



HOW TO FIT A TREE IN A BOX

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PART I  
TREES & DRAWINGS

# DRAWINGS OF TREES



TREE



ROOTED TREE



PLANE DRAWING



UPWARD DRAWING

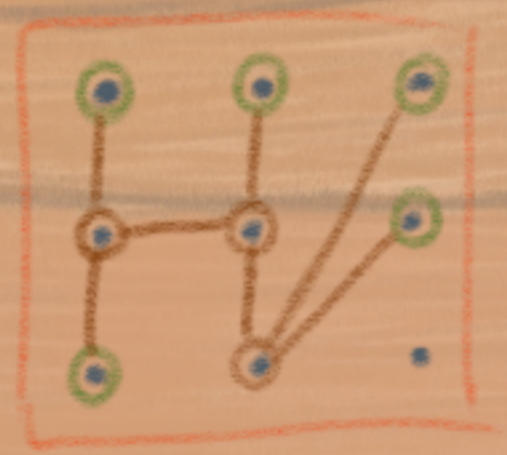
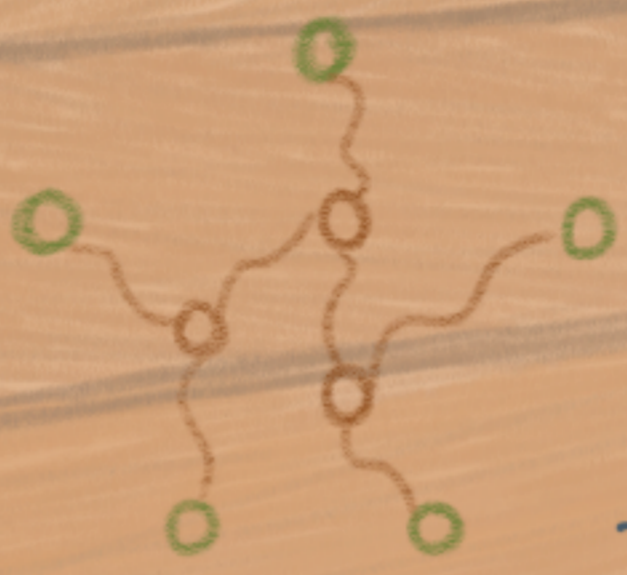


GRID DRAWING

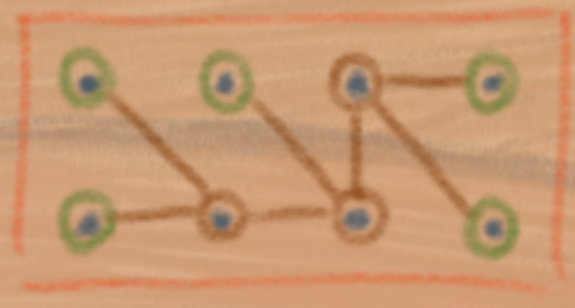


PLANE GRID DRAWING

# COMPACT PLANE GRID DRAWINGS



3x3



4x2



4x4  
(UPWARD)

