple polygon P with a single point of interest p inside.

Question: Can we place one feed link so that the worst dilation from p to any point on P is minimised?



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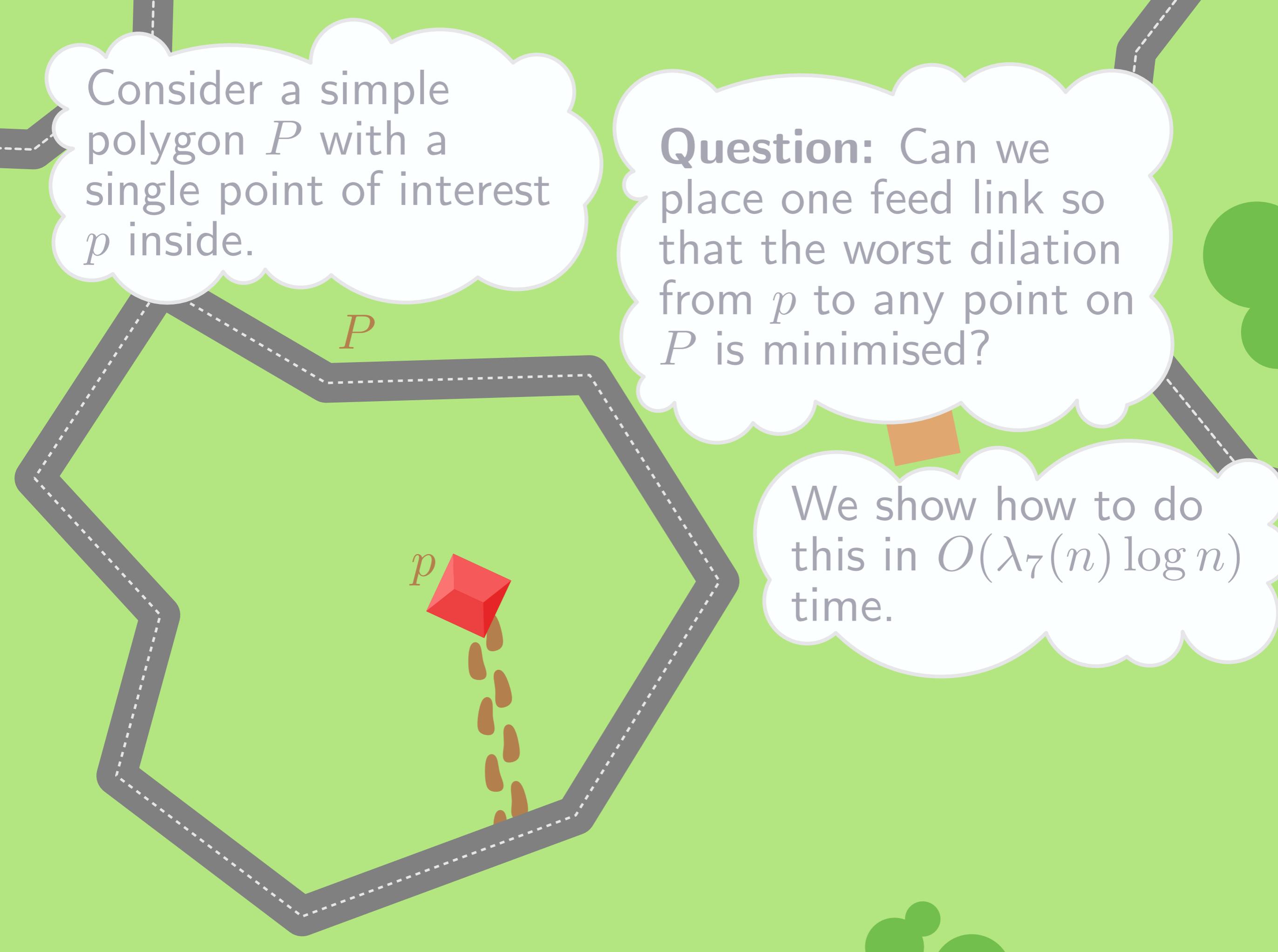
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We show how to do this in $O(\lambda_7(n) \log n)$ time.

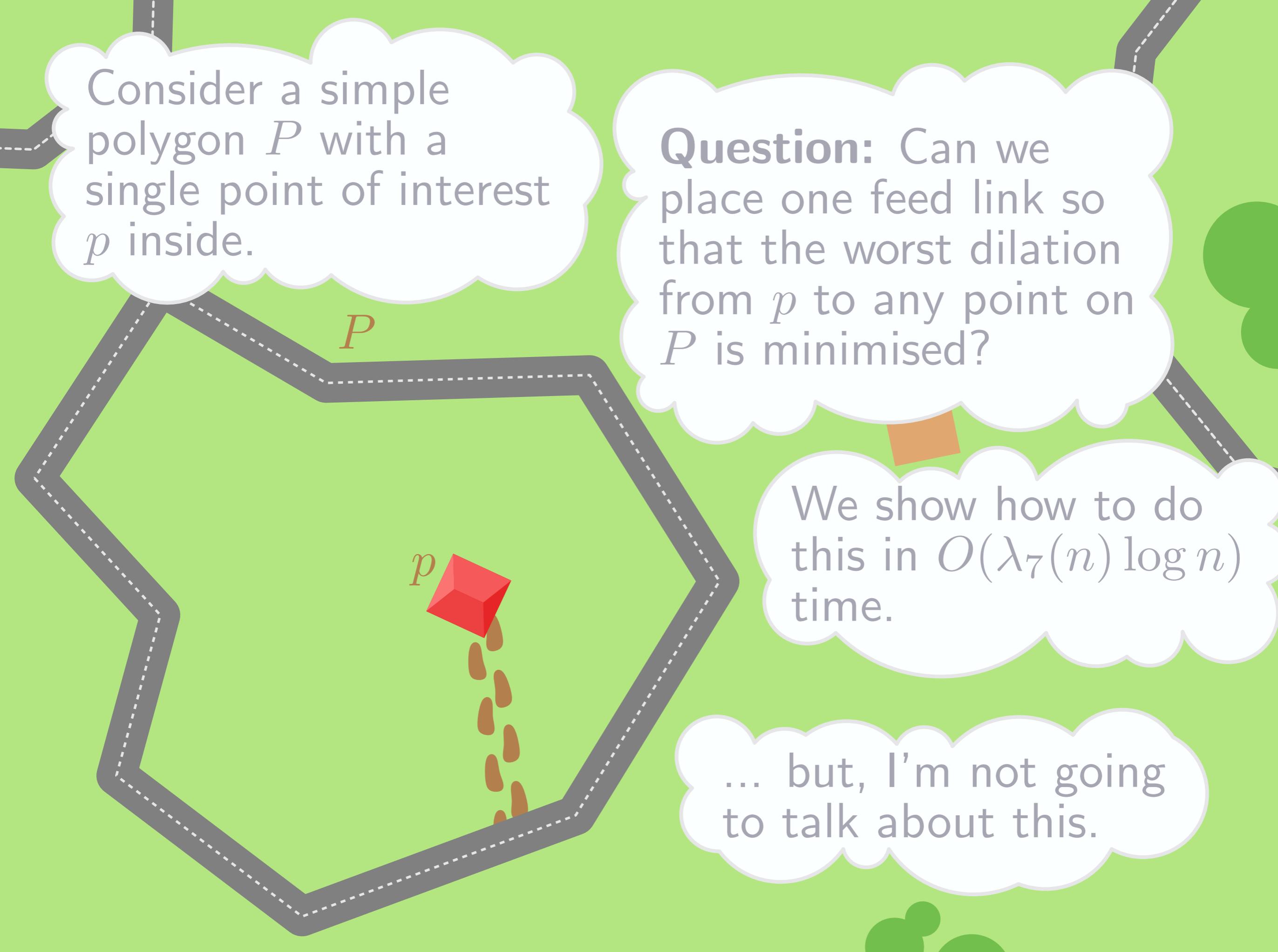


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... but, I'm not going to talk about this.

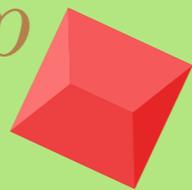
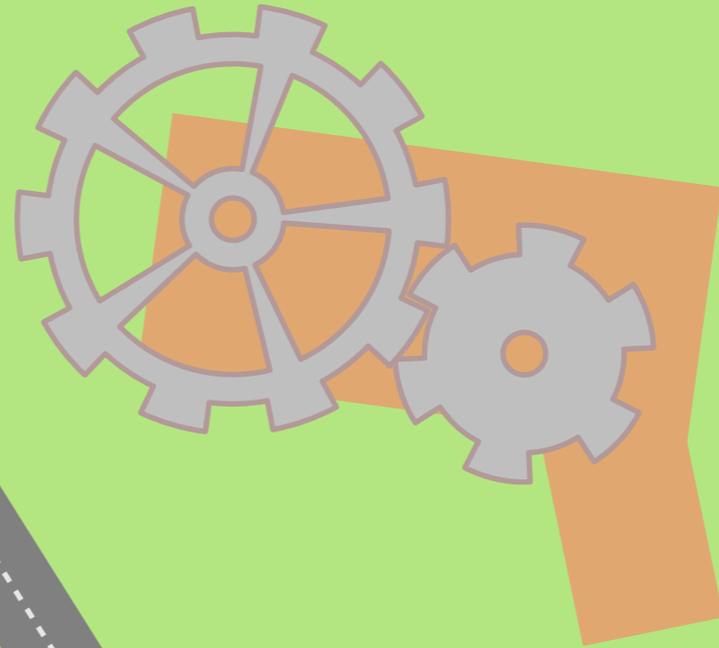




Instead, let's consider a different problem.

P

p



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We then show how to extend the solution to the original scenario.



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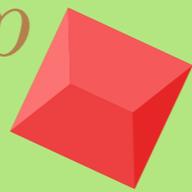
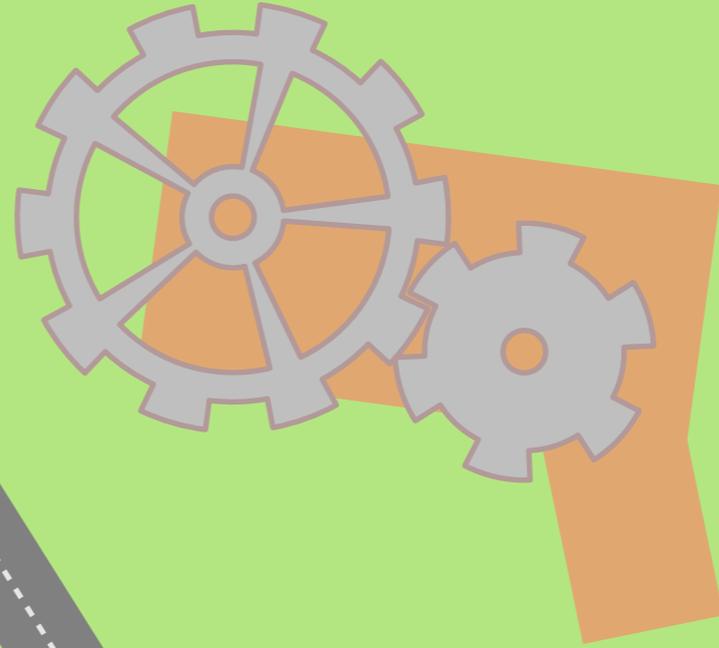
However, this problem is also interesting in its own right.



Idea: Consider a fixed point r_i on P .

P

p

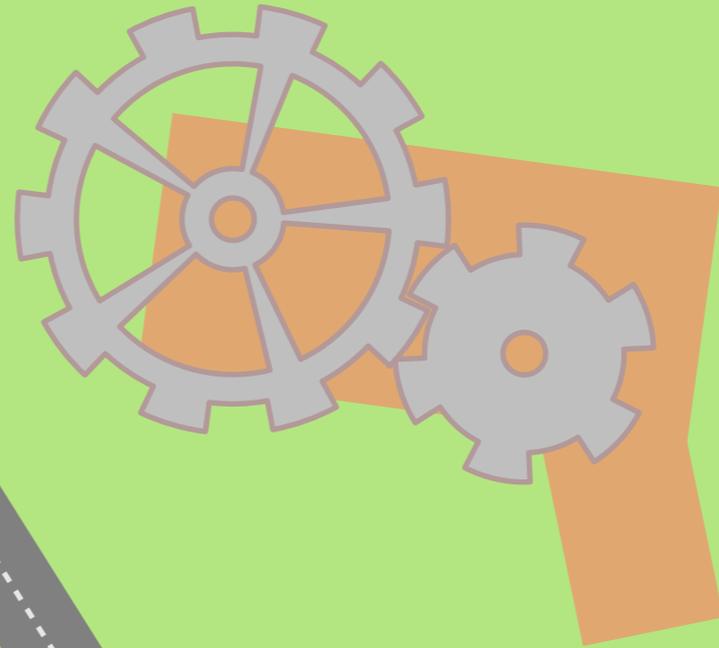


Idea: Consider a fixed point r_i on P .

P

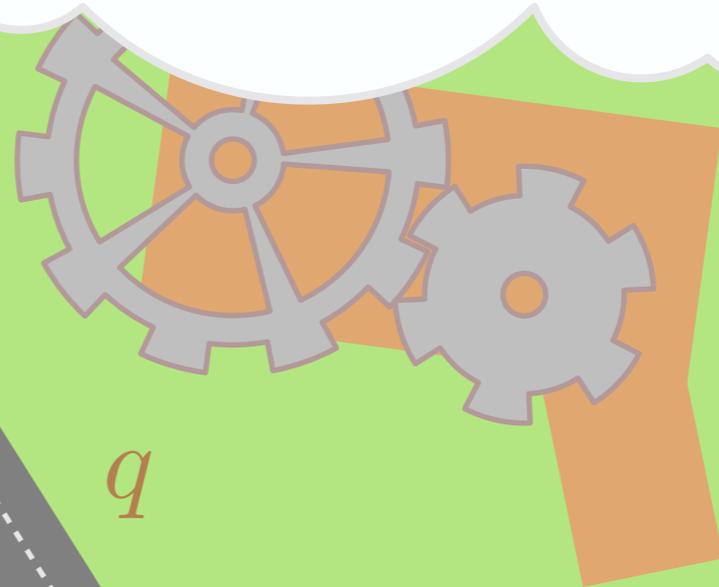
p

r_i



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Now, let q be the point the feed link attaches to.



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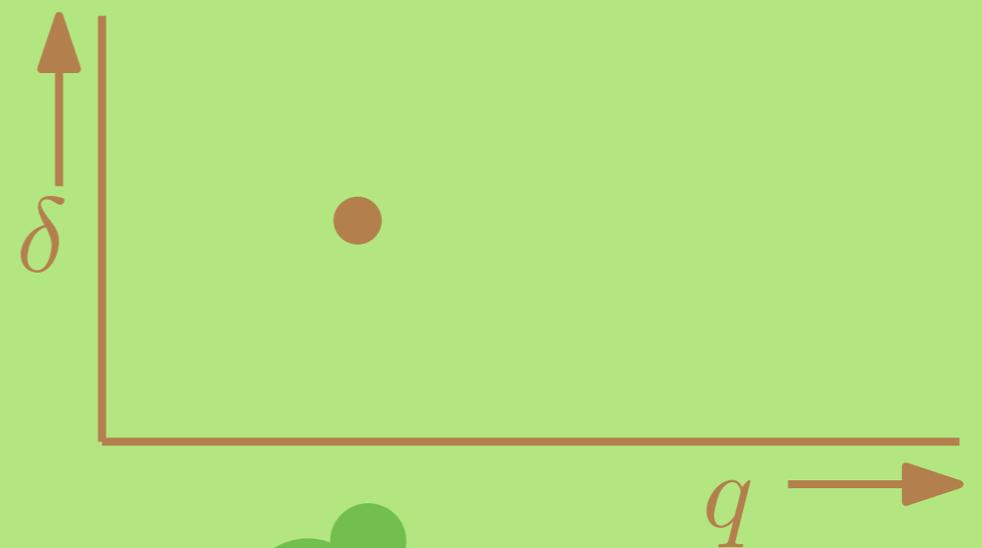
As q moves around, the dilation δ_i from p to r_i varies.



