Region-based approximation algorithms for visibility between imprecise locations

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ACT I

Region-based approximation what?

Problem statement

Compute the probability that two random points can see each other.

Monkey behaviour

Do wandering monkey groups alter their course when they see each other?

Convexity measure

Can we parameterise the extent to which a polygon is convex?

[Eric Willems, 2013]
ACT I

REGION-BASED APPROXIMATION WHAT?
PROBLEM STATEMENT

COMPUTE THE PROBABILITY THAT TWO RANDOM POINTS CAN SEE EACH OTHER

CONVEXITY \( M \)
MONKEY BEHAVIOUR

DO WANDERING MONKEY GROUPS ALTER THEIR COURSE WHEN THEY SEE EACH OTHER?

ASURE

THE EXTENT CONVEX?

[ERIK WILLEMS, 2013]
CONVEXITY MEASURE

CAN WE PARAMETERISE THE EXTENT TO WHICH A POLYGON IS CONVEX?

[GUENTER ROTTE, 2012]
ACT II

WAIT, ISN'T THAT A SOLVED PROBLEM?

POINT SABRATION

JUST TRY TWO AND TEST THEIR RANDOM HATS
POINT SAMPLING

JUST TRY TWO RANDOM POINTS, AND TEST THEIR VISIBILITY!

(AND THEN DO IT AGAIN, AND AGAIN.)

ACKS
ILLUSTRATION

PLEASE CHECK YOUR SEAT FOR ANY RANDOM HATS
DRAWBACKS

- YOU NEED A RELIABLE SOURCE OF RANDOMNESS
- YOU NEED TO TAKE A LOT OF SAMPLES
- MOST LIKELY YOU STILL WON'T GET THE RIGHT ANSWER
- YOU JUST NEVER KNOW FOR SURE!
YOU NEED TO TAKE A LOT OF SAMPLES
MOST LIKELY YOU STILL WON'T GET THE RIGHT ANSWER
YOU JUST NEVER KNOW FOR SURE!

ACT III
A FUNDAMENTAL APPROACH, WITH INTEGRALS AND STUFF

BLEM
POLYGONS OF COLLECTION
FORMAL PROBLEM

Given two convex polygons of complexity $N$ and a collection of obstacles of complexity $M$, compute the probability that two points, taken from both polygons, are mutually visible.
VARIANTS

SOME POLYGONS OVERLAP.
OTHERS DON'T.
POINT-LINE DUALITY

WE WANT TO ARGUE ABOUT THE SPACE OF ALL LINES THROUGH TWO POLYGONS
THE CALCULATION

\[ I = \iiint_C \left( \frac{x_2(\alpha, \beta) \cdot x_4(\alpha, \beta)}{x_1(\alpha, \beta) \cdot x_3(\alpha, \beta)} \right) (x_2 - x_1) \, dx_2 \, dx_1 \, d\alpha \, d\beta = \iiint_C \frac{(x_2 - x_1)}{C} \, dx_2 \, dx_1 \right) \, d\alpha \, d\beta \]

\[ I_{ij} = \iiint_C x_i(x_1) \cdot x_j(x_1) \, dx_1 \, d\alpha \, d\beta = \iiint_C \frac{(b_i \beta - c_i)(b_j \beta - c_j)}{(b_i \alpha + a_i)(b_j \alpha + a_j)^2} \, dx_1 \, d\alpha \, d\beta = \sum_i \sum_j \]

\[ F_{ij}(x) = \sum_i \sum_{A_i \alpha + B_i} \frac{(b_i \beta - c_i)(b_j \beta - c_j)}{(b_i \alpha + a_i)(b_j \alpha + a_j)^2} \, dx = \frac{\log(a_i + a_j \beta)}{12 b_i^2 b_j^2} \left[ u(A_i \alpha + B_i b_j^2) + \frac{1}{2 u b_i b_j^2} \left[ 3 x^2 \right] + \cdots \right] \]
\[= \iint_{\mathcal{C}} \frac{(x_2-x_1)(x_4-x_3)(x_3+x_4-x_1-x_2)}{2} \, dx \, dy = \frac{1}{2} \iint_{\mathcal{C}} (-x_1^2 + x_2^2) \, dx = \sum_{\mathcal{C}_v \subset \mathcal{C}} \int_{\alpha_1} \int_{\alpha_2} \frac{(b_i \beta - c_i)(b_j \beta - c_j)}{(b_i \alpha + a_i)(b_j \alpha + a_j)} \, dx \, dy \]