## KNOWN RESULTS

EVERY TREE HAS A DRAWING WITH

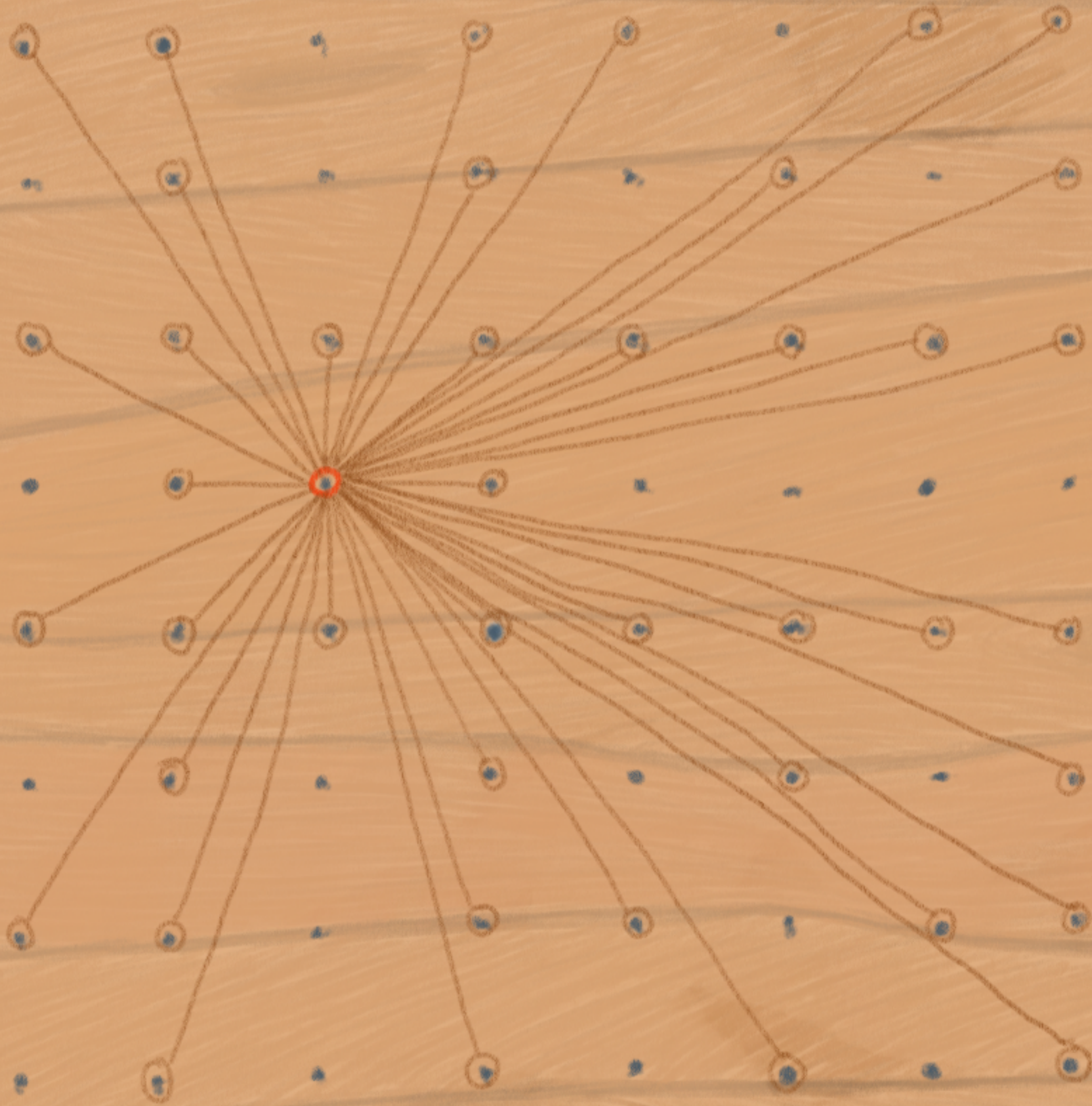
- $O(n \log n)$  AREA [FOLKLORE]
- $n \cdot 2^{O(\sqrt{\log \log n \log \log \log n})}$  [CHAN 2018]

EVERY TREE OF DEGREE  $< \sqrt{n}$  HAS DR. WITH

- $O(n)$  AREA [GARG & RUSU 2004]

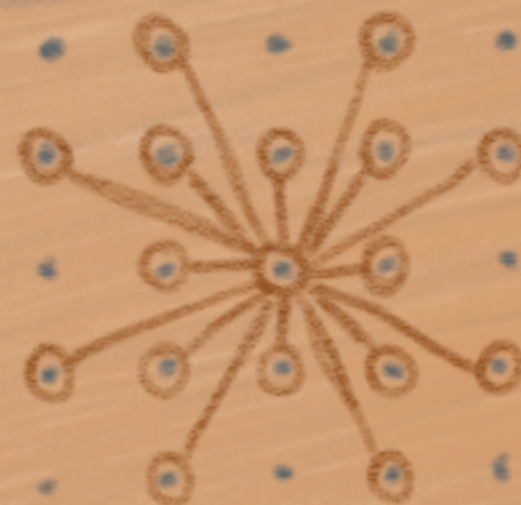
UPWARD DRAWINGS REQUIRE

- $\Theta(n \log n)$  AREA [CRES., DiBAT. & PIP. 1992]
- $O(n)$  AREA FOR PERFECTLY BALANCED TREES
- $O(n \sqrt{\log n} (\log \log n)^c)$  AREA FOR WEAK. [CHAN 2018]
- TESTING IS NP-HARD [BIEDL & MONDAL 2017]