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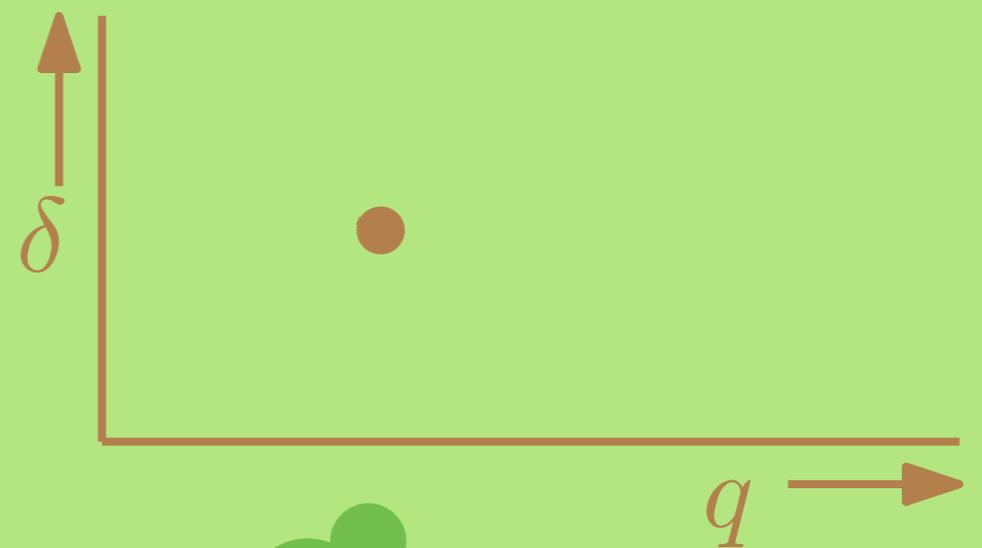
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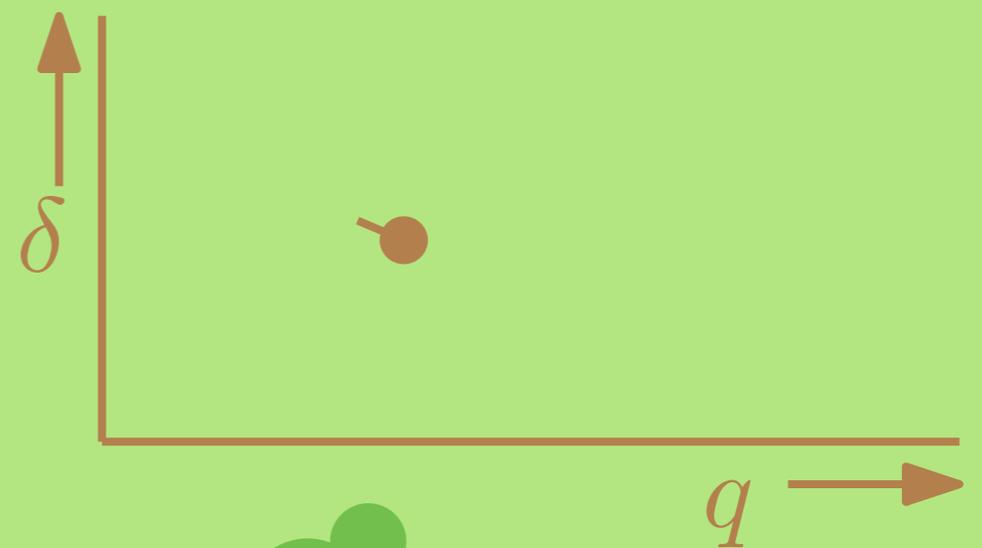
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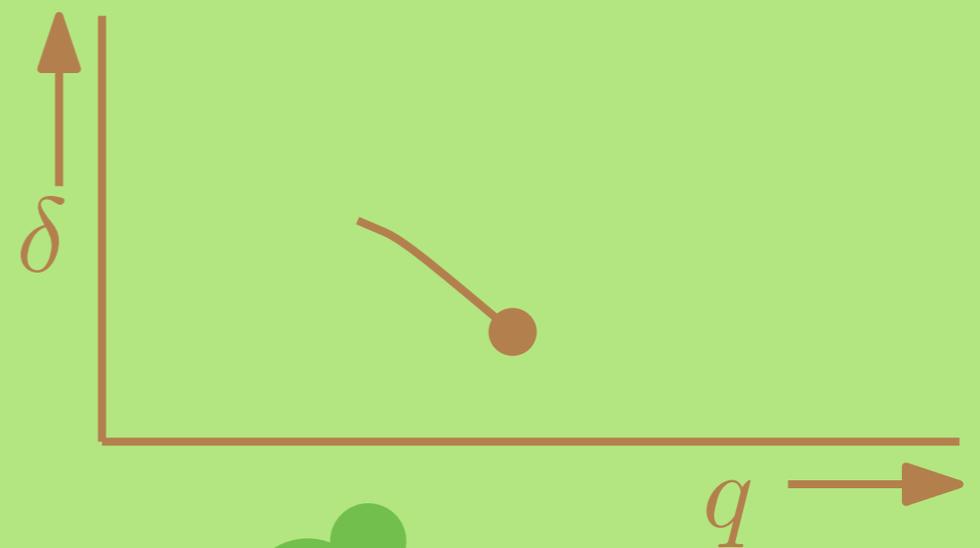
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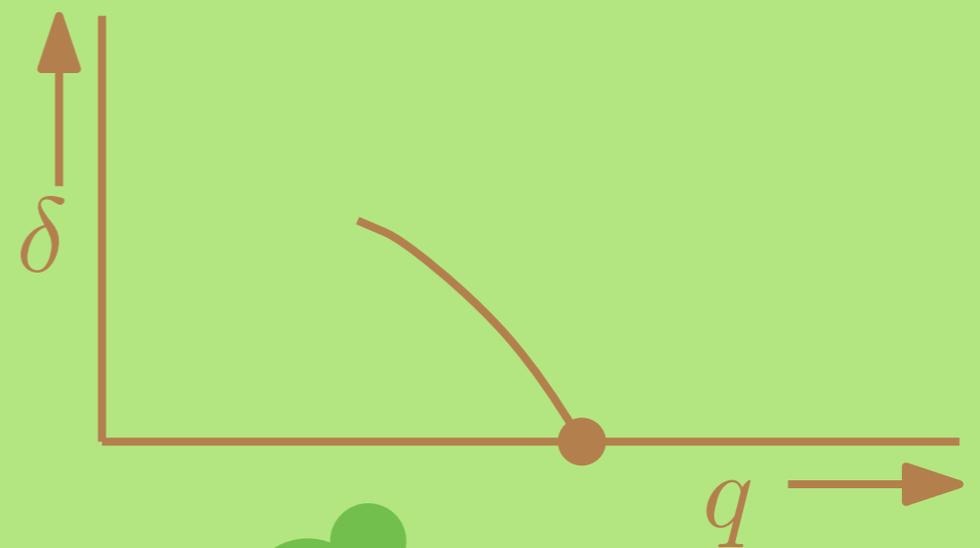
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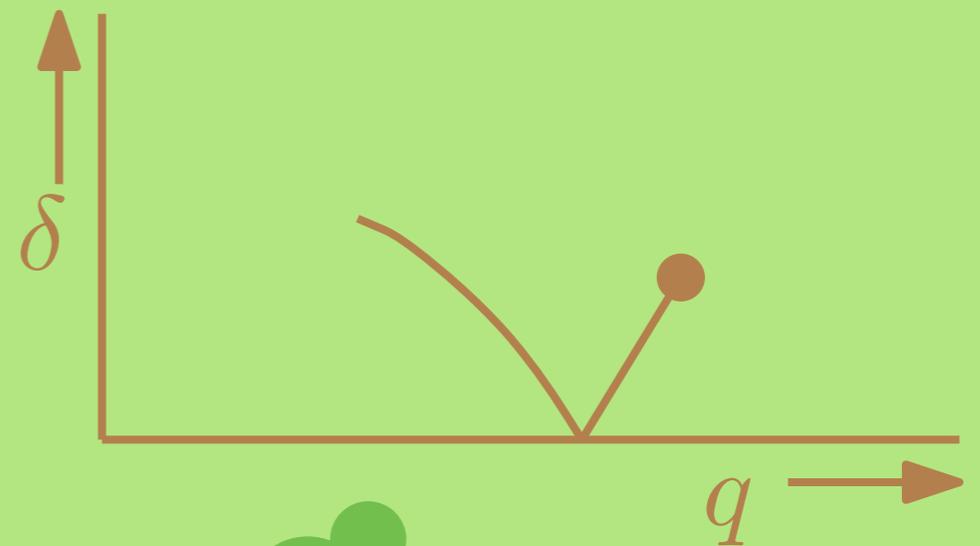
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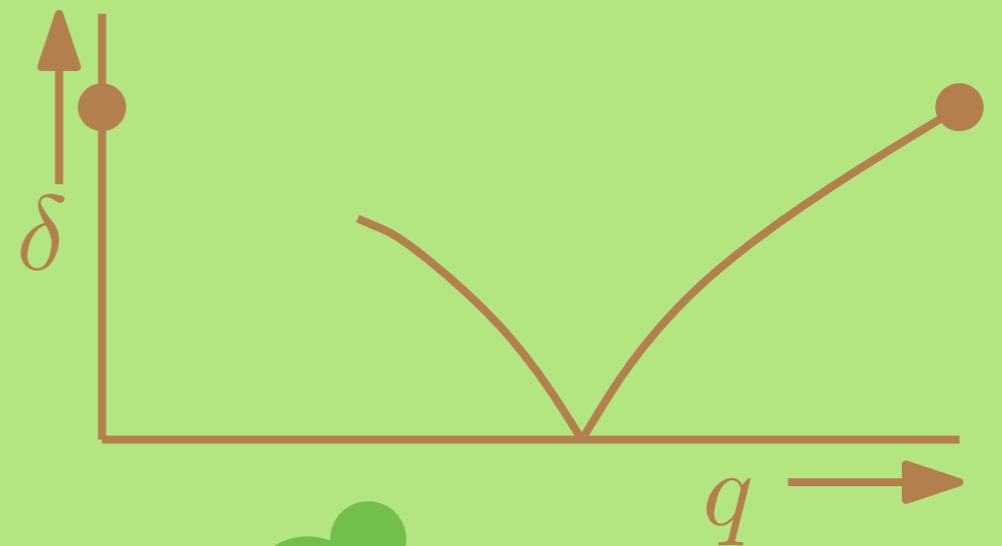
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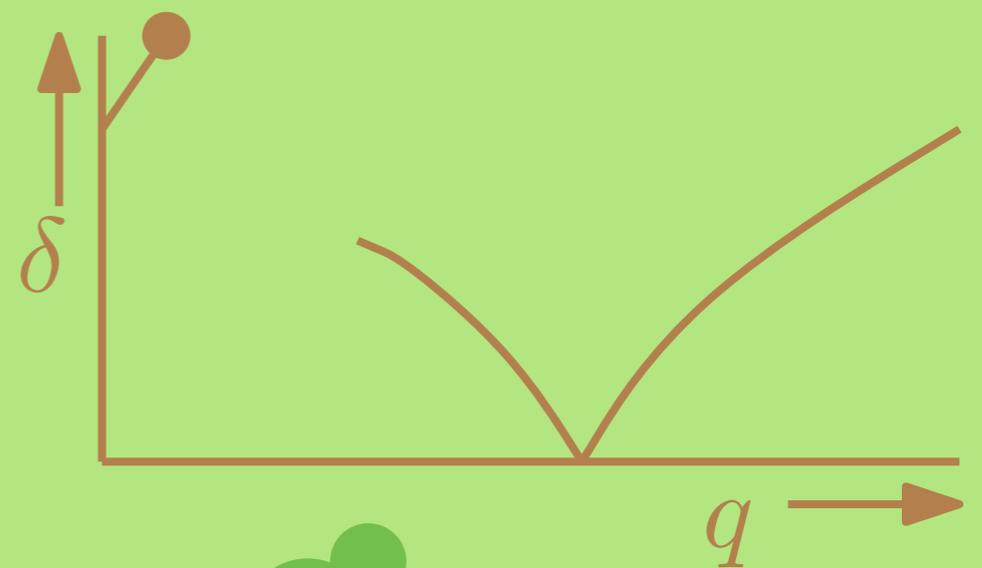
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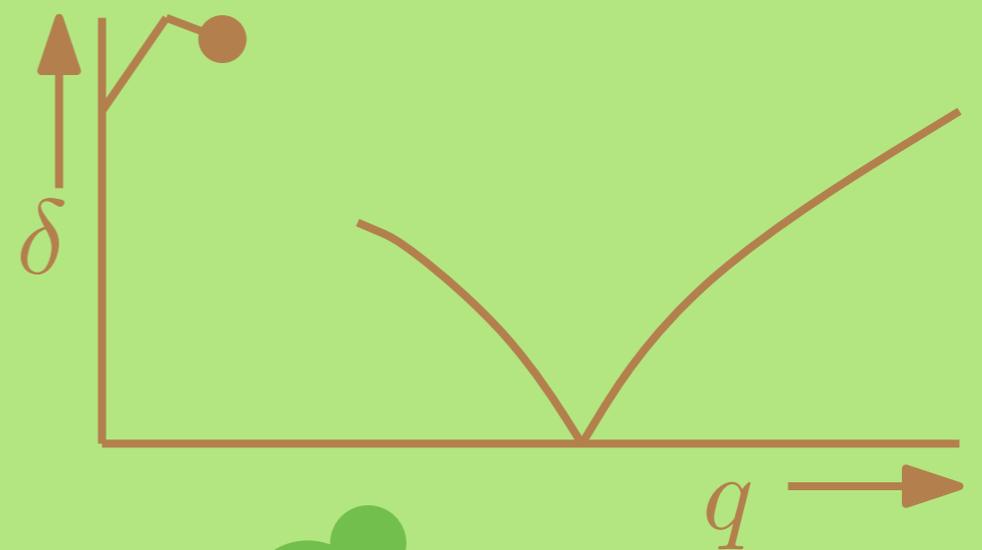
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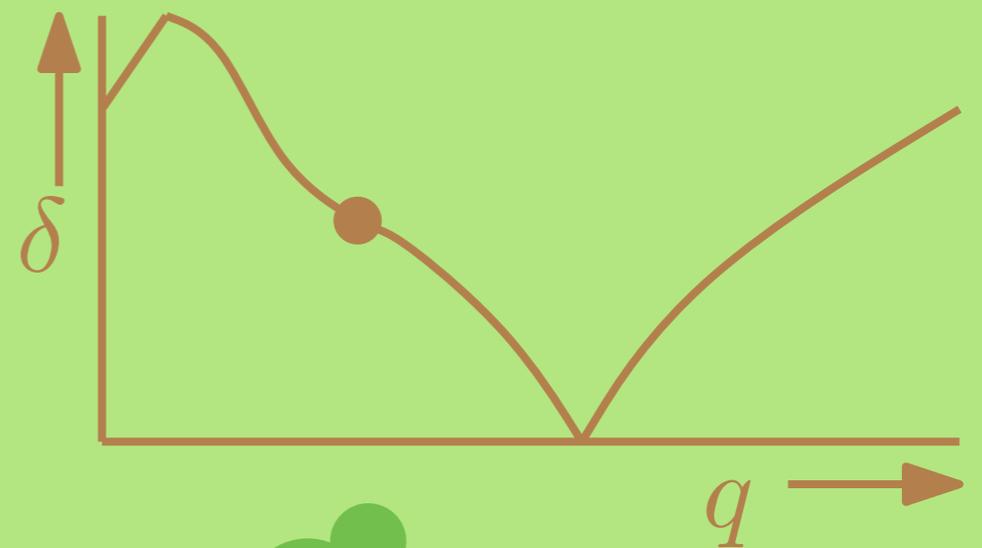
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Idea: Consider a fixed point r_i on P .

We can draw similar curves for the other points.

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r_i



Idea: Consider a fixed point r_i on P .

We can draw similar curves for the other points.

The lowest point on the upper envelope of these curves gives the solution.

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r_i

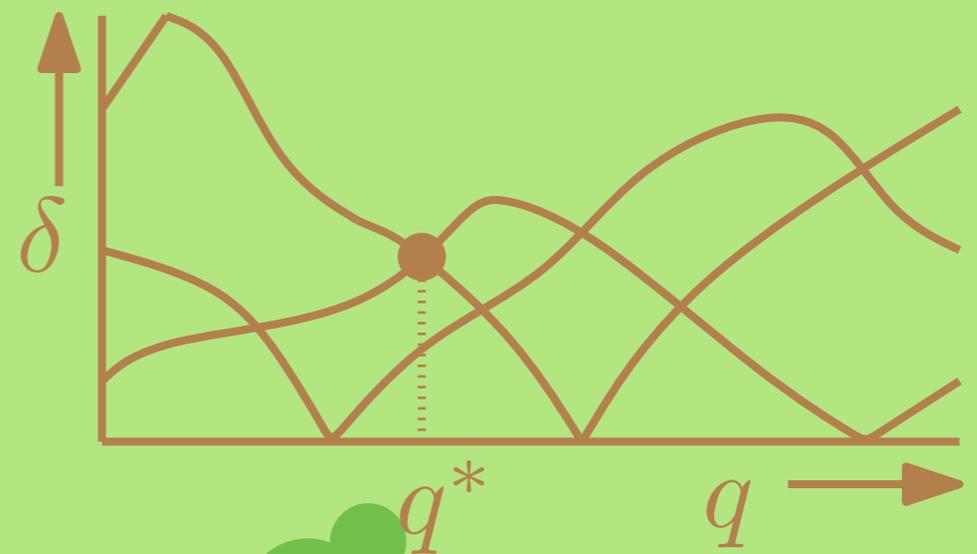
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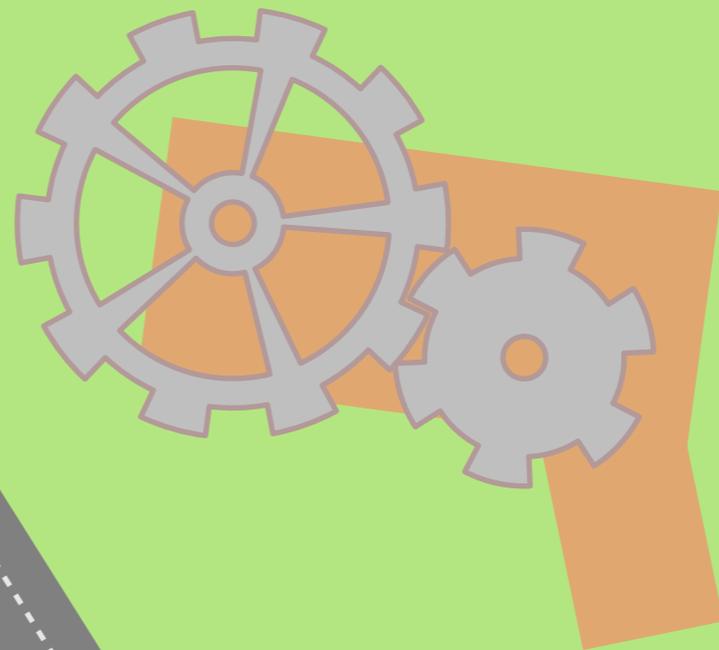
r_i



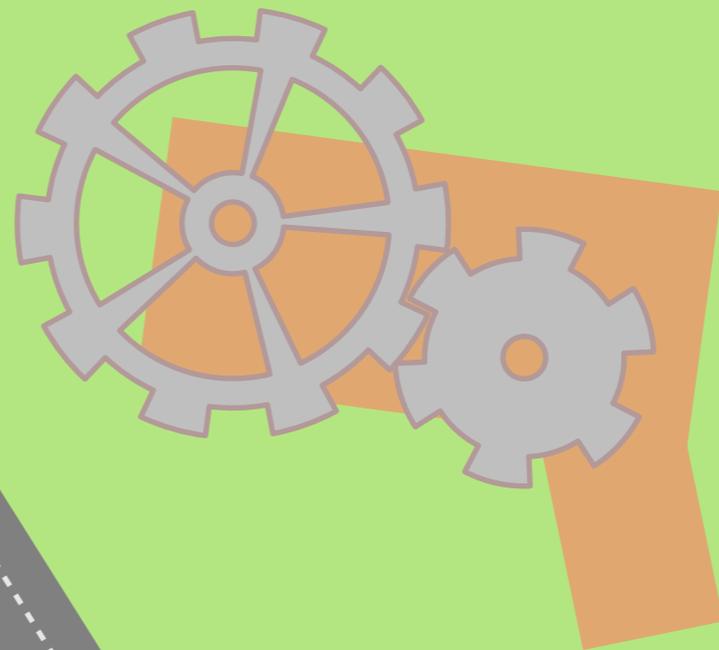
Consider again a fixed point r_i on P .

P

p



Consider again a fixed point r_i on P .



P

p



r_i



Consider again a fixed point r_i on P .

As before, let q be the point the feed link attaches to.



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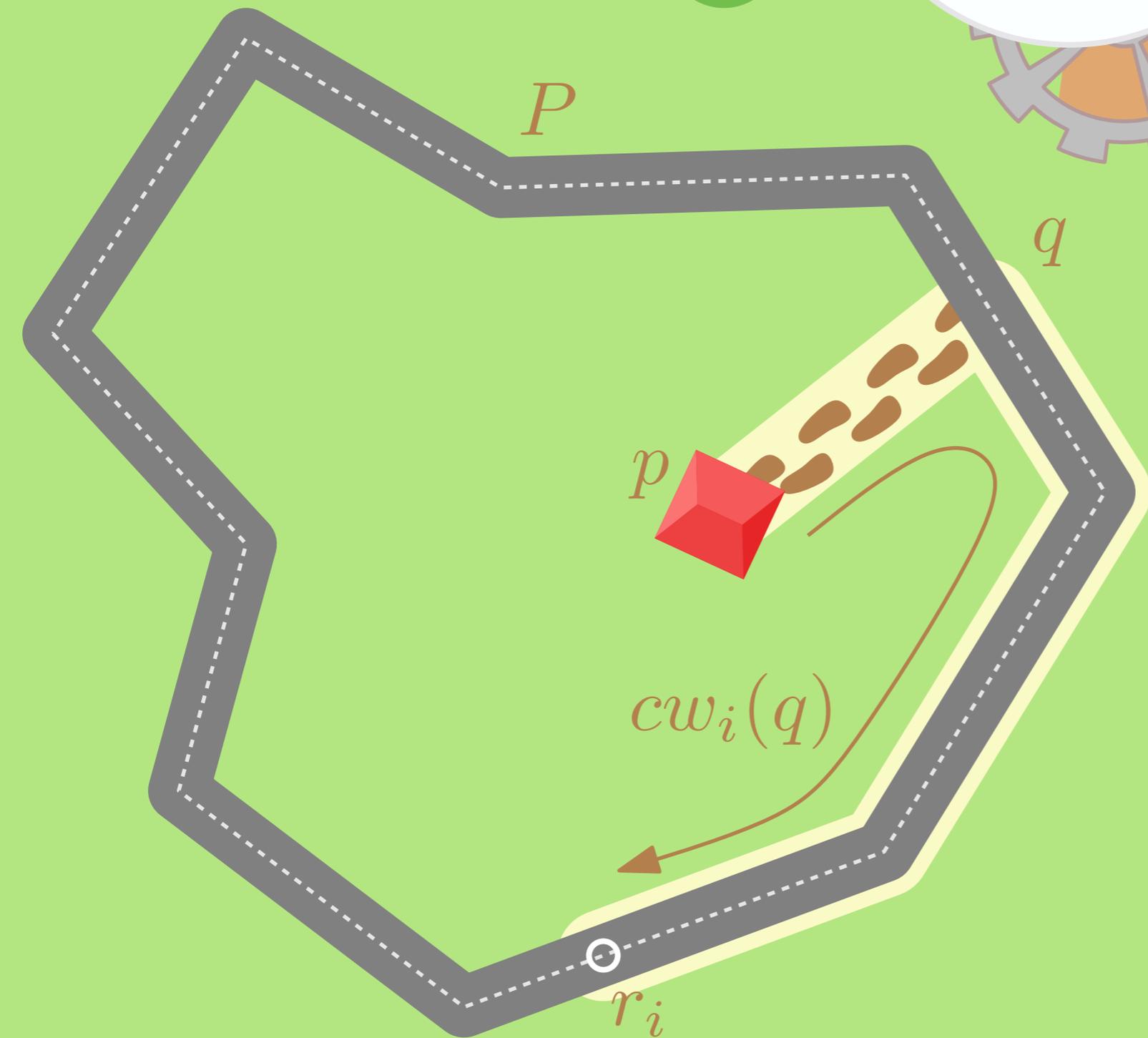
We define $cw_i(q)$ as the length of the path clockwise from p to r_i .



Consider again a fixed point r_i on P .

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Consider again a fixed point r_i on P .

As before, let q be the point the feed link attaches to.

We define $cw_i(q)$ as the length of the path clockwise from p to r_i .

Similarly, let $ccw_i(q)$ be the length of the counterclockwise path.



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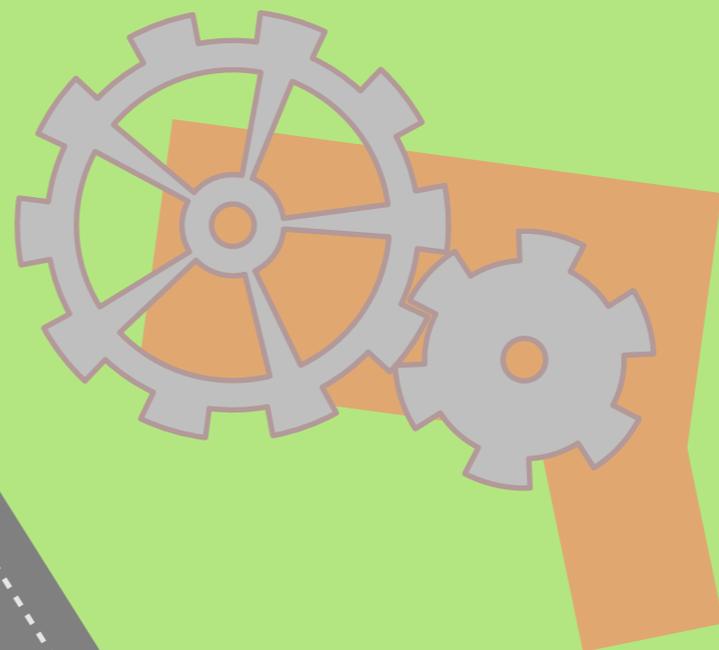
We define $cw_i(q)$ as the length of the path clockwise from p to r_i .

Similarly, let $ccw_i(q)$ be the length of the counterclockwise path.

As q moves around, $cw_i(q)$ and $ccw_i(q)$ increase or decrease monotonely.