\[= \sum_{\mathcal{C}_v \subset \mathcal{C}} \int_{\alpha_1} \int_{\alpha_2} \left( \frac{A_2 x + B_2}{A_1 \alpha + B_1} \right) \, dx \, dy \]

\[+ \frac{\alpha b_i}{b_i^2} \left[ u \left( A_2^2 - A_1^2 \right) (a_j b_i - a_i b_j) (b_i c_j - b_j c_i) + 3 \left( A_2^u - A_1^u \right) \right] \]

\[+ \frac{\alpha b_j}{b_j^2} \left[ 3 \alpha^2 \left( A_2^u - A_1^u \right) b_i b_j + \alpha \left( (A_2^u - A_1^u) (12 a_j b_i - 6 a_i b_j) \right) \right] \]
\[ I_{ij} = \iint_C x_i x_j \, dx \, dy = \iint_C \frac{\beta_i \beta_j - \beta_i - \beta_j + \beta_i \beta_j}{\beta_i + \beta_j} \, dx \, dy = \sum \left( \frac{\beta_i \beta_j - \beta_i - \beta_j + \beta_i \beta_j}{\beta_i + \beta_j} \right) \, dy \, dx \]

\[ F_{ij}(x) = \int \left( \frac{\beta_i \beta_j - \beta_i - \beta_j + \beta_i \beta_j}{\beta_i + \beta_j} \right) \, dx \, dy = \log(\beta_i + \beta_j) \]

\[ + \frac{1}{2} \left( \frac{3 \beta_i^2}{2} + \frac{3 \beta_j^2}{2} \right) \]
THEOREM

THE DESIRED PROBABILITY CAN BE COMPUTED IN TIME

$O(M \cdot (M+N)^2)$

ACT IV
THEOREM

THE DESIRED PROBABILITY CAN BE COMPUTED IN TIME $O(M \cdot (M+N)^2)$

ACT IV

SURELY WE CAN DO BETTER?
SEGMENT SPACE

CONSIDER THE SPACE OF LINE SEGMENTS WHOSE END POINTS TOUCH AN OBSTRUCTION
LAYERS OF SEGMENTS

Each line may consist of $M$ maximal segments, adding a discrete dimension to the segment space.
THEOREM

THE DESIRED PROBABILITY CAN ALSO BE COMPUTED IN JUST $O((M+N)^2)$ TIME
THE DESIRED PROBABILITY CAN ALSO BE COMPUTED IN JUST $O((M+N)^2)$ TIME

ACT

BUT WHAT ABOUT REAL DISTRIBUTIONS?

DON'T
GAUSSIANS

REAL RANDOM POINTS DON'T HAVE UNIFORM DISTRIBUTIONS

FOR NORMAL DISTRIBUTIONS, OUR INTEGRAL HAS NO CLOSED-FORM SOLUTION!
APPROXIMATION BY DISKS

WE CAN APPROXIMATE A GAUSSIAN DISTRIBUTION WITH A BUNCH OF UNIFORM DISTRIBUTIONS ON CONCENTRIC DISKS

WITH POLYGONS
APPROXIMATION WITH POLYGONS

AND THEN, WE CAN IN TURN APPROXIMATE THE DISKS BY REGULAR POLYGONS
THEOREM

A normal distribution can be $\varepsilon$-approximated with polygons of total complexity $O(\sqrt{\varepsilon})$.
ACT VI

ISN'T THIS SUPPOSED TO BE AN EXPERIMENTAL PAPER?
SETTINGS

WE PERFORMED TESTS IN A FOREST AND AN URBAN ENVIRONMENT
THE END

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