# BINARY TREES

HIGH-DEGREE TREES SOMETIMES NEED  
A LARGE AREA



DEGREE-2 TREES CAN BE EMBEDDED  
VERY COMPACTLY



WHAT ABOUT DEGREE-3 TREES?

PART II

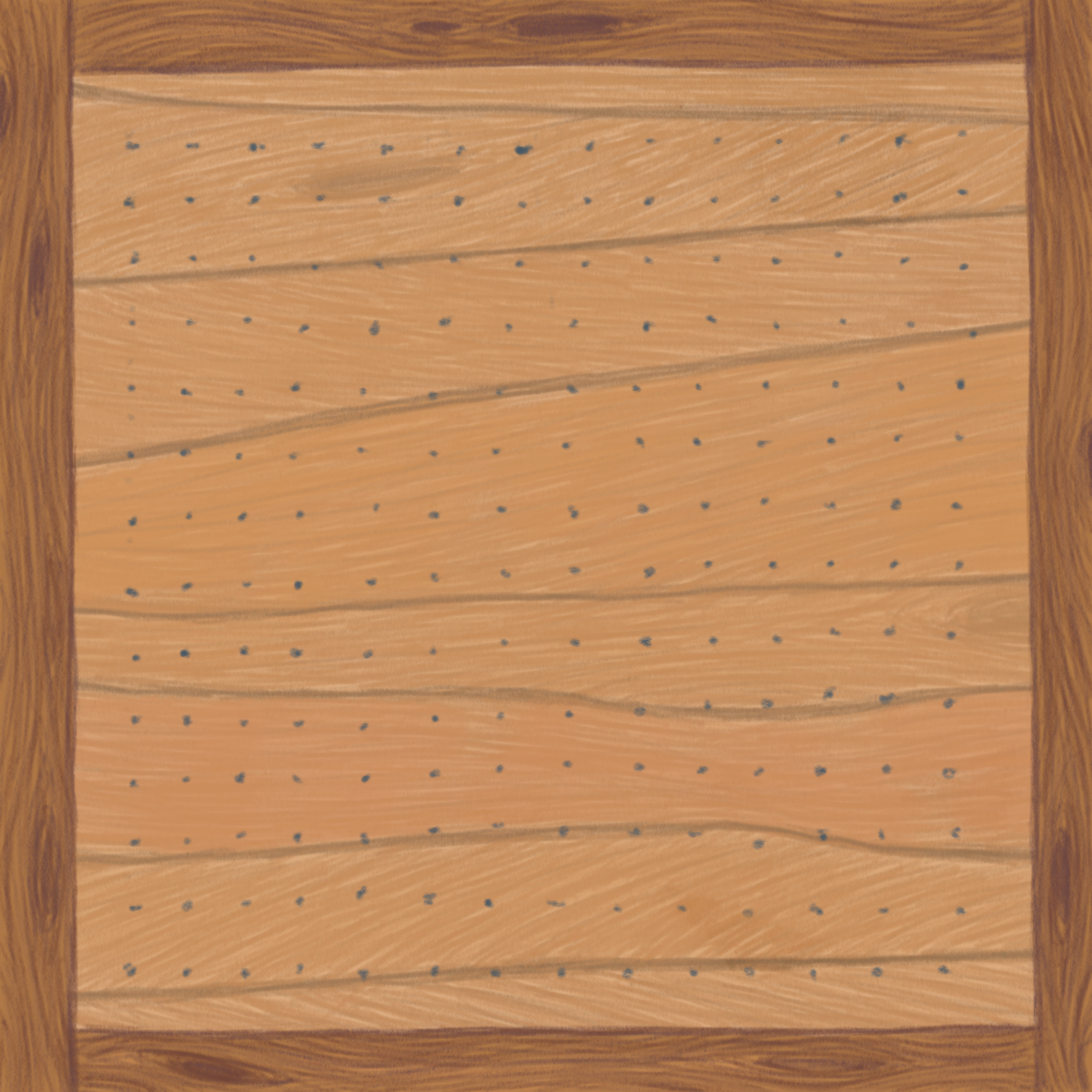
UPWARD TREES

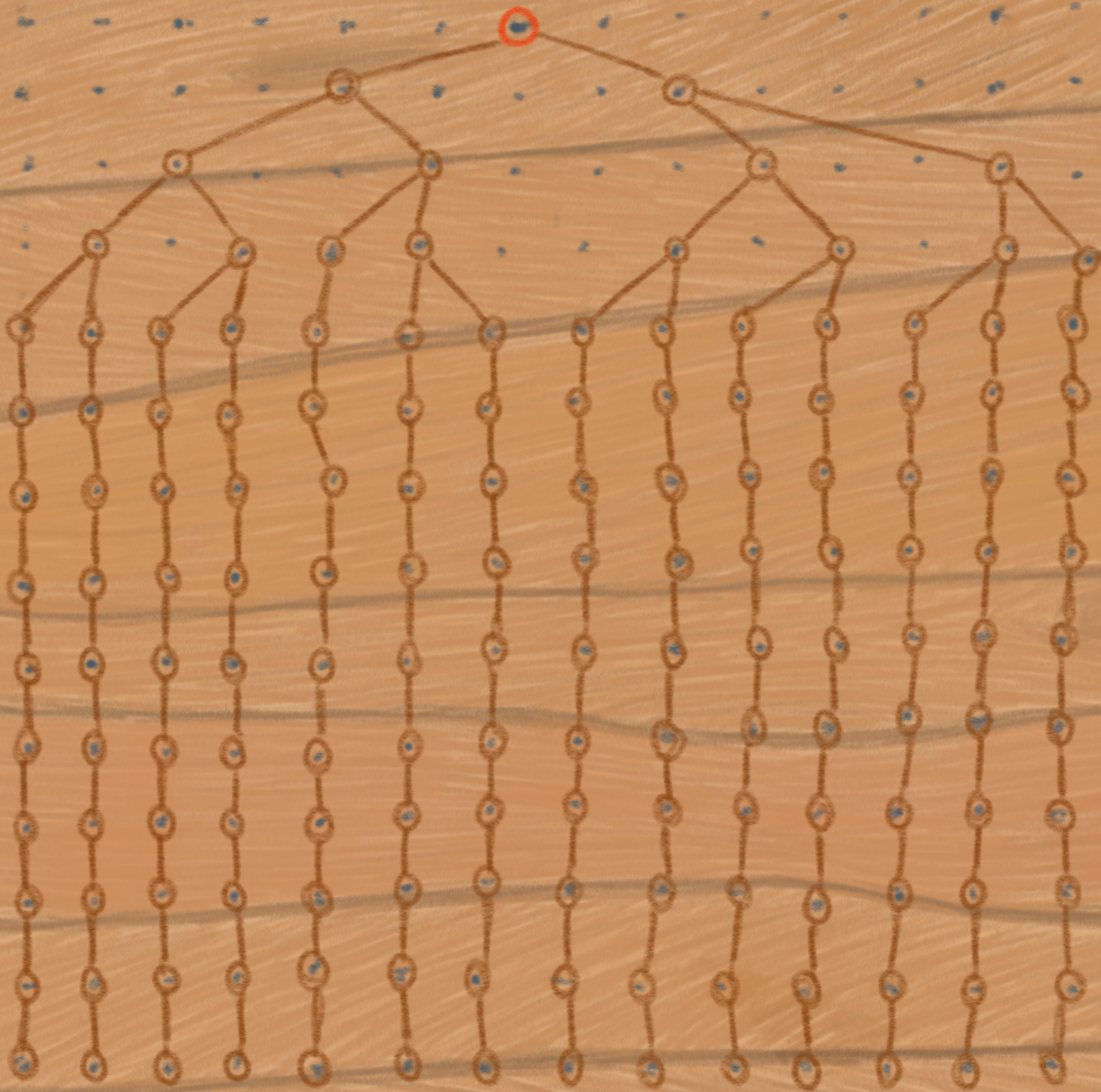