Now, we can express the dilation δ_i in terms of cw_i or ccw_i , as well as the constant $|pr_i|$



P

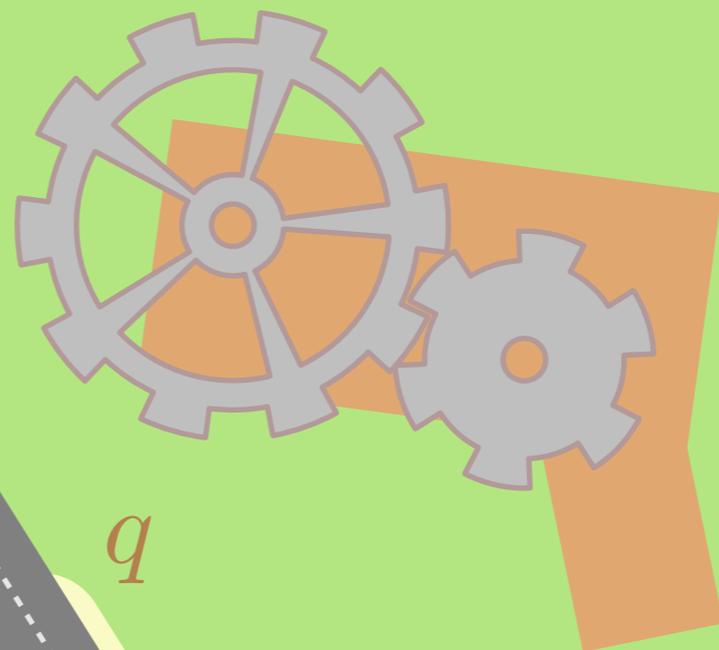
p



r_i

Now, we can express the dilation δ_i in terms of cw_i or ccw_i , as well as the constant $|pr_i|$

$$\delta_i(q) = \frac{cw_i(q)}{|pr_i|}$$

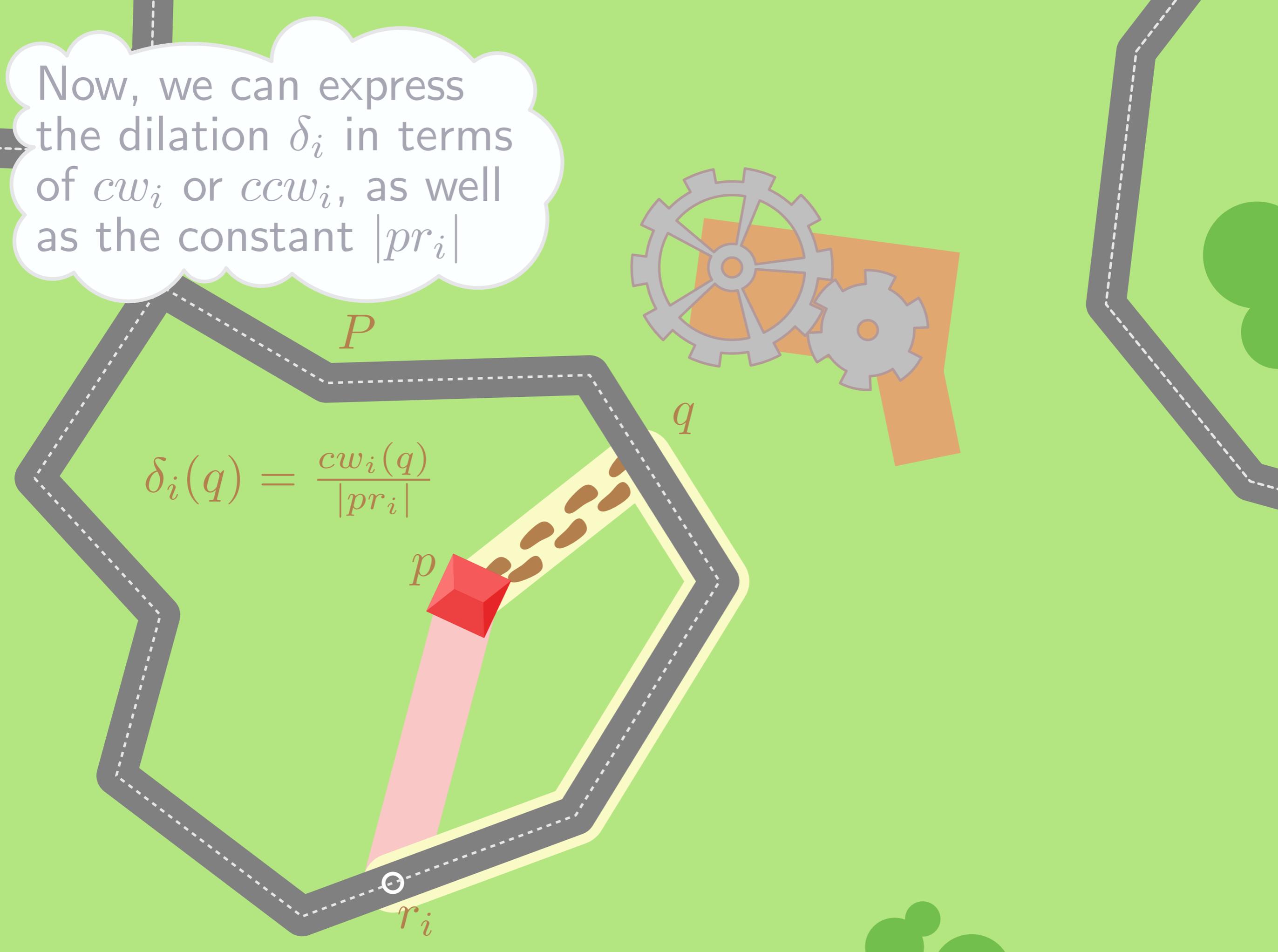


P

q

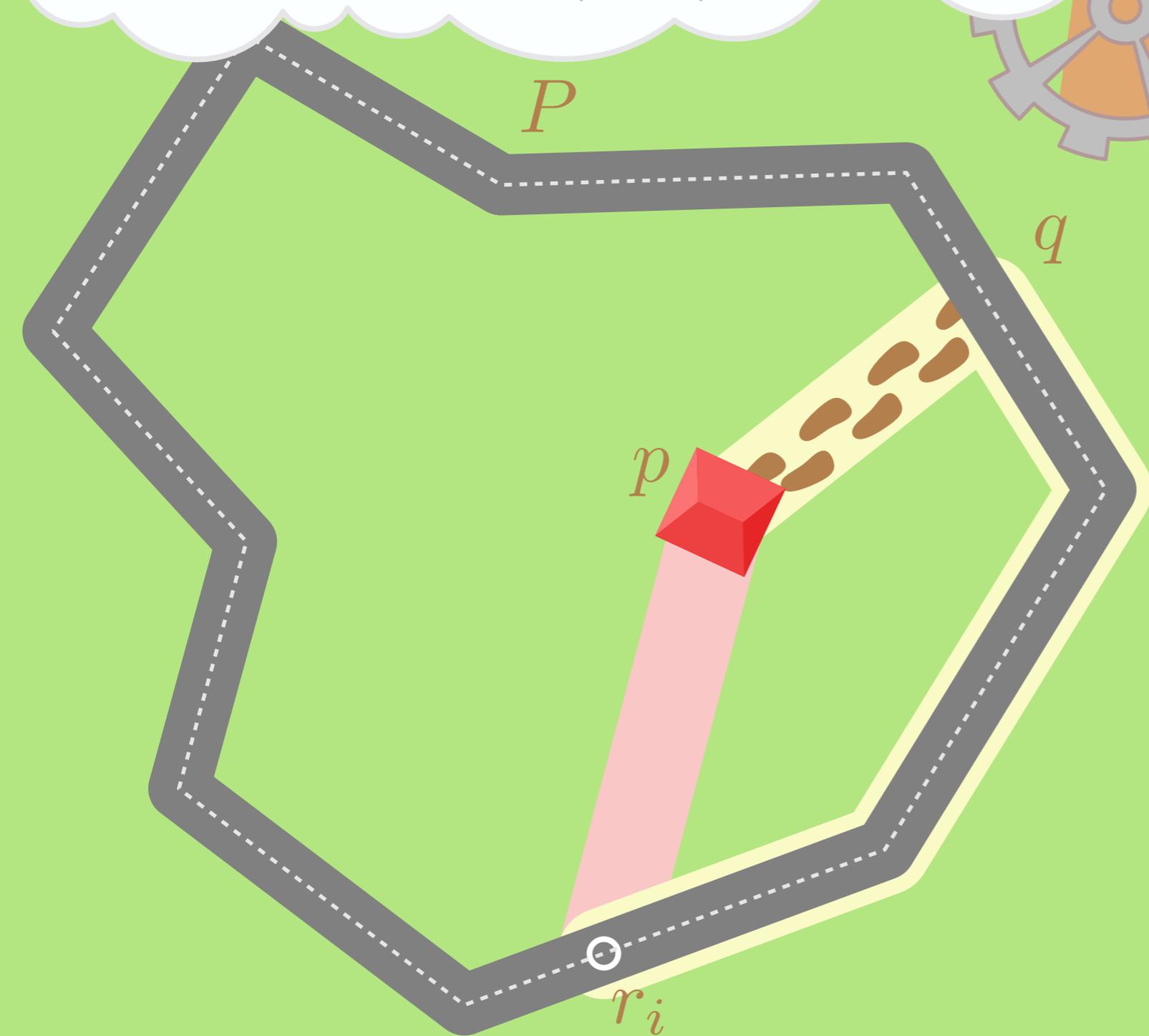
p

r_i



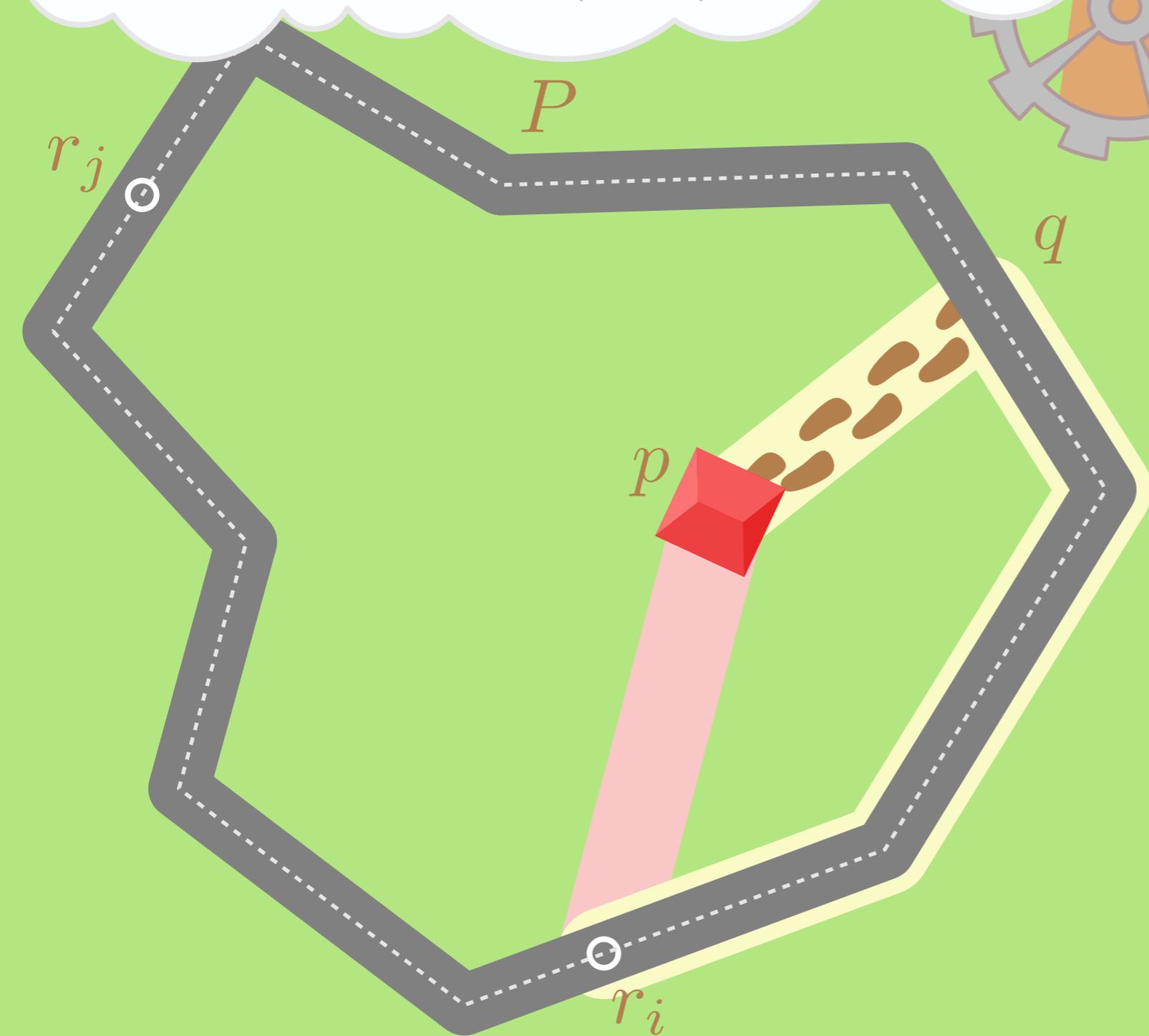
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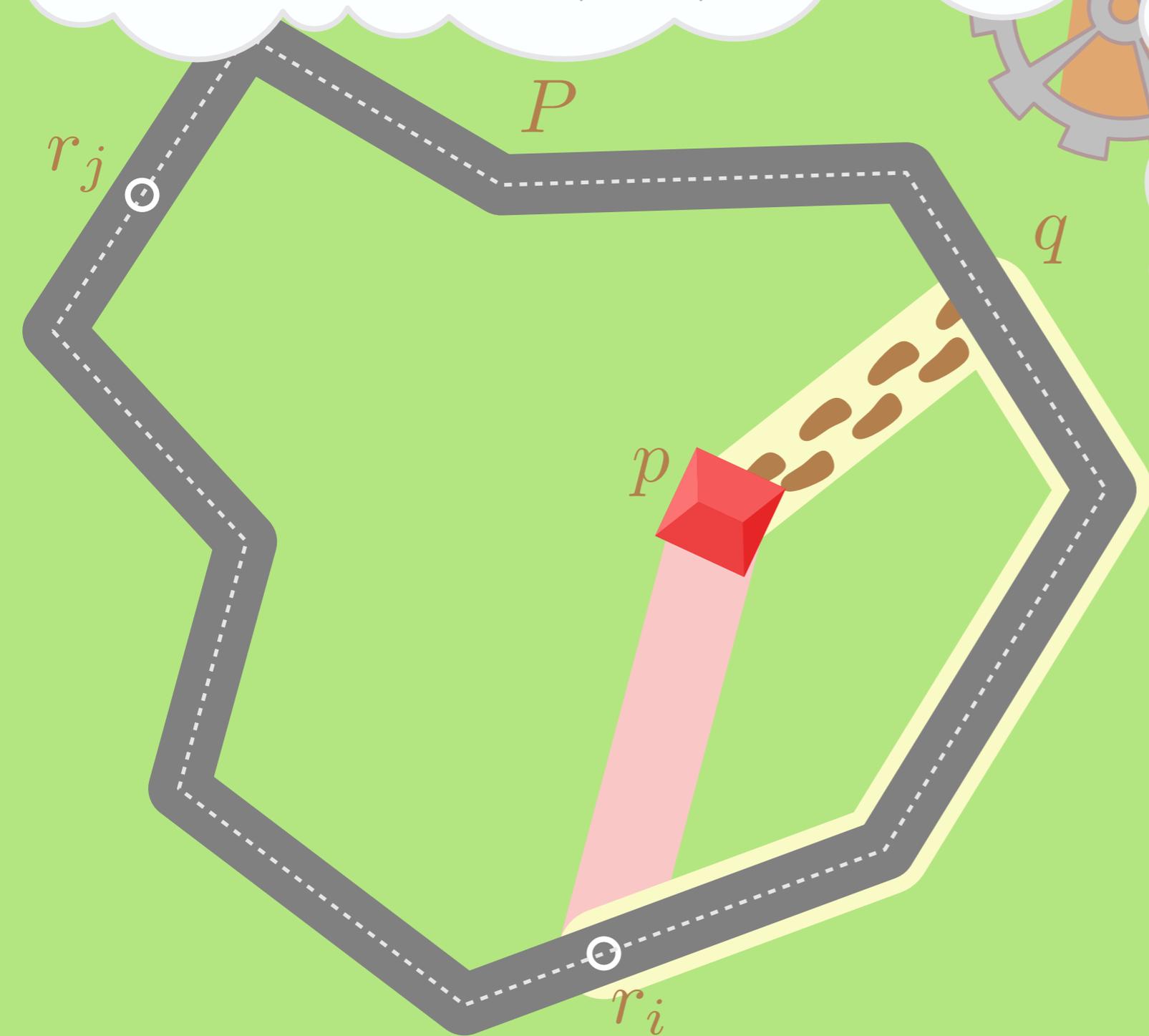
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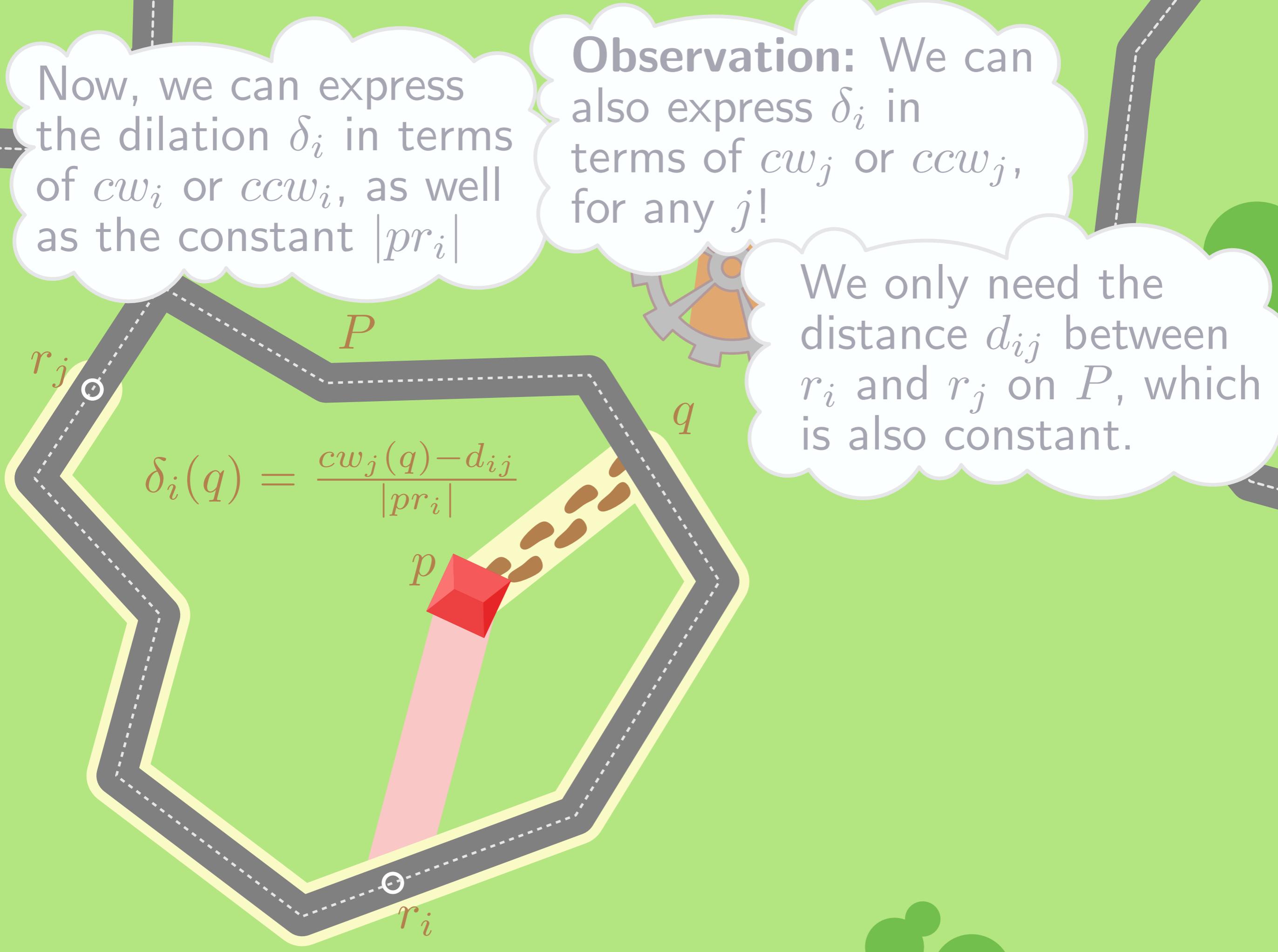


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$$\delta_i(q) = \frac{cw_j(q) - d_{ij}}{|pr_i|}$$