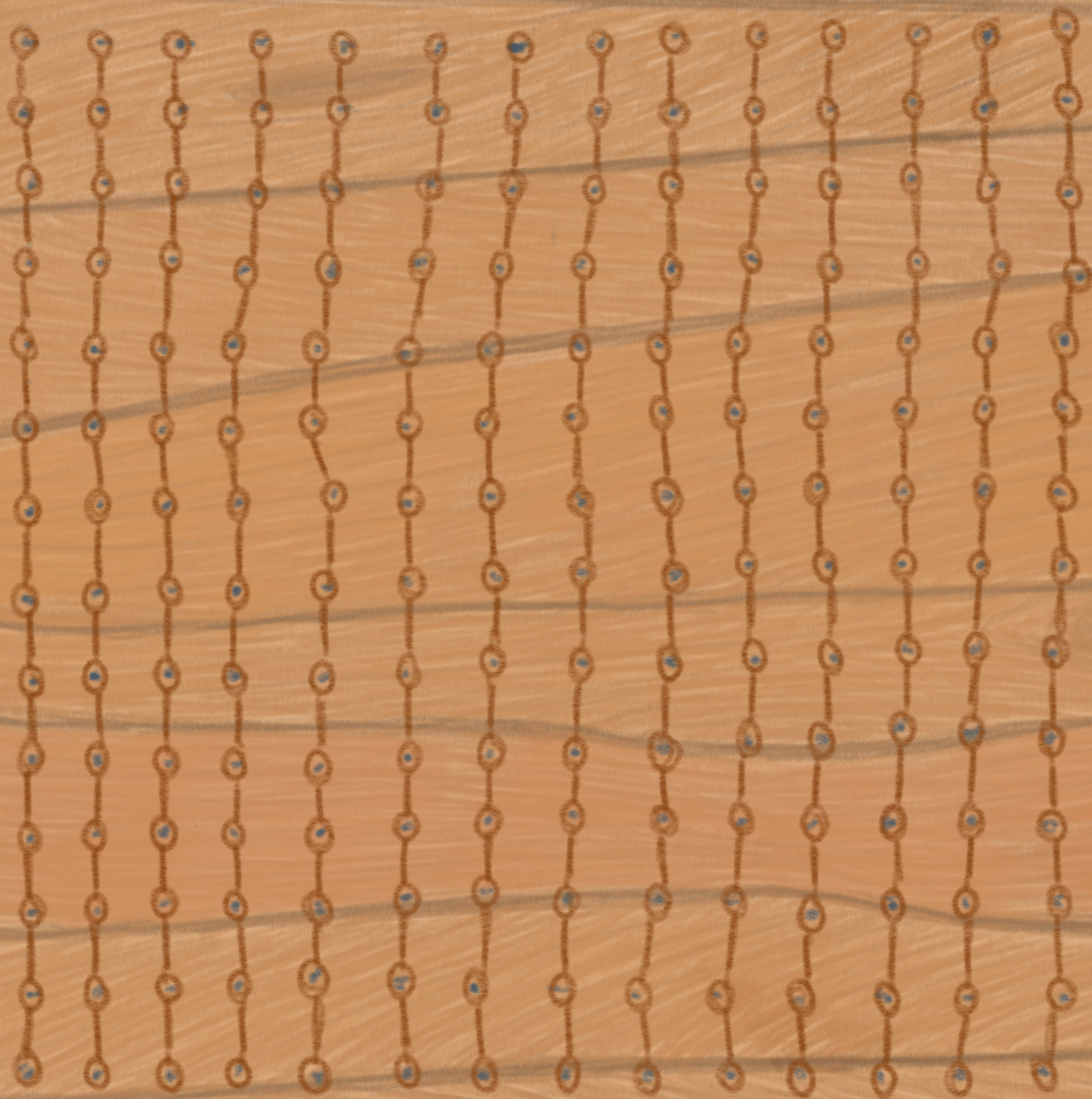
## THEOREM

GIVEN A ROOTED BINARY TREE  $T$   
WITH A GIVEN COMBINATORIAL EMBEDDING  