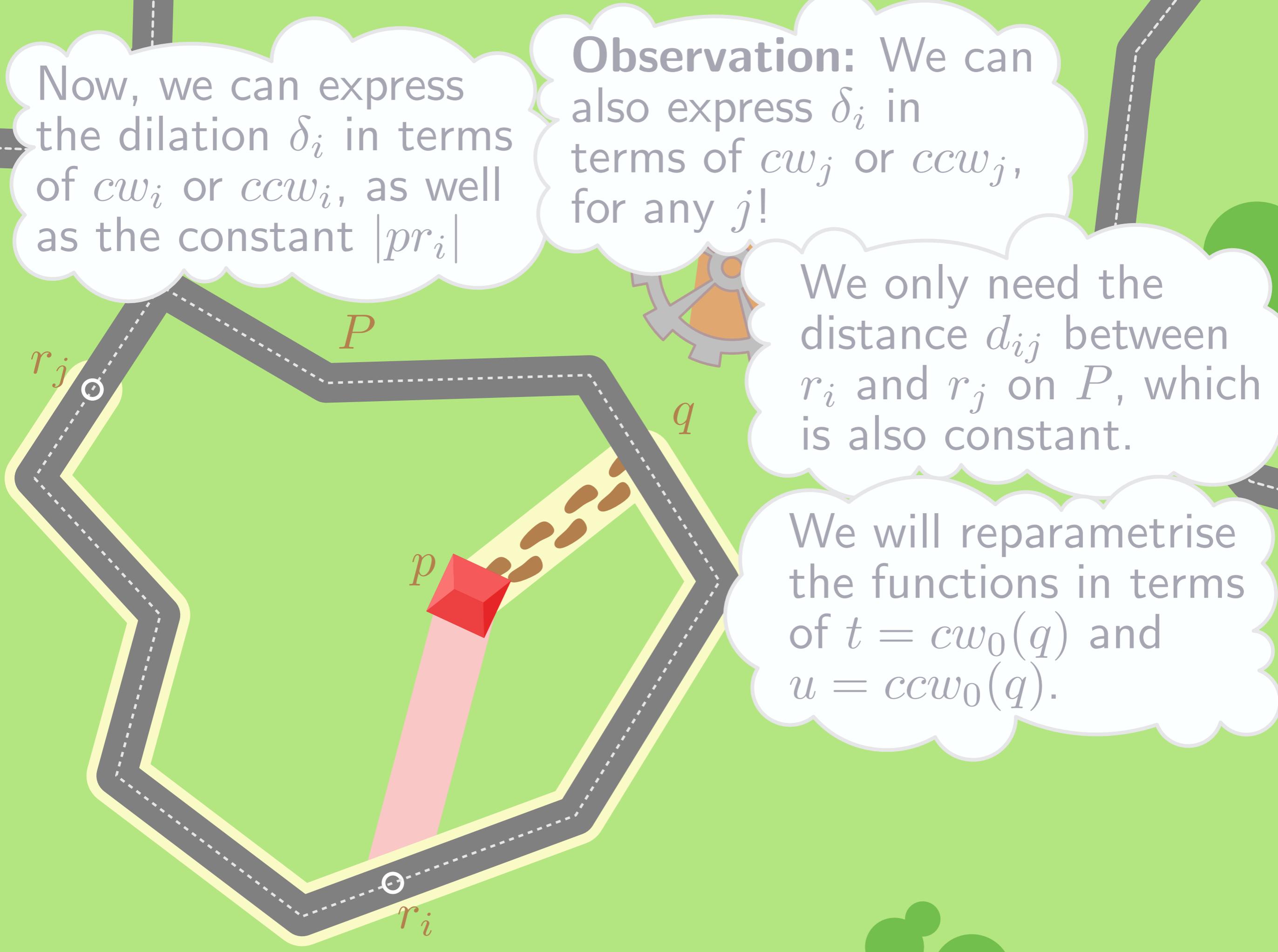


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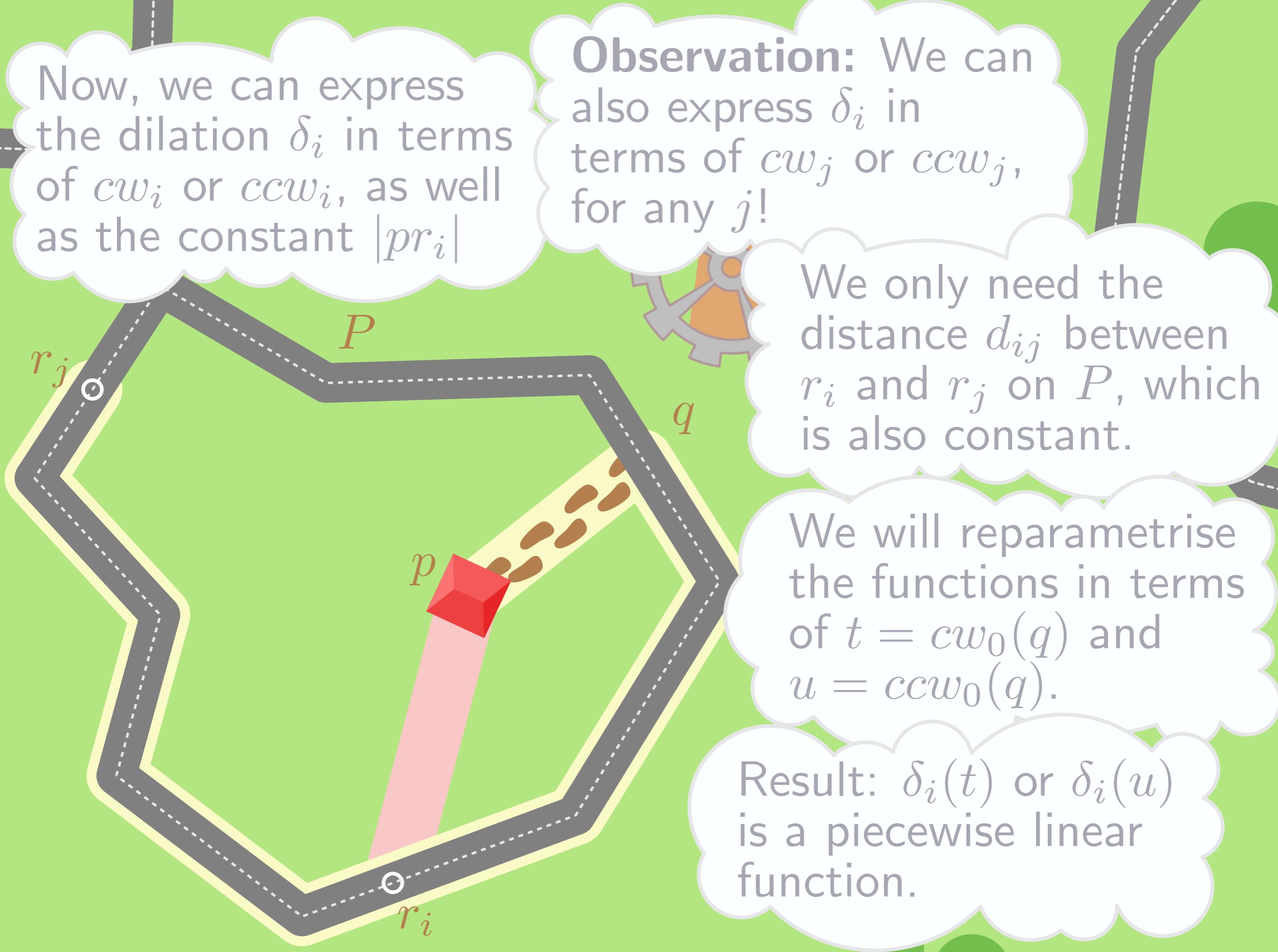
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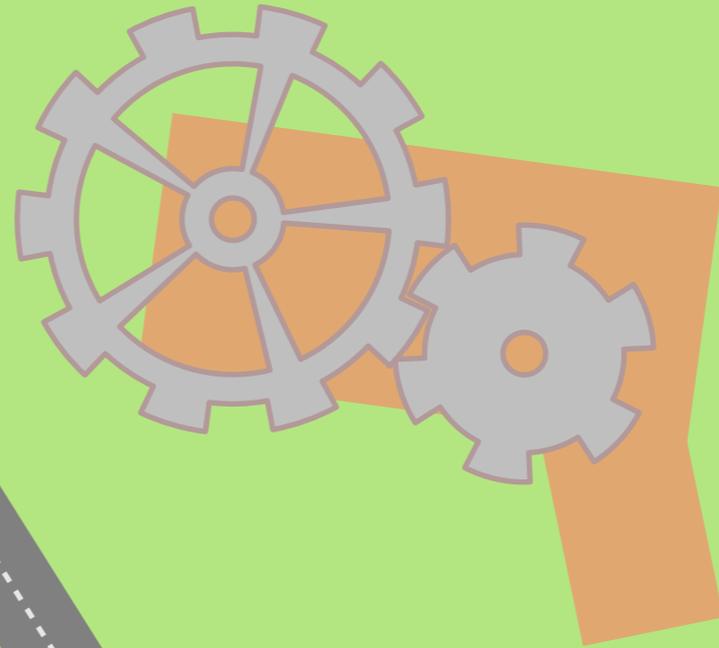
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Result: $\delta_i(t)$ or $\delta_i(u)$ is a piecewise linear function.

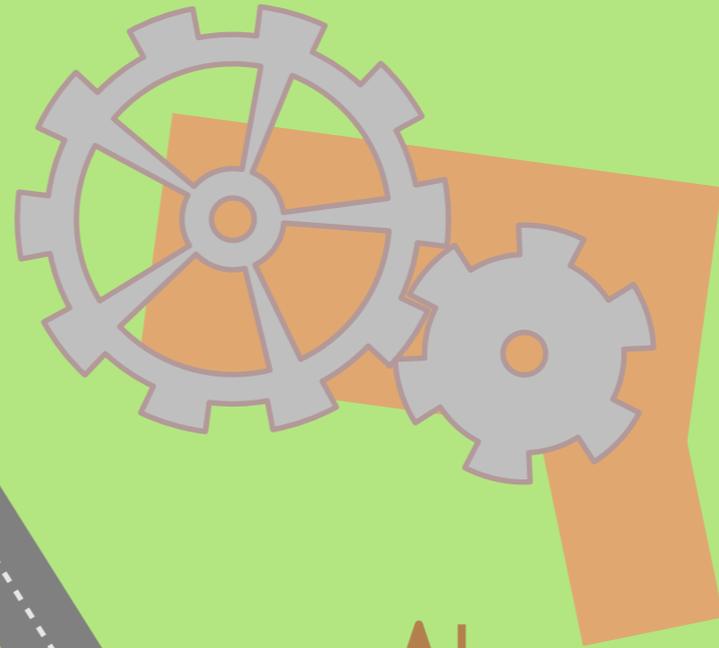




Now, we draw the graphs of $\delta_i(t)$ and $\delta_i(u)$.

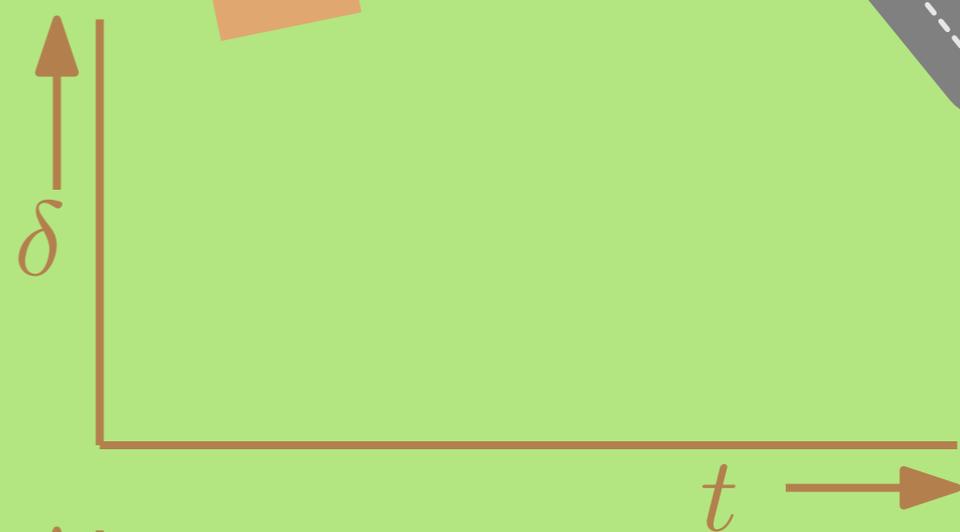


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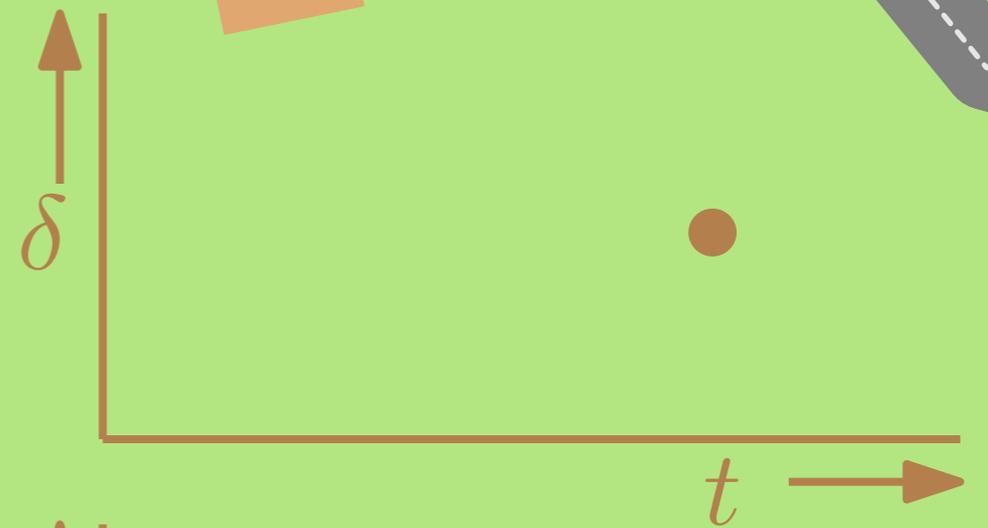
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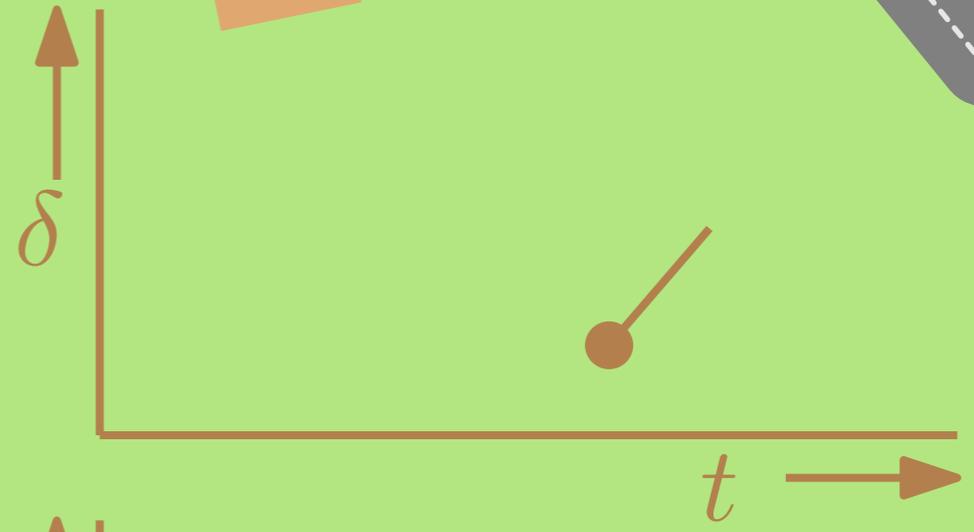
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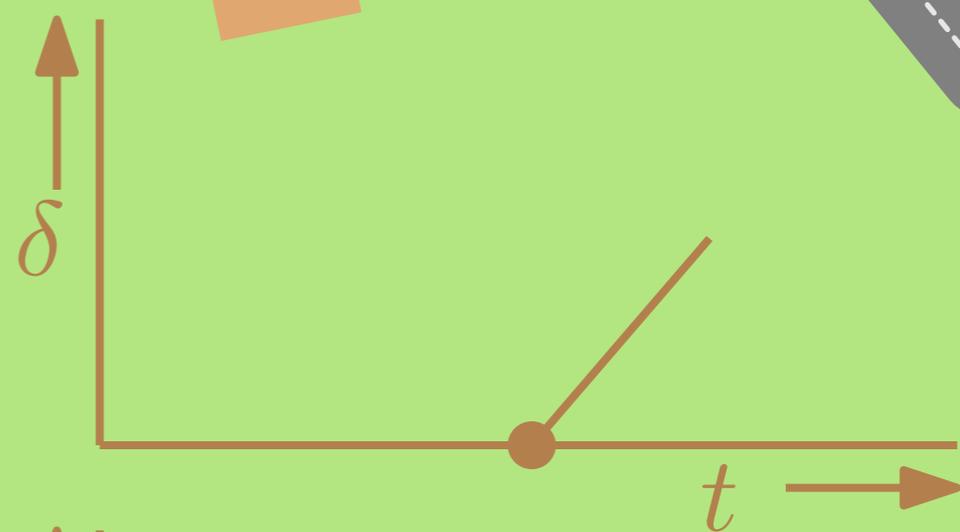
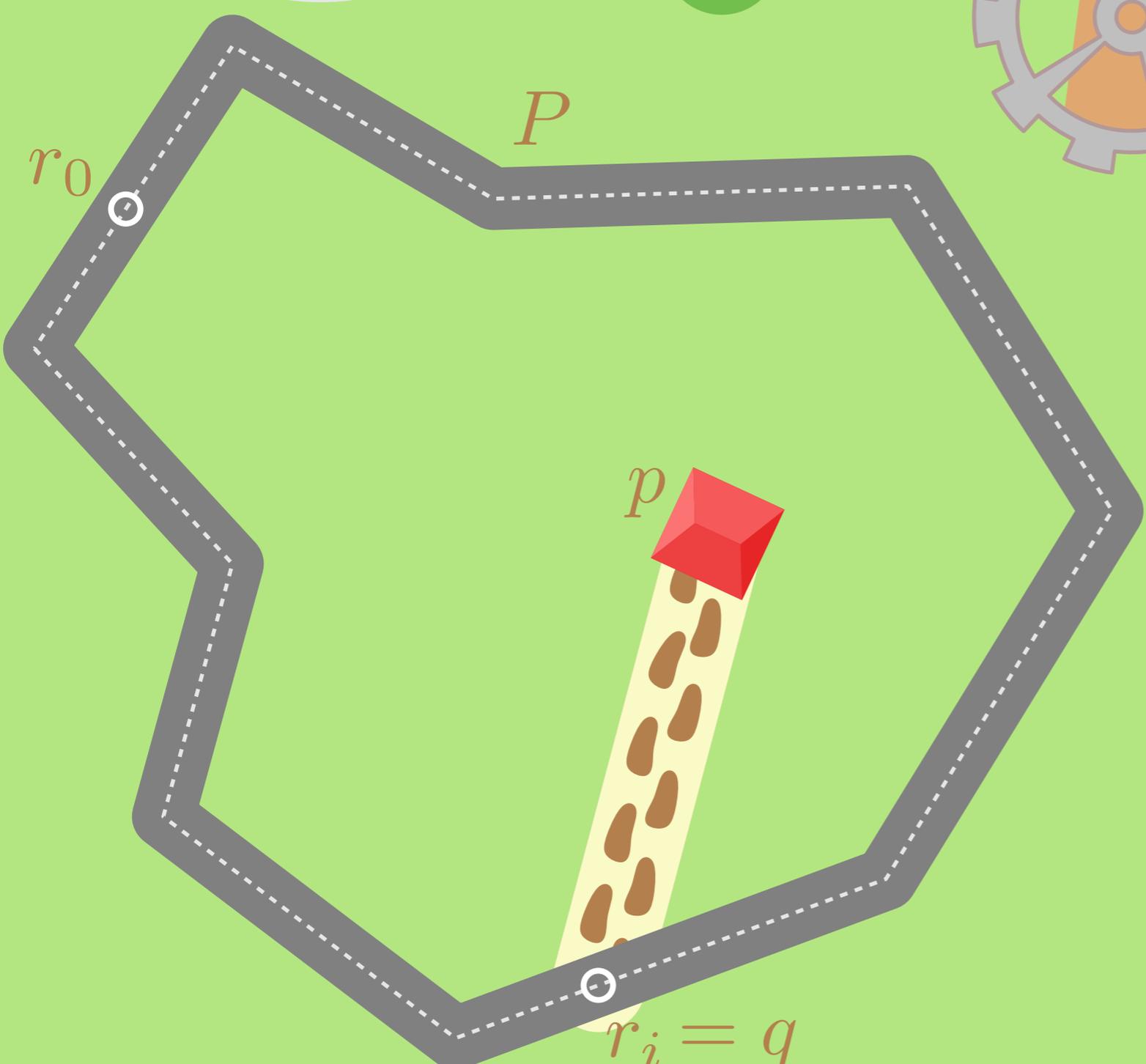
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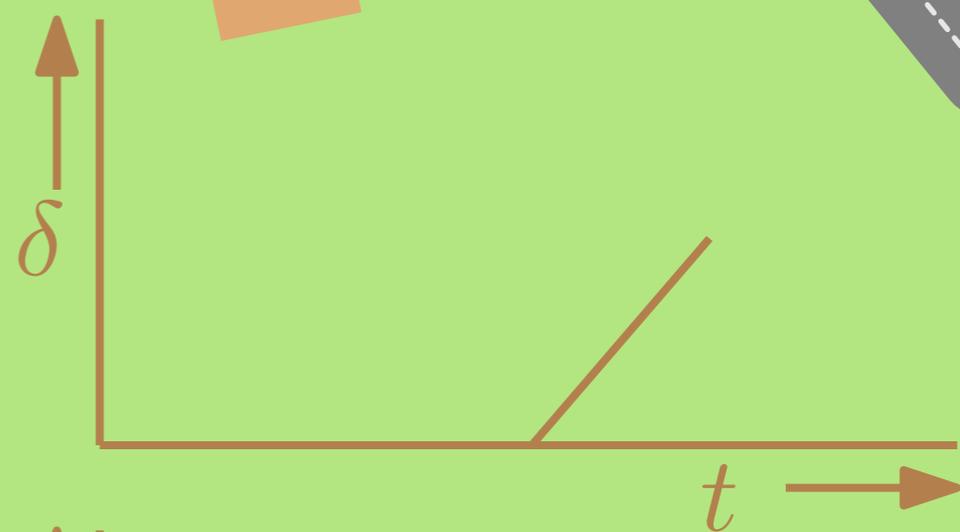
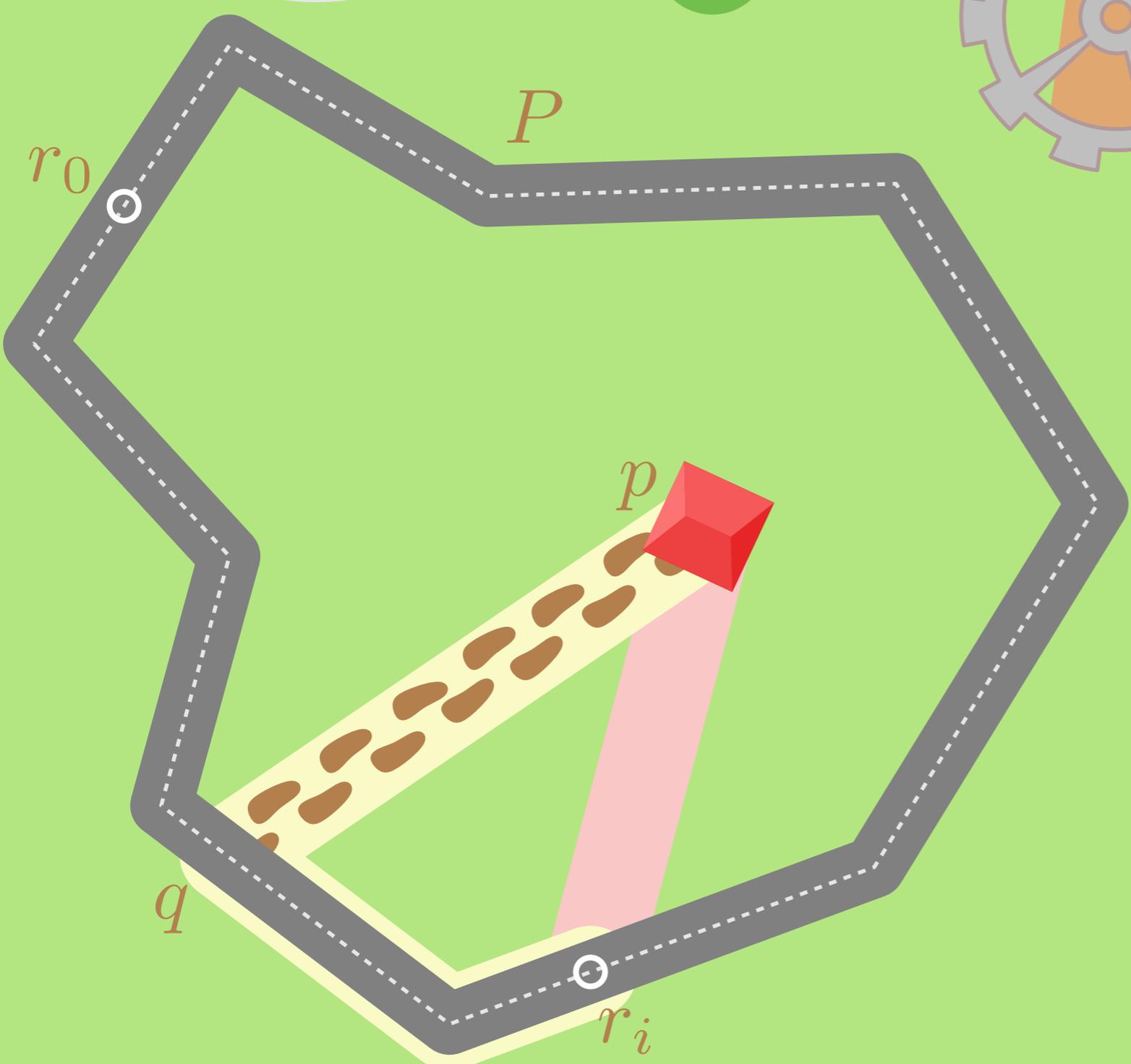
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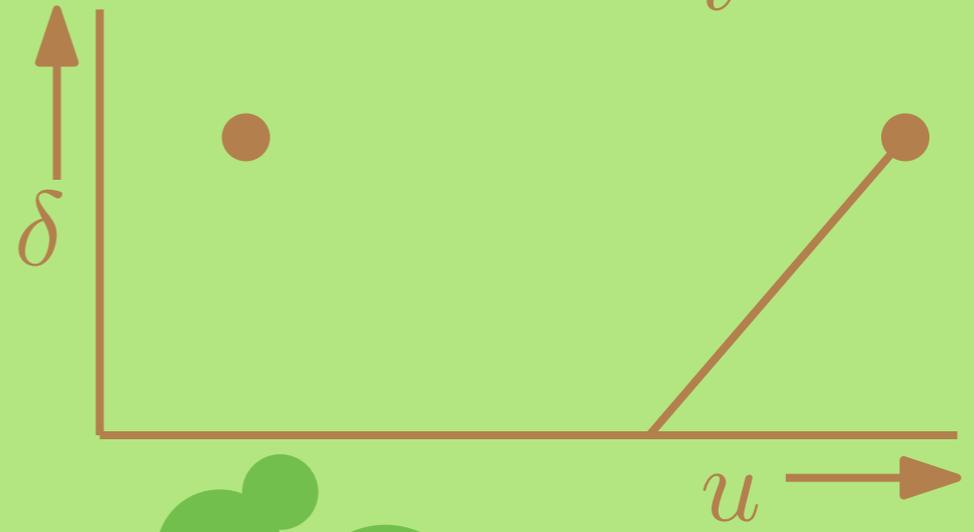
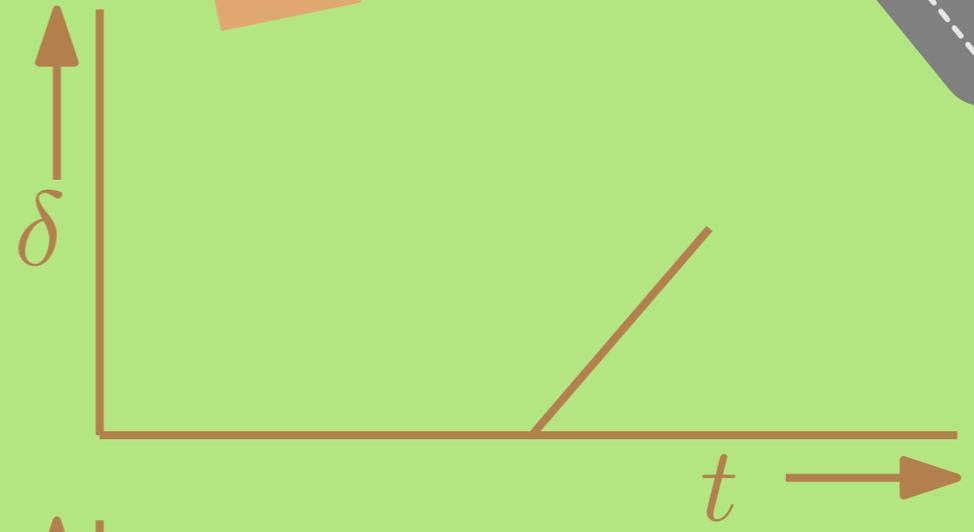
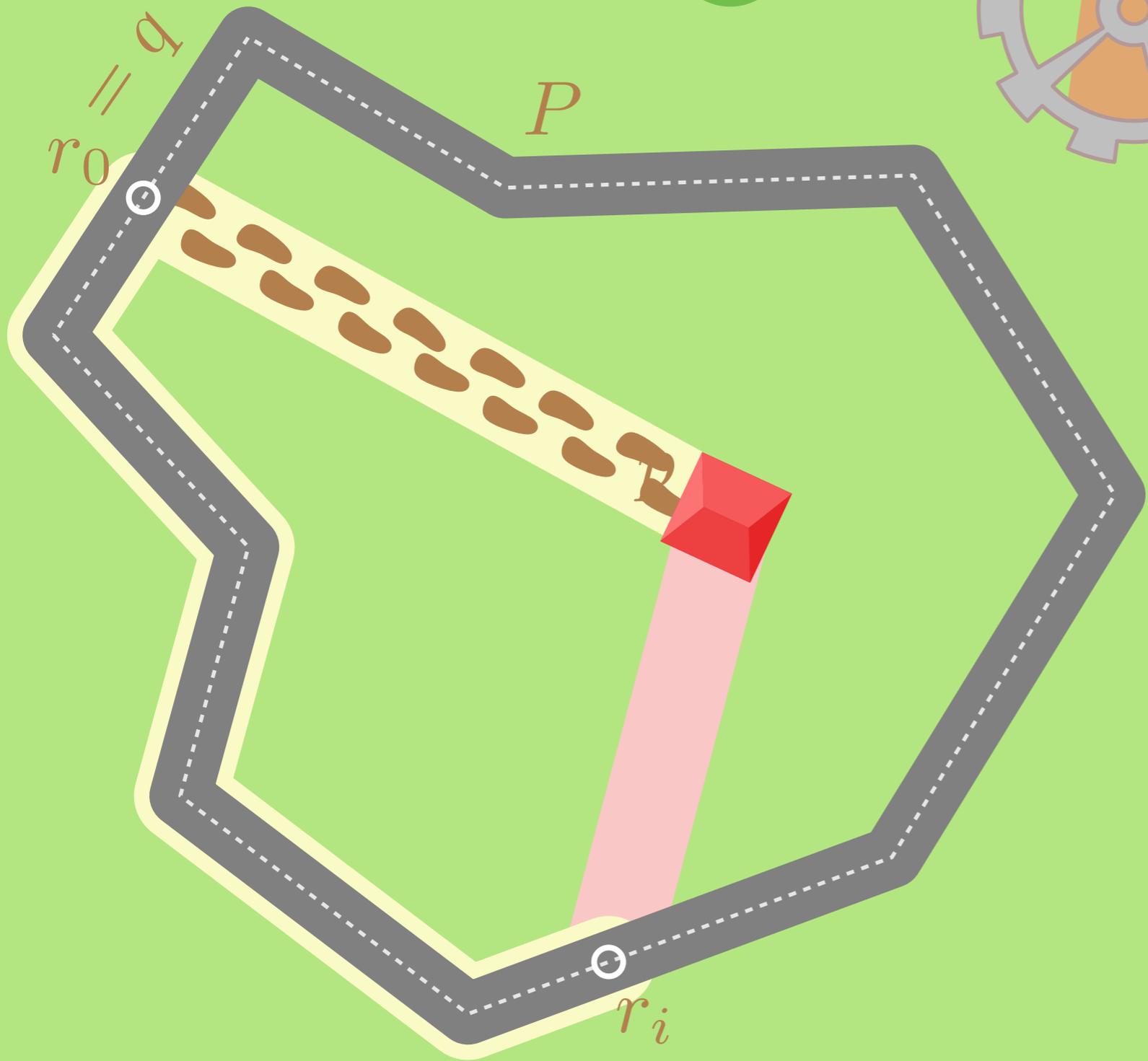
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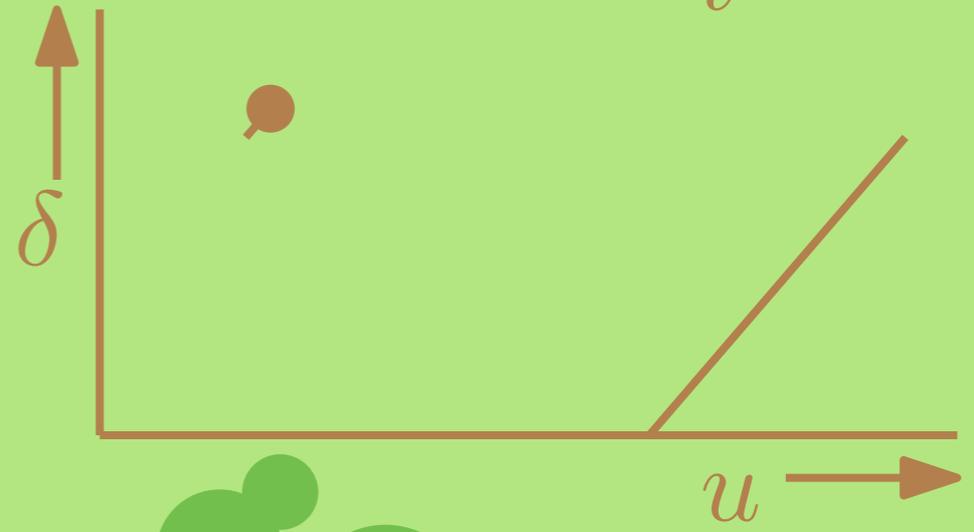
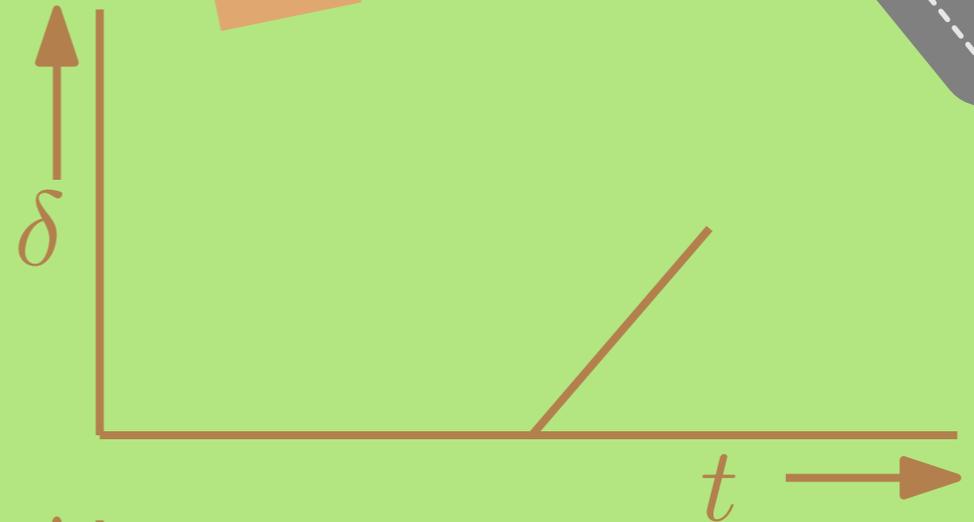
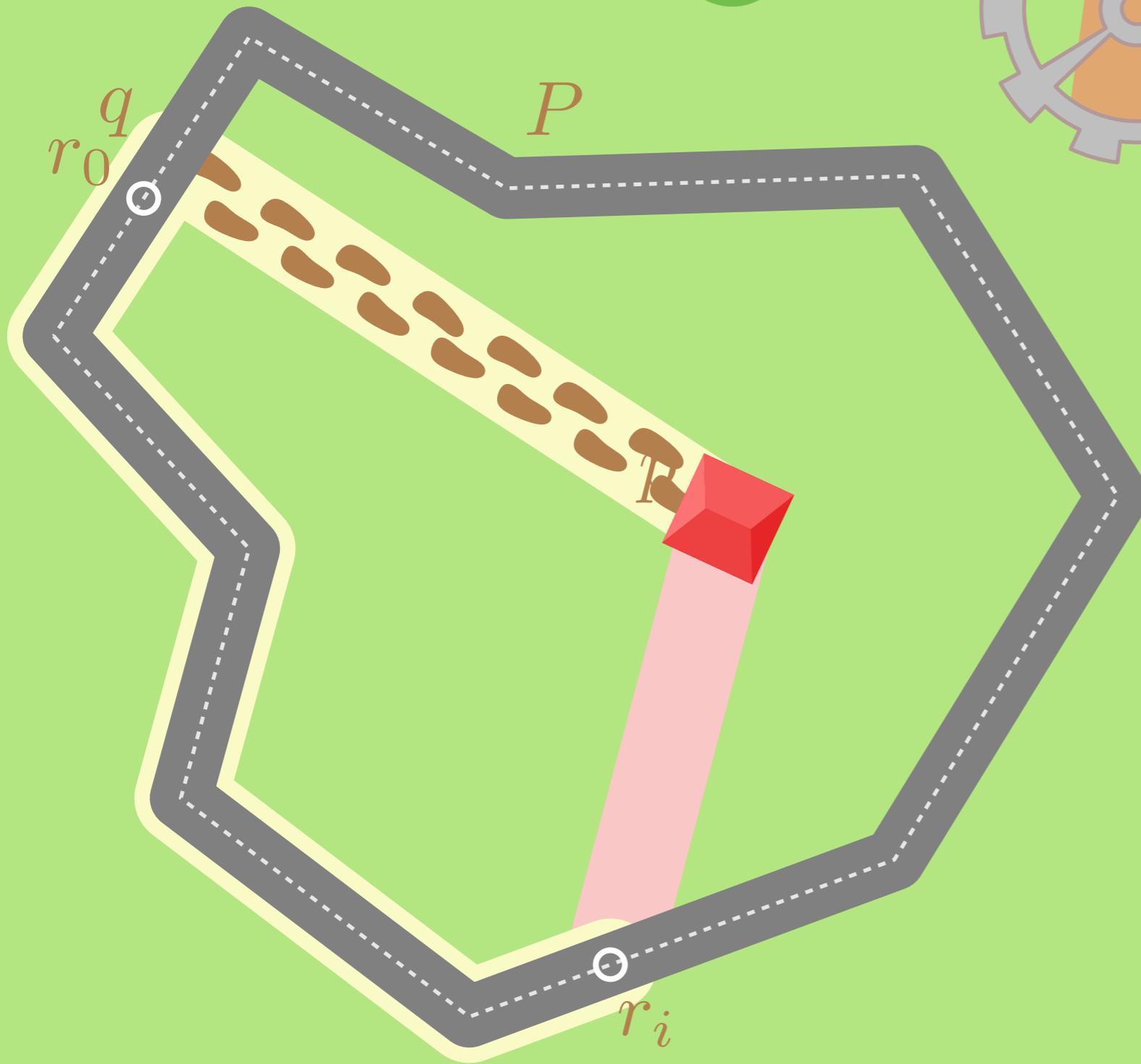
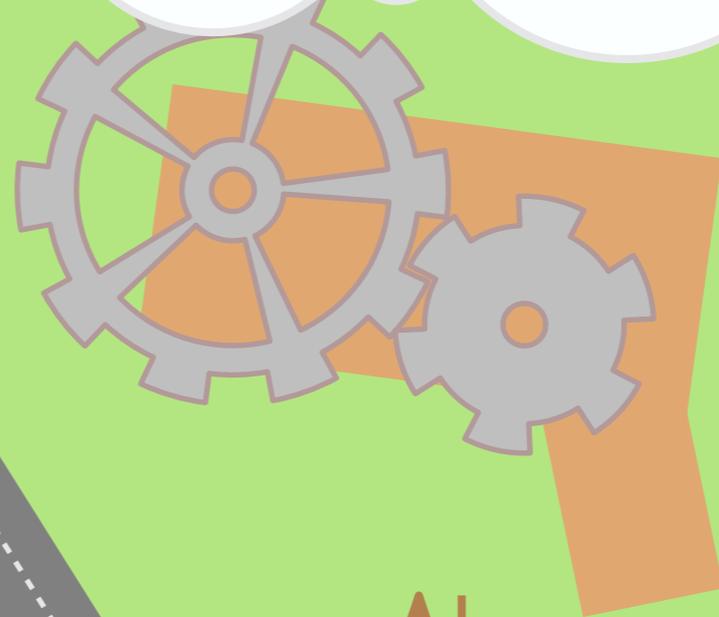
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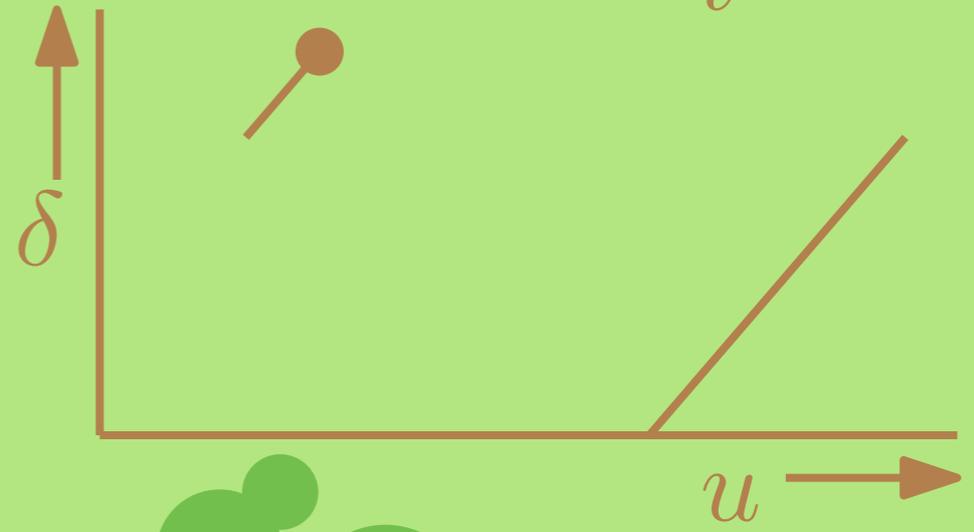
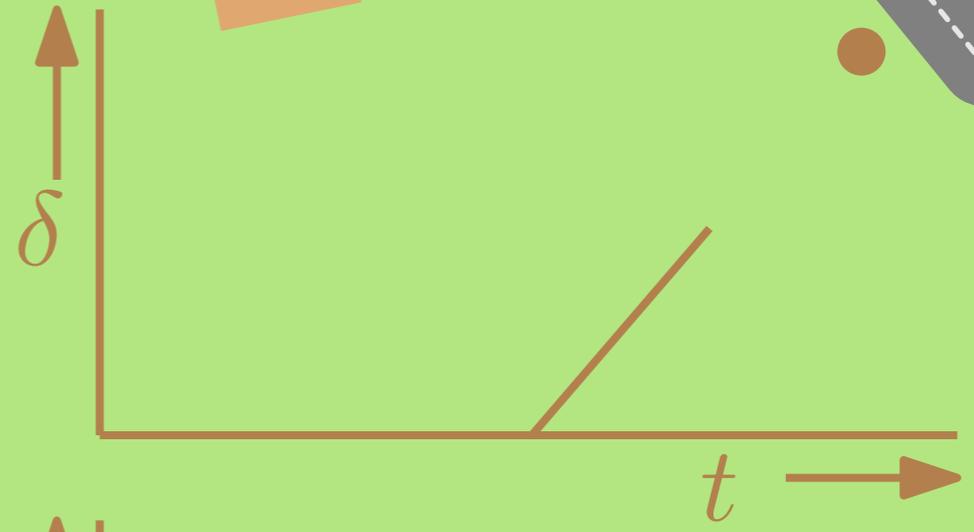
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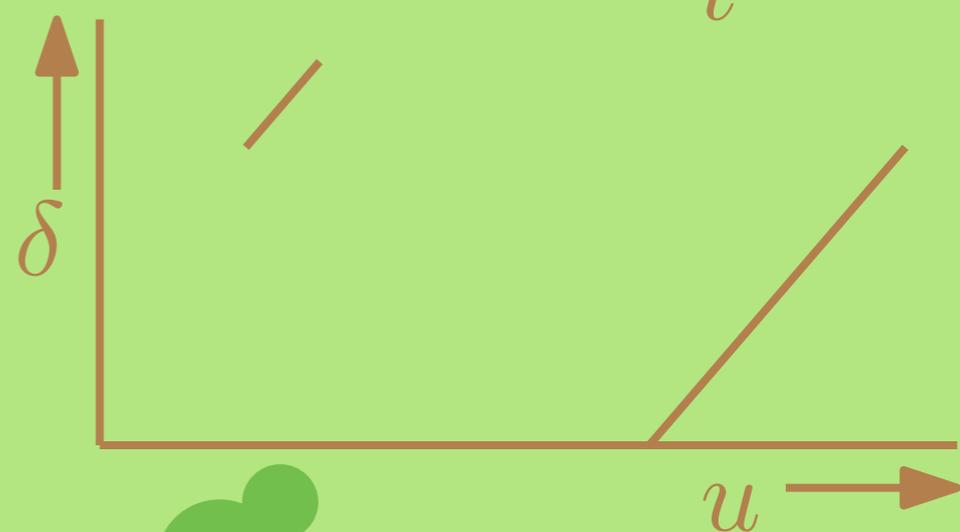
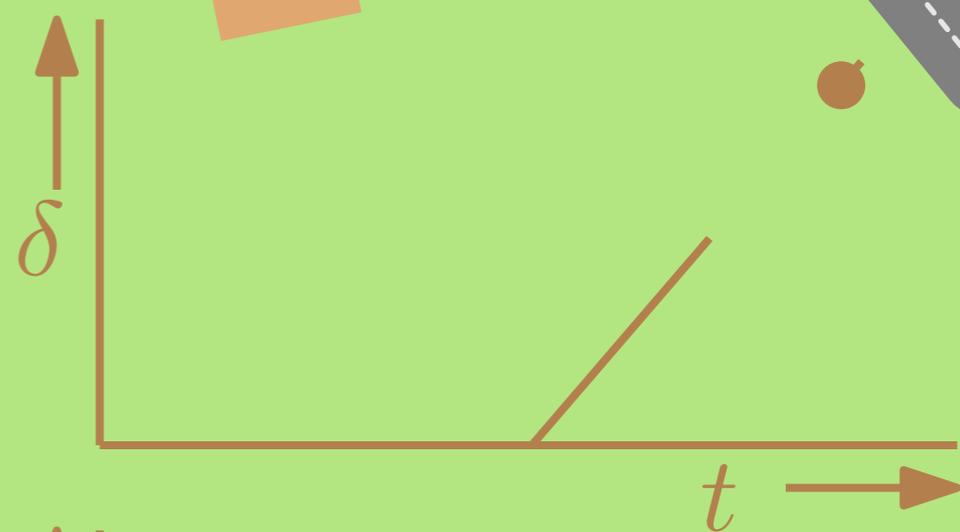
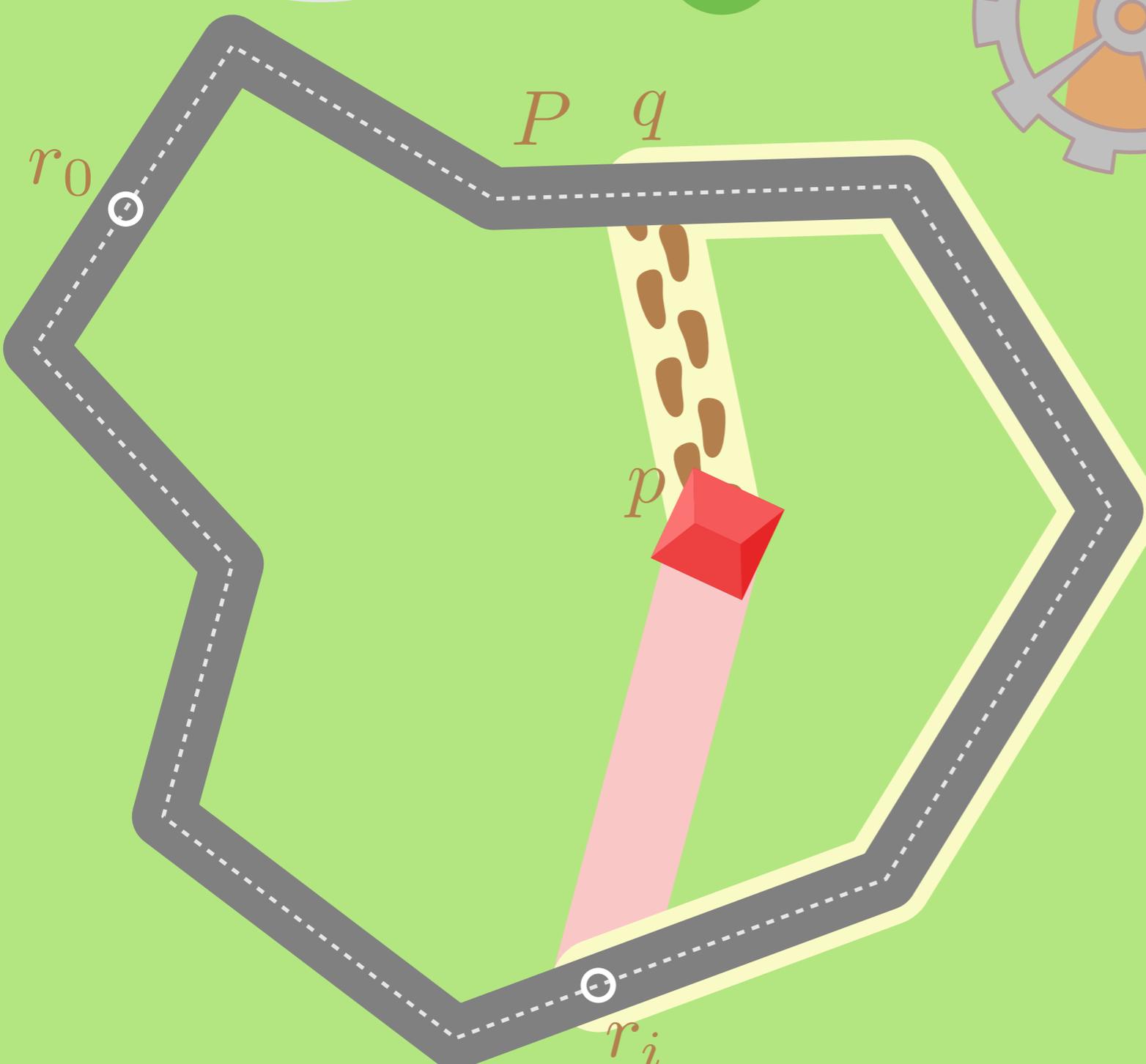
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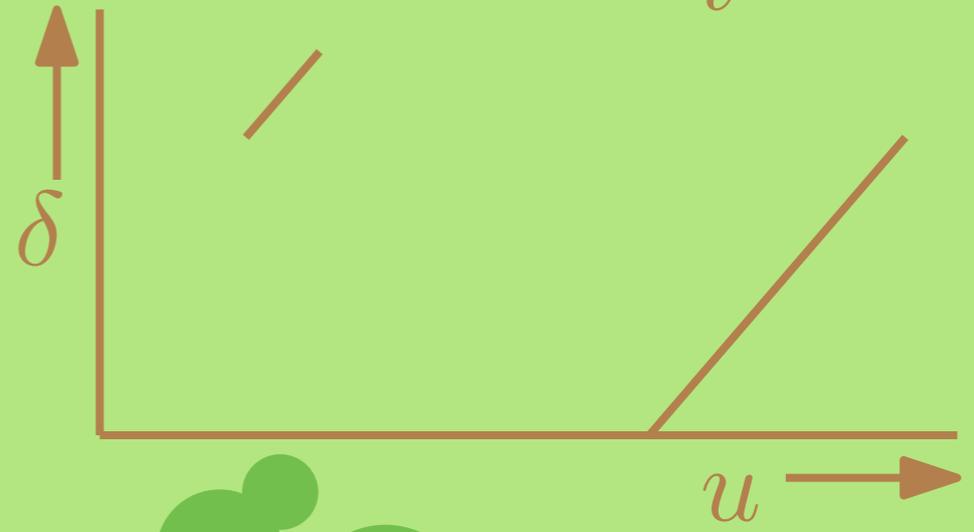
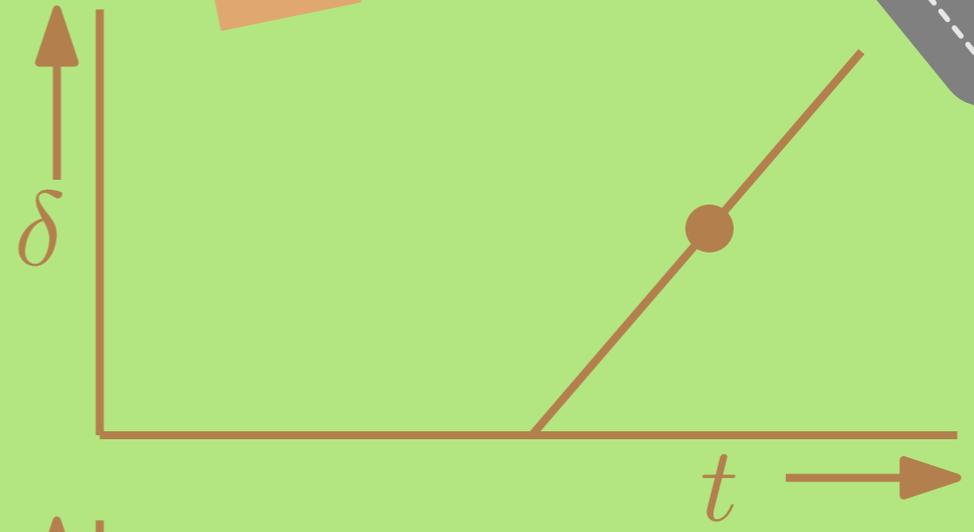
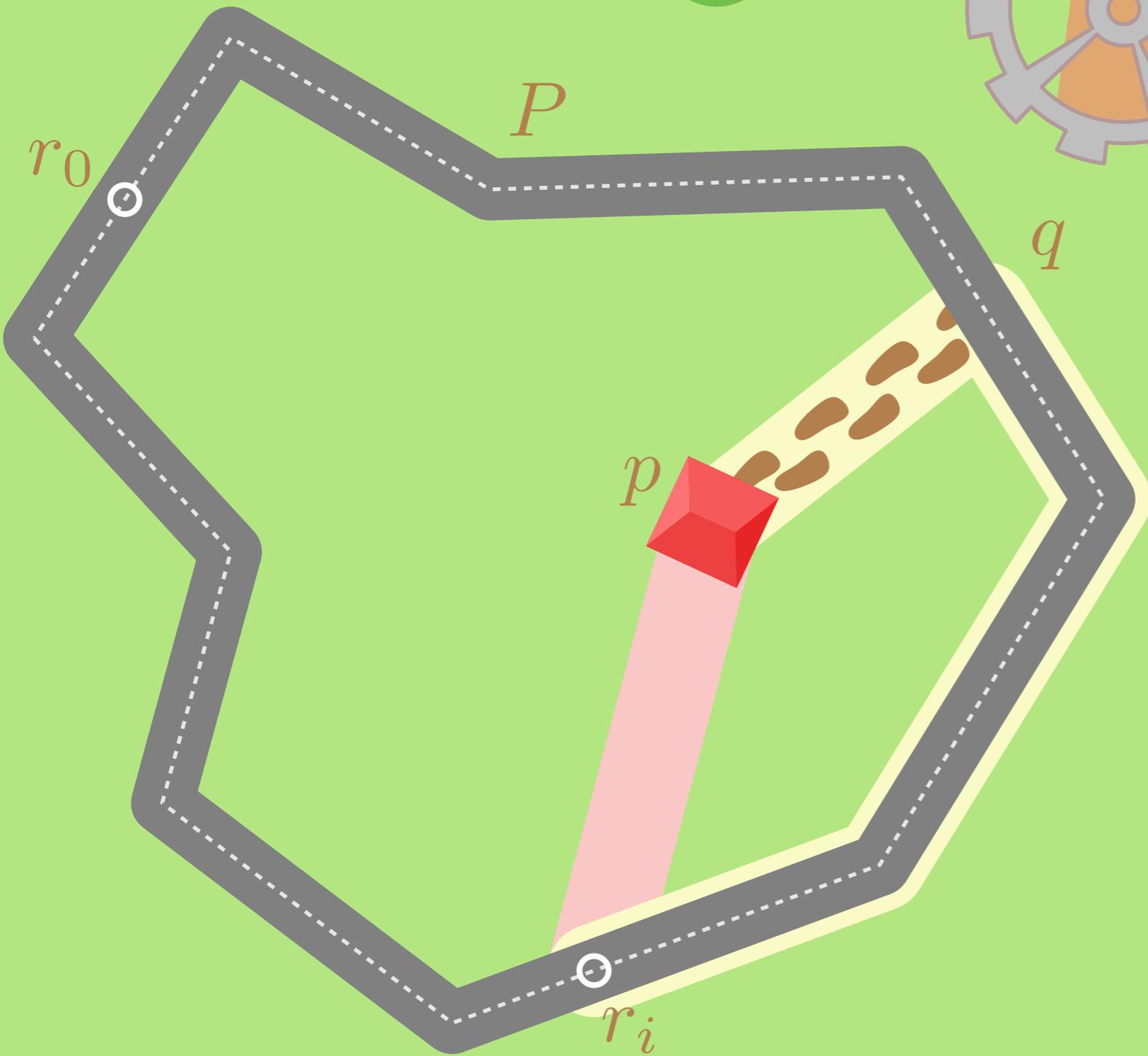
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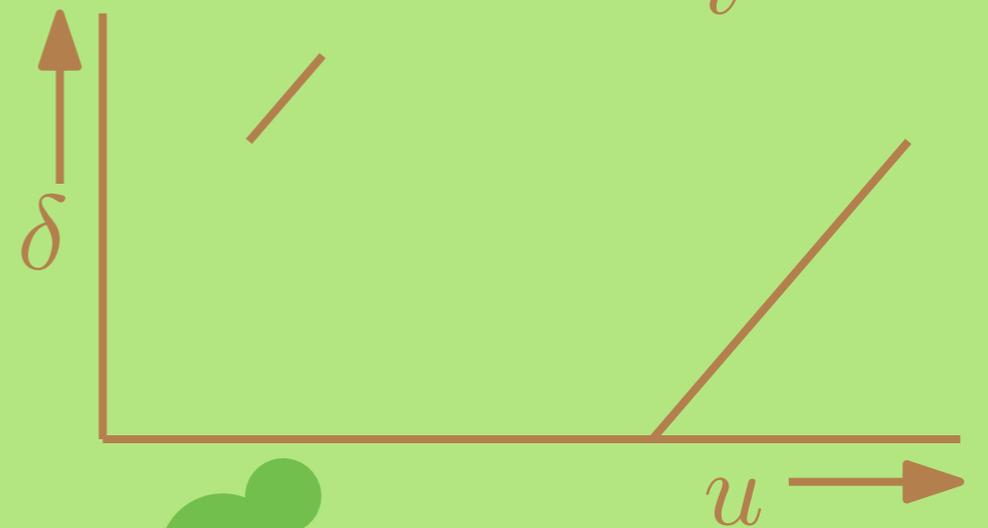
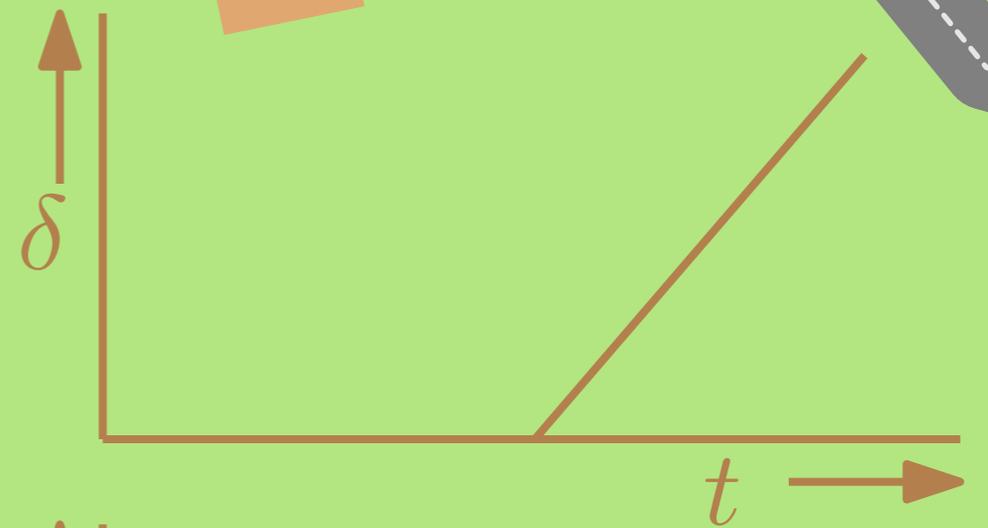
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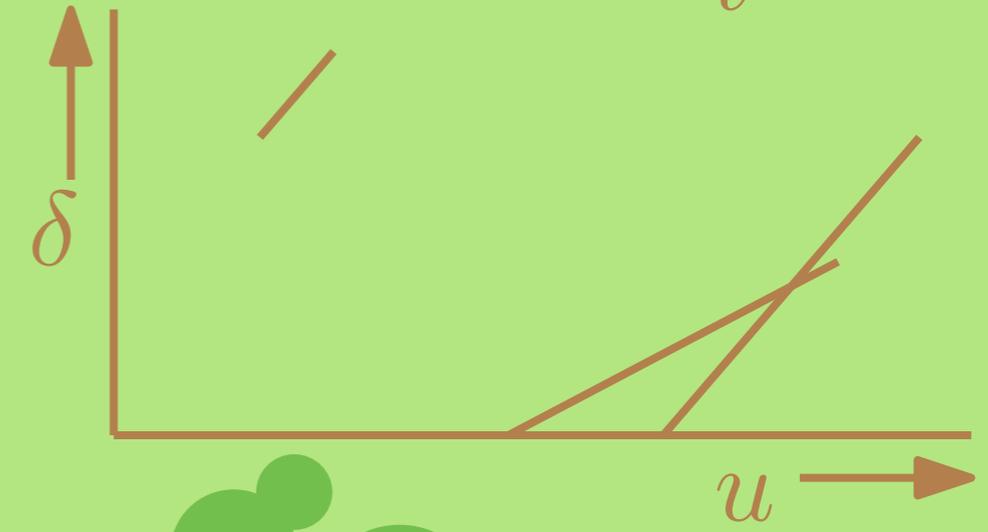
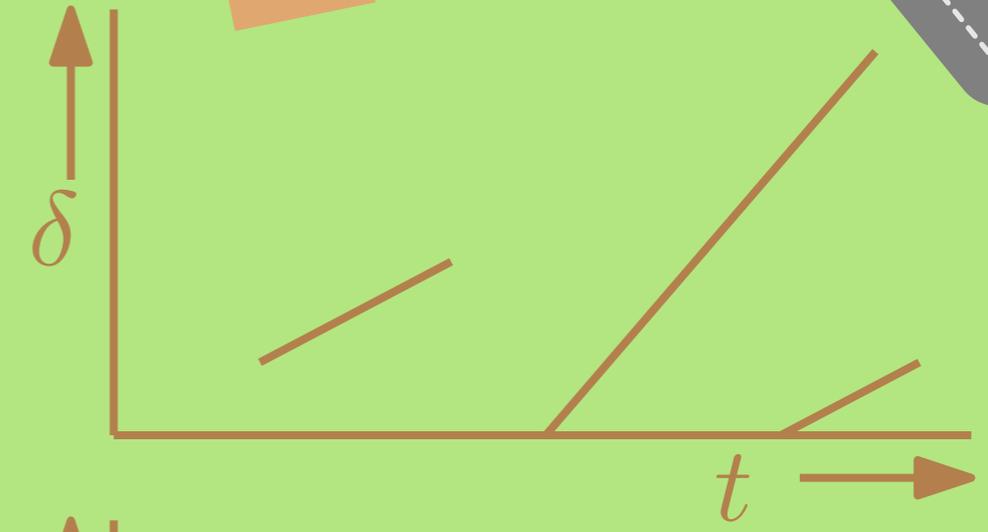
We do the same for the other points.



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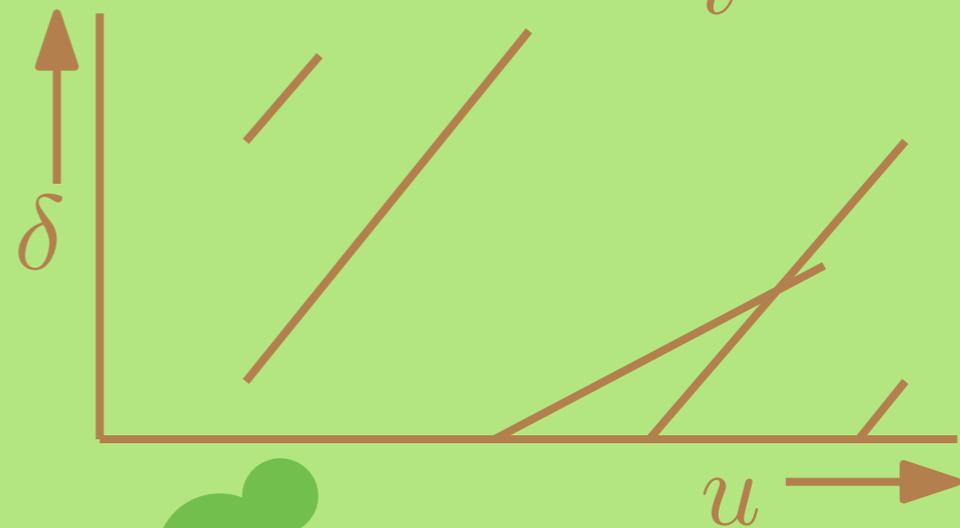
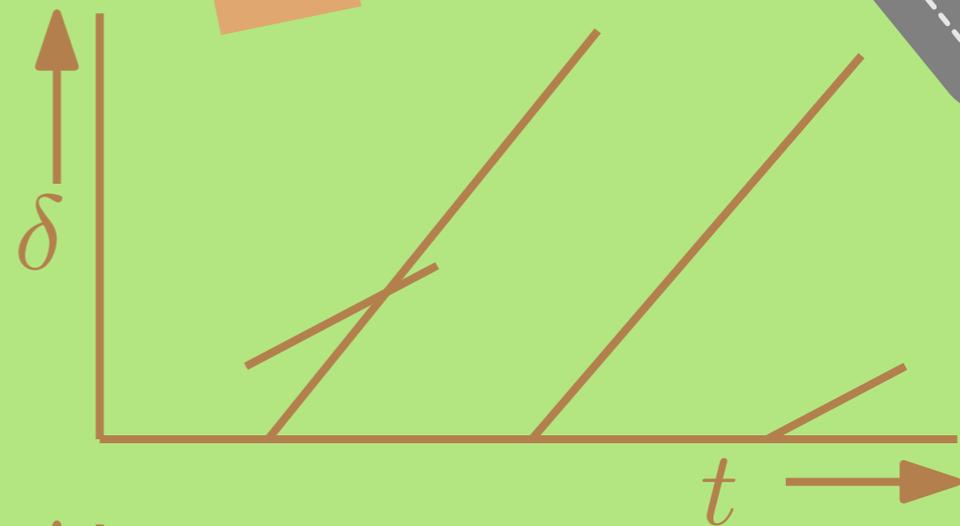
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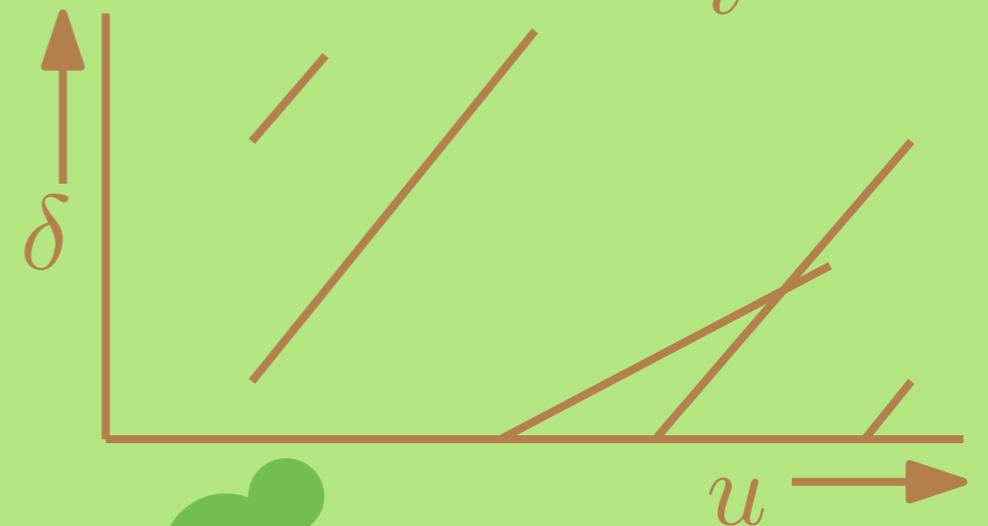
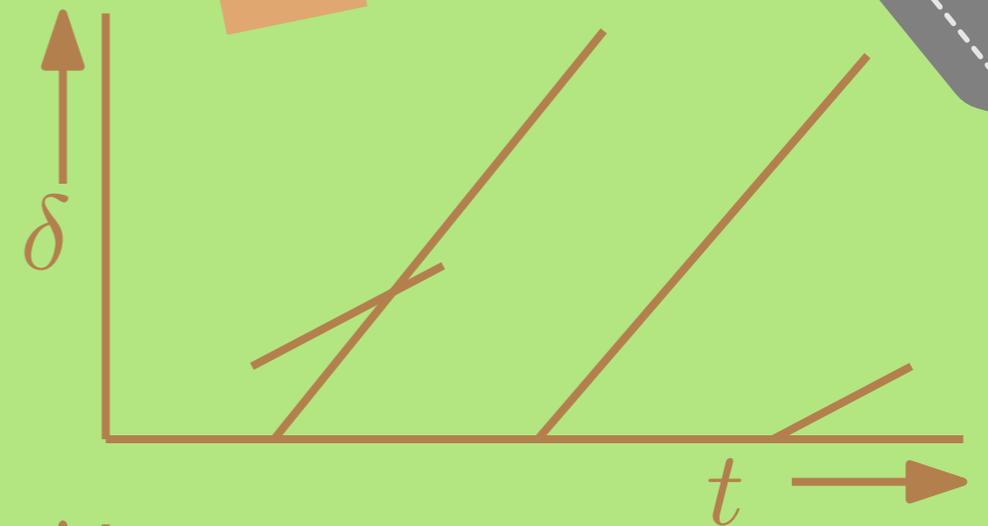
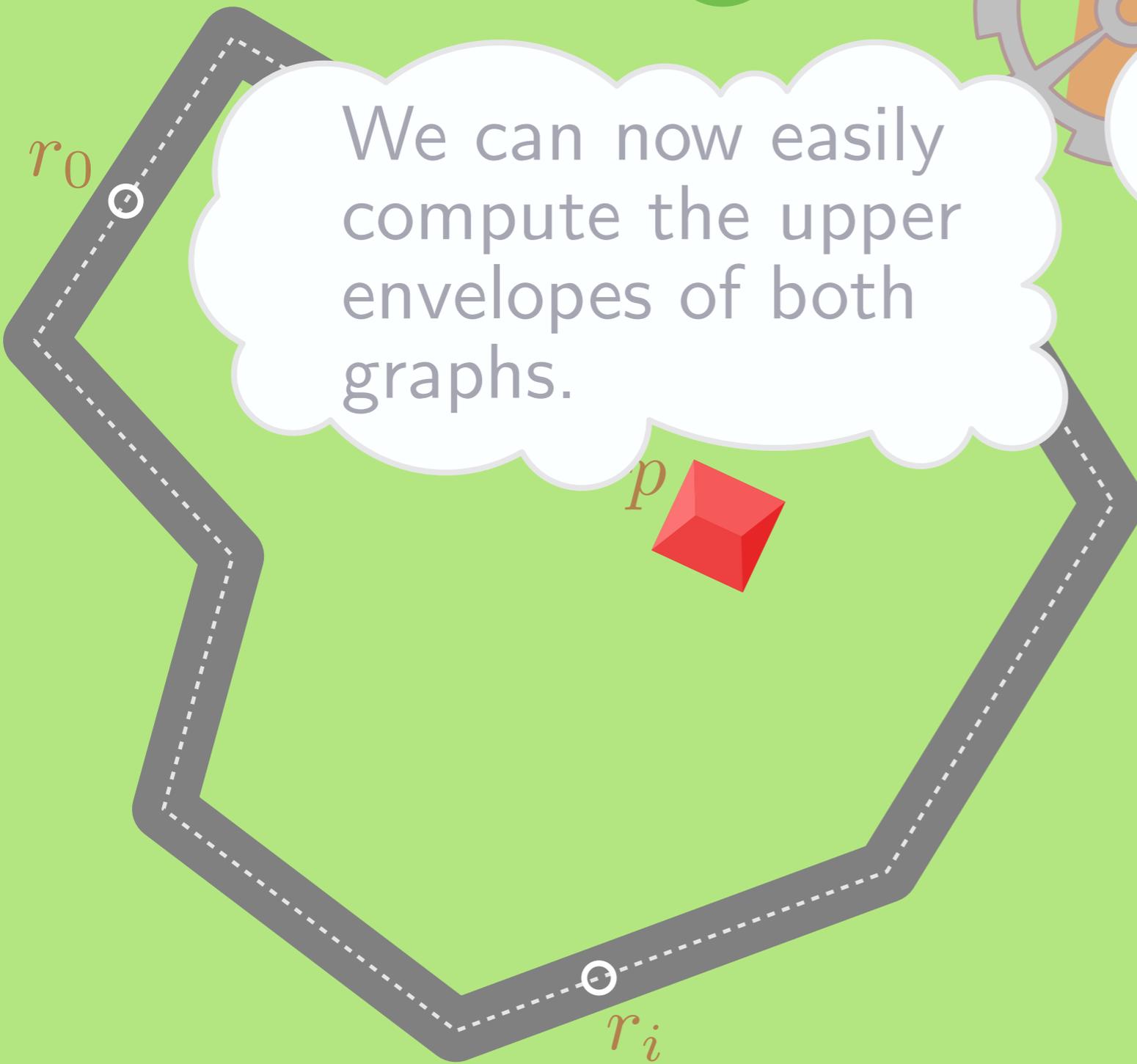


Now, we draw the graphs of $\delta_i(t)$ and $\delta_i(u)$.

Recall: $t = cw_0(q)$ and $u = ccw_0(q)$.

We can now easily compute the upper envelopes of both graphs.

We do the same for the other points.



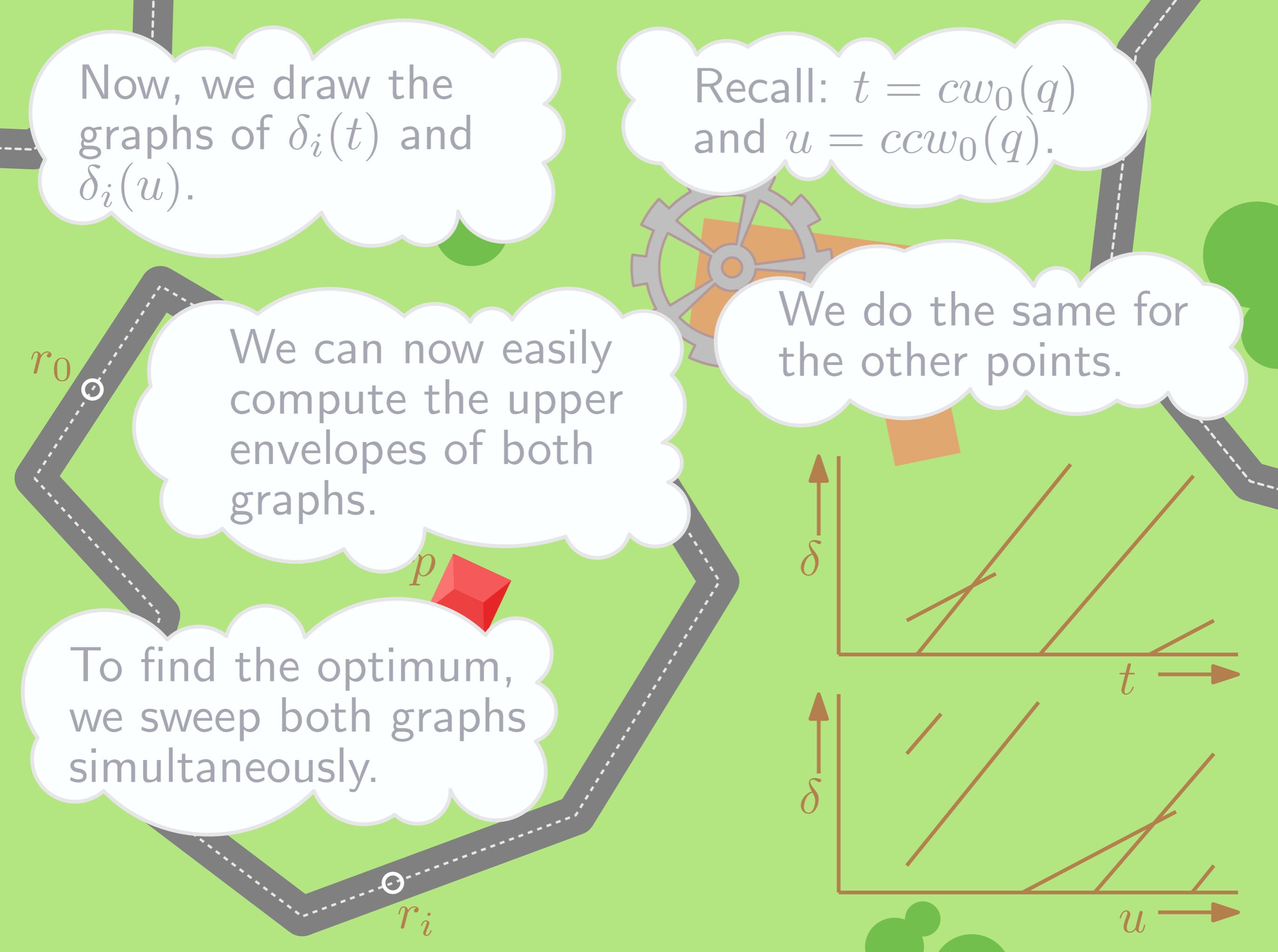
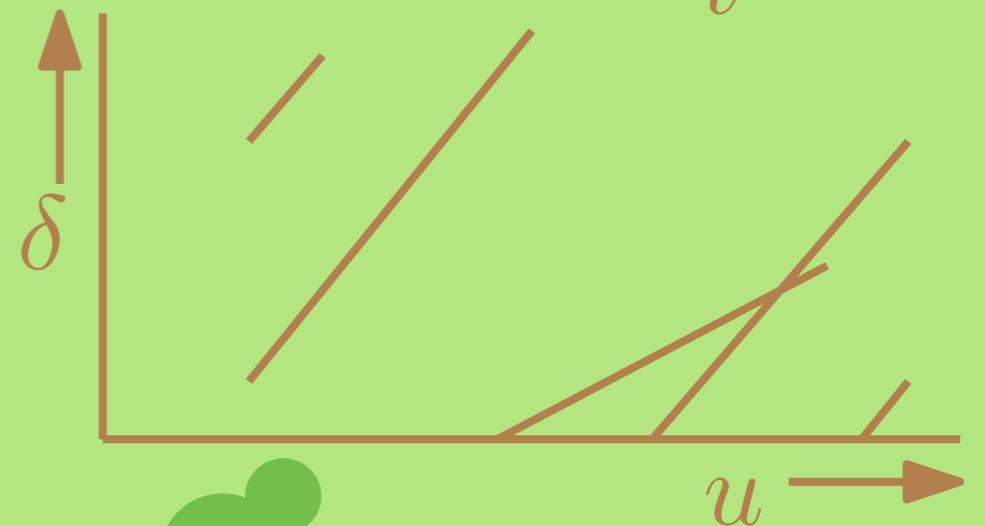
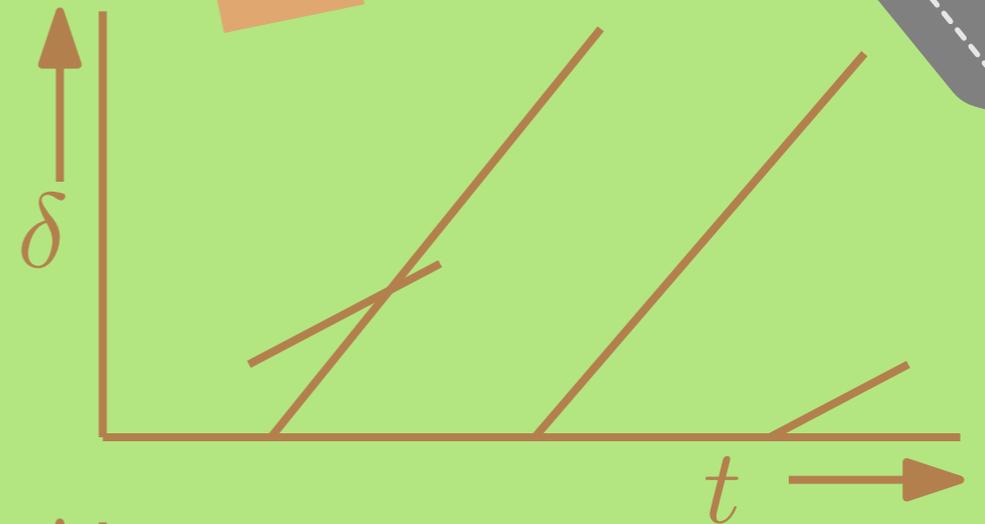
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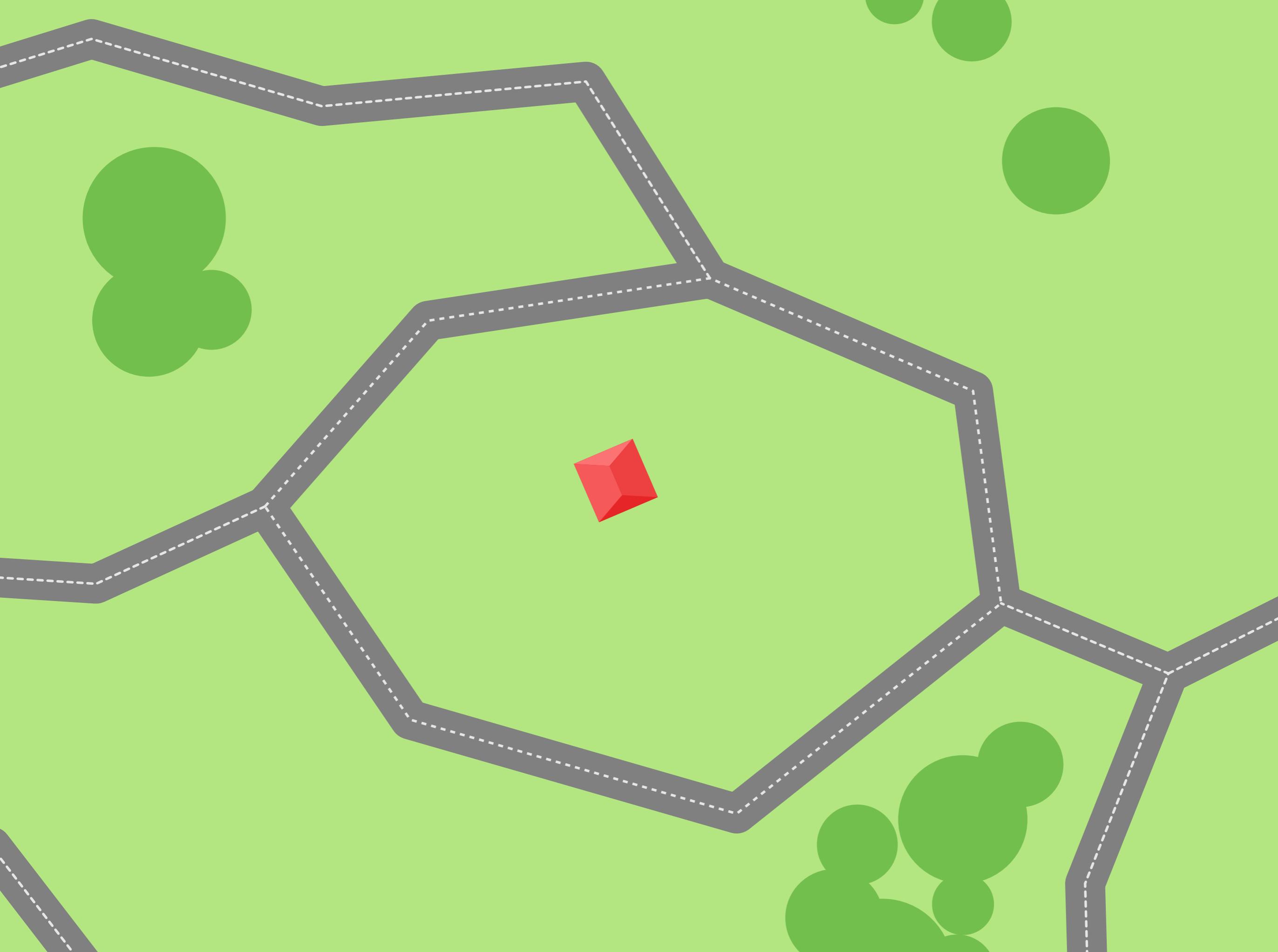
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We can now easily compute the upper envelopes of both graphs.

We do the same for the other points.

To find the optimum, we sweep both graphs simultaneously.





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the dilation is often
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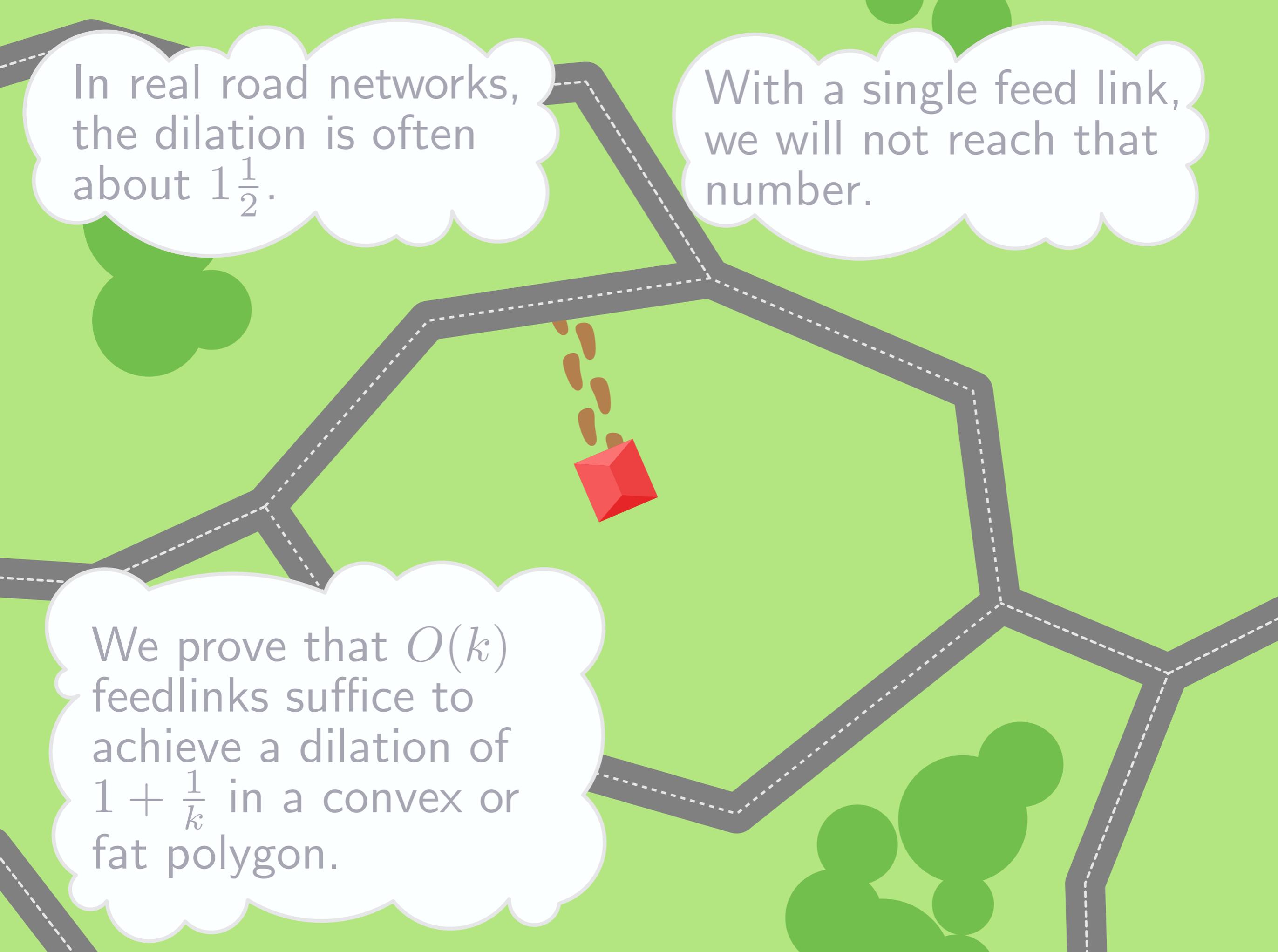
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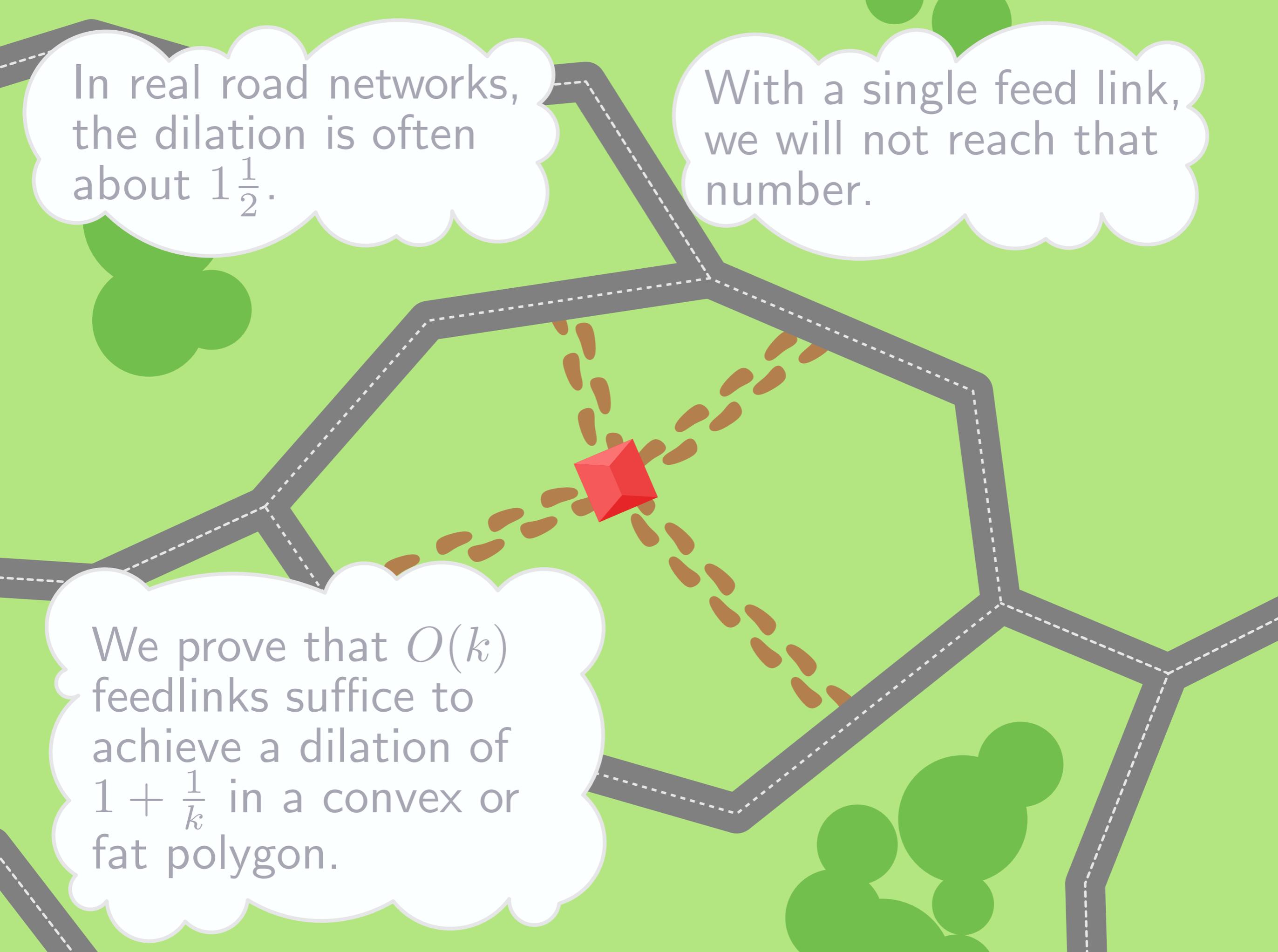


The diagram shows a network of grey roads with dashed white center lines. A red cube is positioned in the center, with a dashed brown path leading from it towards the network. The background is light green with stylized green trees.

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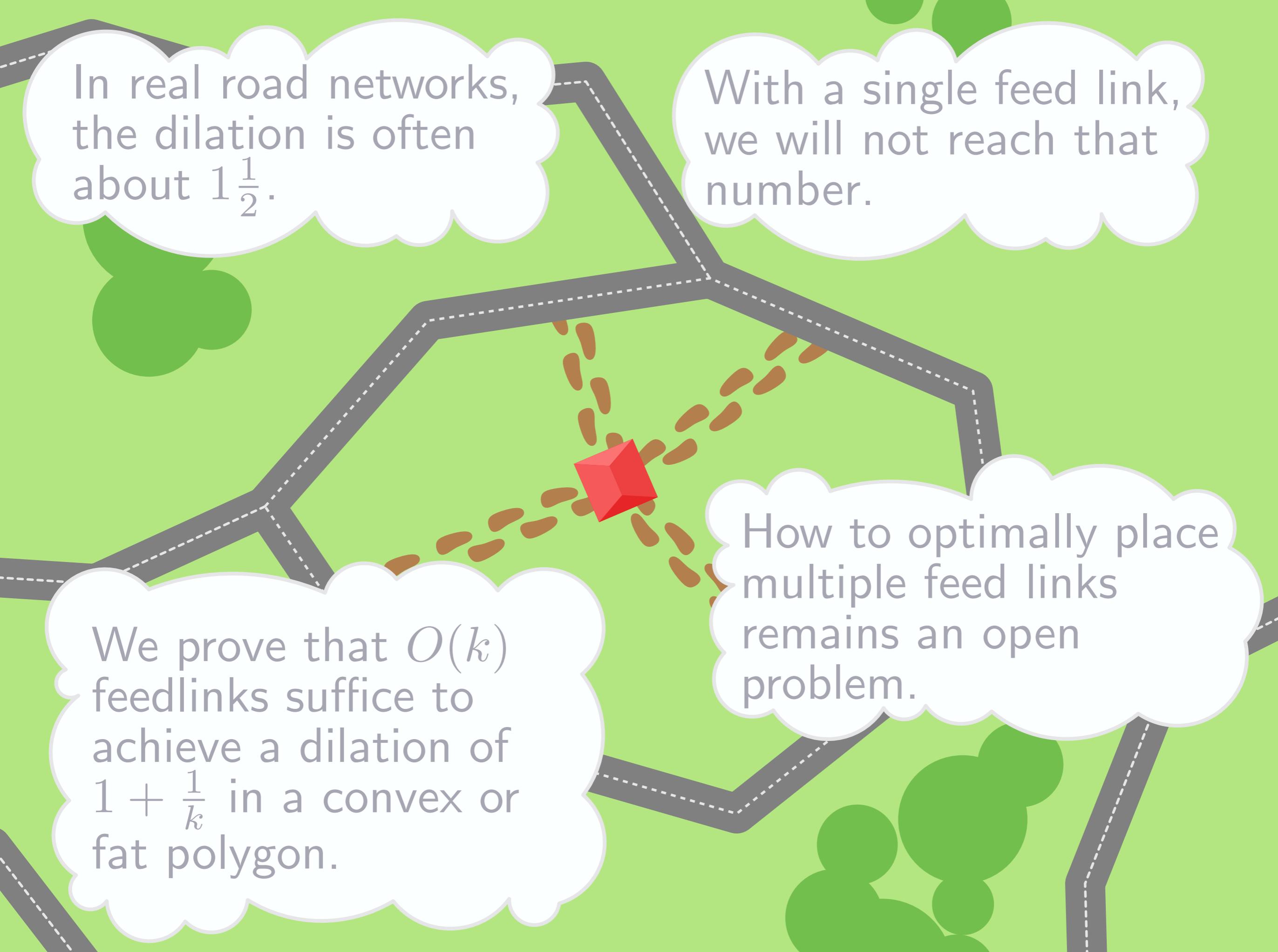
We prove that $O(k)$ feedlinks suffice to achieve a dilation of $1 + \frac{1}{k}$ in a convex or fat polygon.

The diagram shows a network of grey roads on a green background with stylized green trees. A red 3D cube is positioned in the center of the network. A dashed brown path starts from the cube and branches out to connect to several different road segments. The roads are represented by thick grey lines with dashed white lines inside them. The background is a light green color with several green circular shapes representing trees.

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With a single feed link, we will not reach that number.

We prove that $O(k)$ feedlinks suffice to achieve a dilation of $1 + \frac{1}{k}$ in a convex or fat polygon.

The diagram shows a network of grey roads with dashed white center lines on a green background with stylized green trees. A red 3D cube is positioned in the center of the network. A dashed brown path leads from the cube towards the top-left of the network. Four white thought bubbles with grey outlines contain text related to the network's dilation and feed links.

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With a single feed link, we will not reach that number.

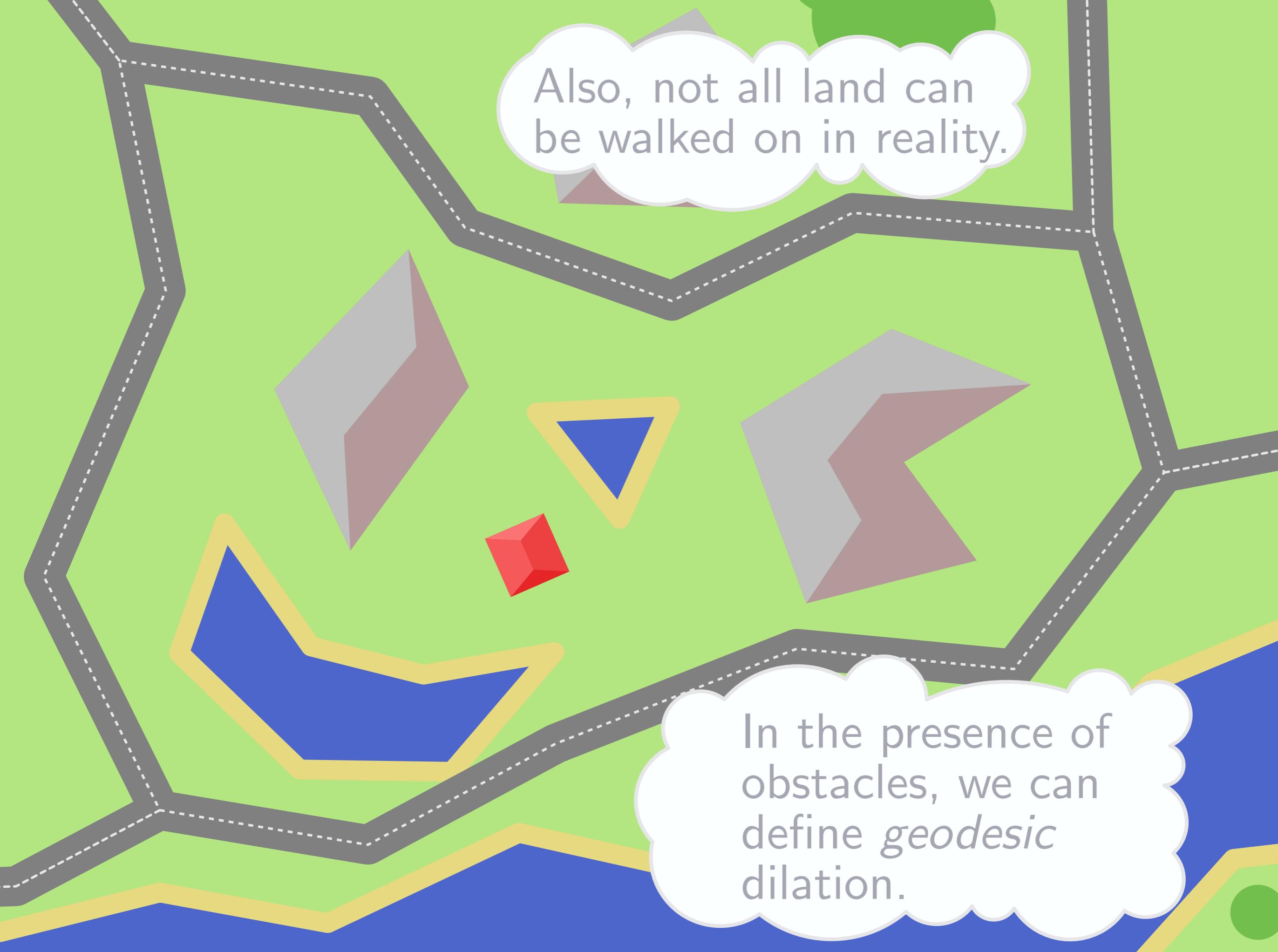
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How to optimally place multiple feed links remains an open problem.



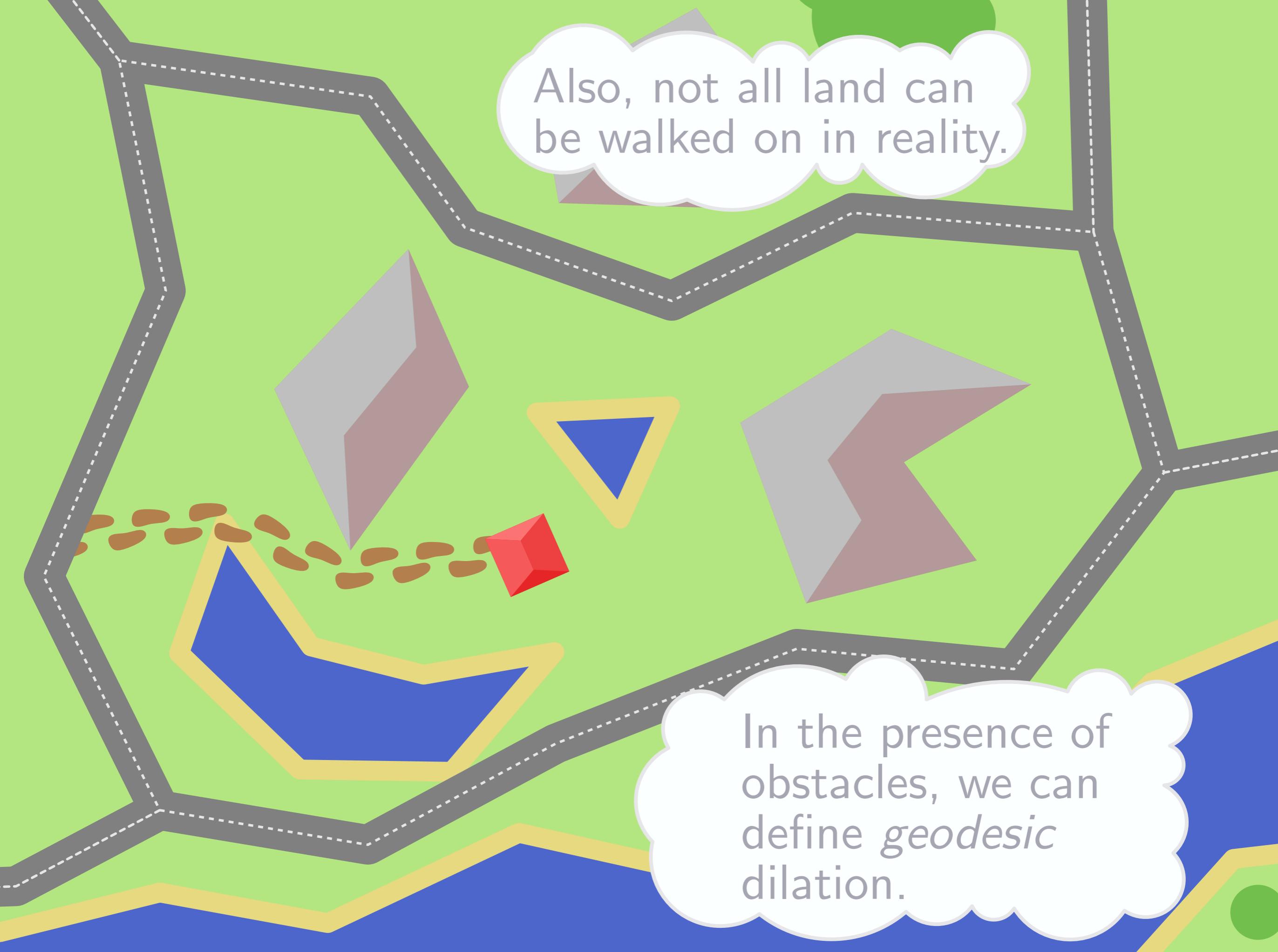
Also, not all land can be walked on in reality.





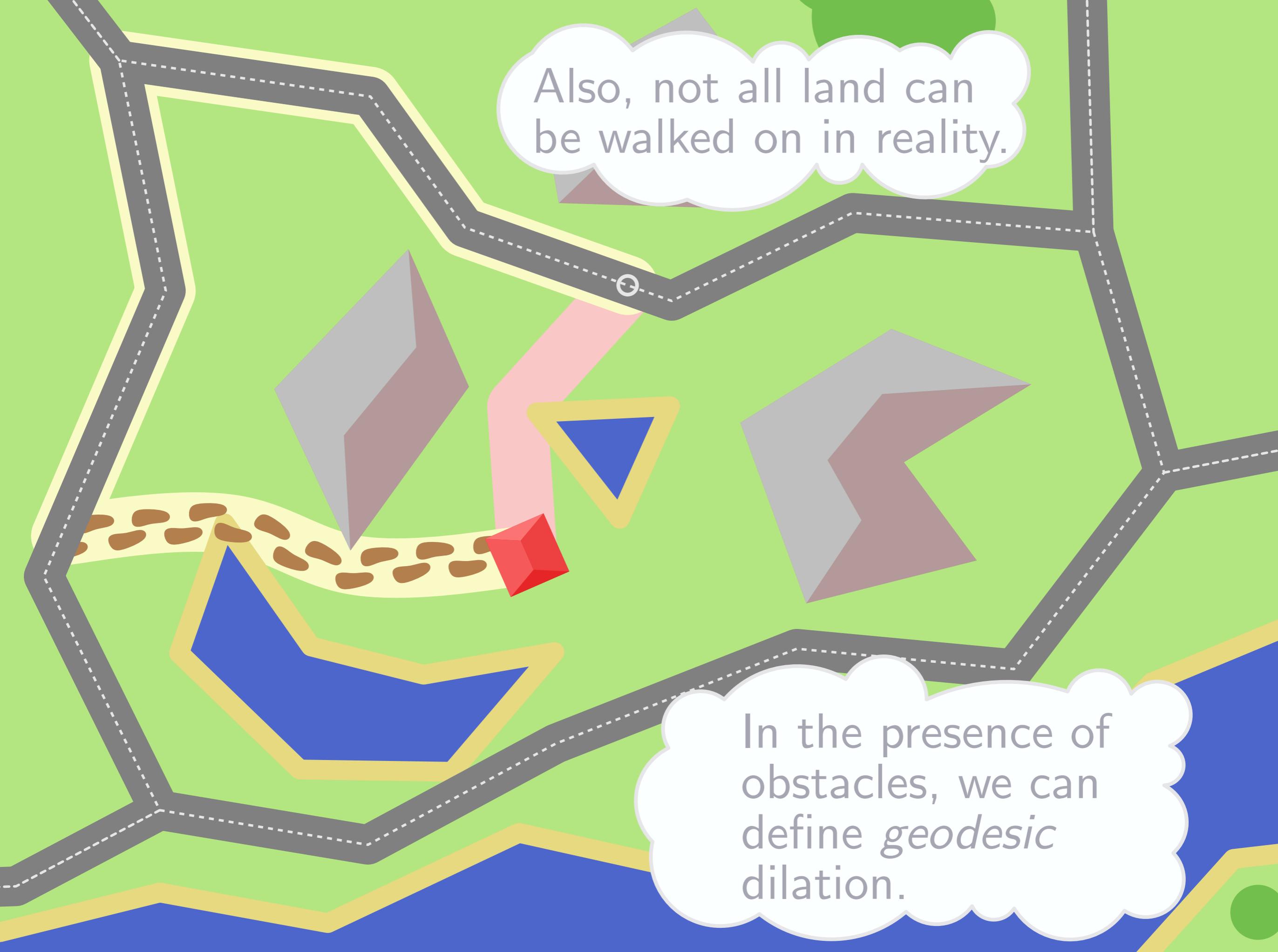
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Our results also generalise to such geodesic dilation.

In the presence of obstacles, we can define *geodesic* dilation.



A stylized landscape illustration. The background is a blue sky at the top. Below it is a green field with a grey road that has a dashed white center line. The road winds through the landscape. There are several red cubes scattered across the green field. In the center, there is a white, cloud-like shape containing the text "Thank you!". The overall style is simple and colorful.

Thank
you!

Boris
Aronov

Bettina
Speckmann

Maike
Buchin

Kevin
Buchin

Thank
you!

Jun
Luo

Rodrigo
Silveira

Marc
van Kreveld

Maarten
Löffler