AND A FIXED  $(w \times h)$  GRID  $G$ ,

DECIDING WHETHER A CORRESPONDING  
UPWARD PLANAR EMBEDDING OF  $T$  ONTO  $G$   
EXISTS IS NP-COMPLETE

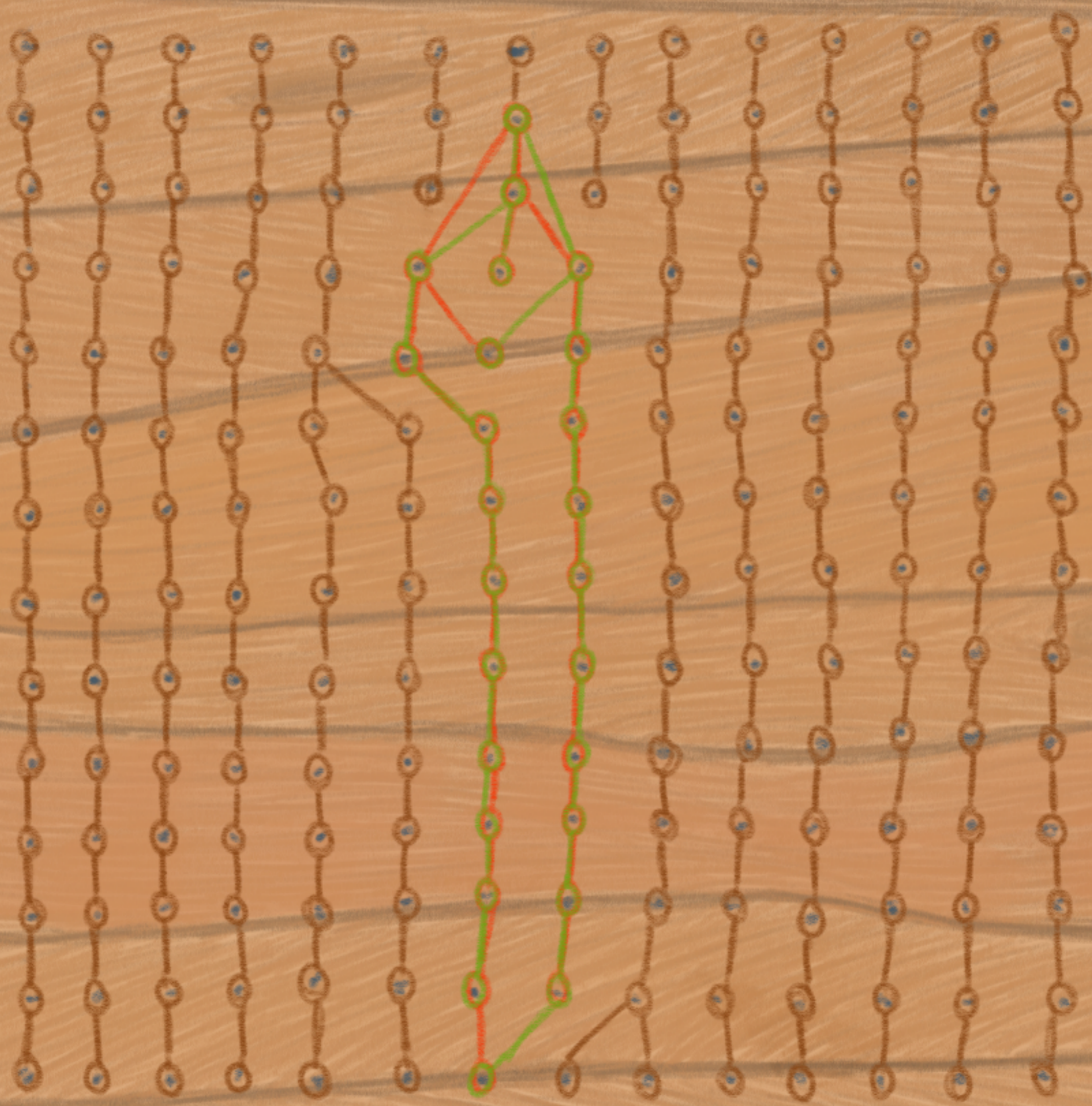


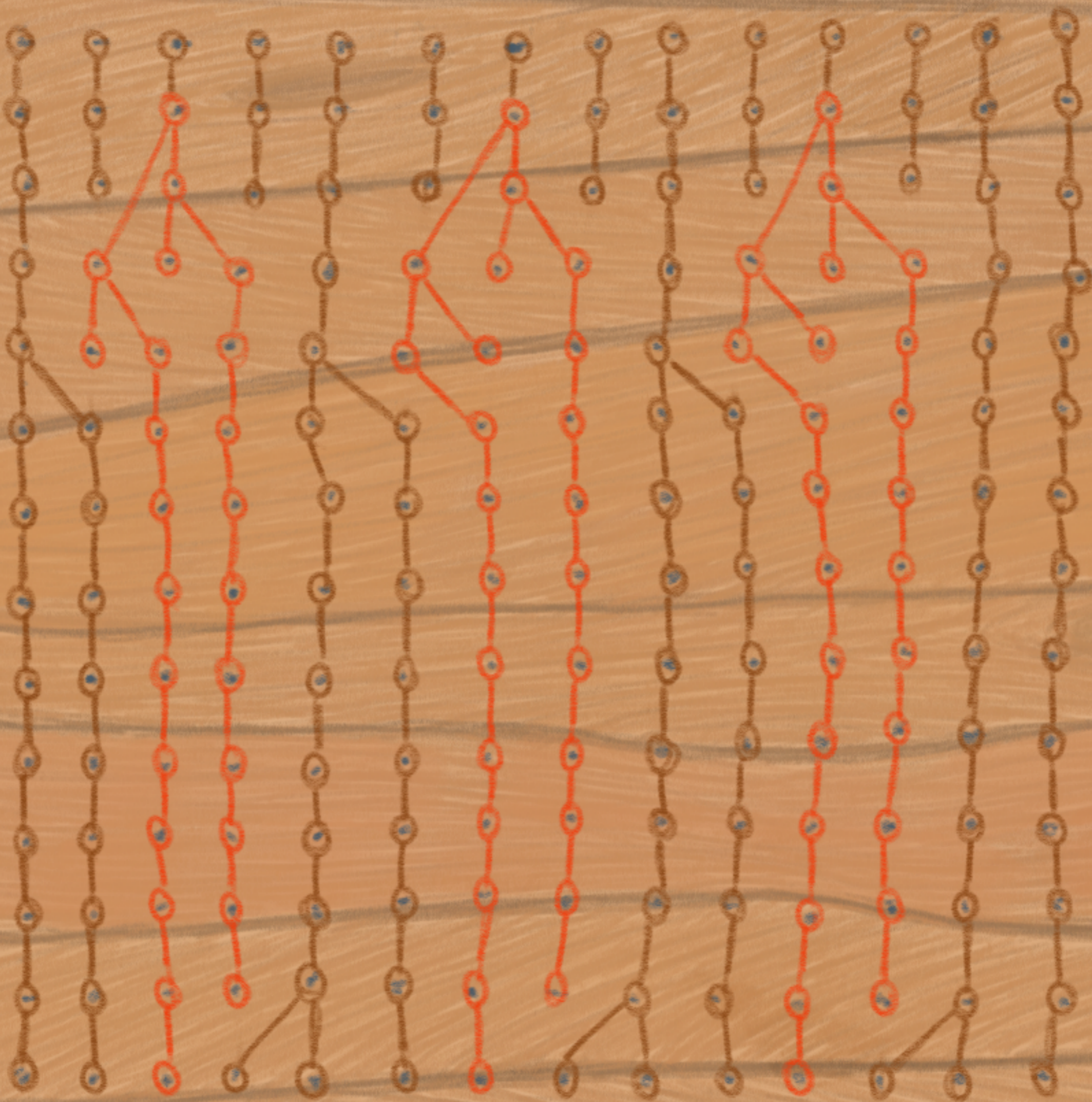




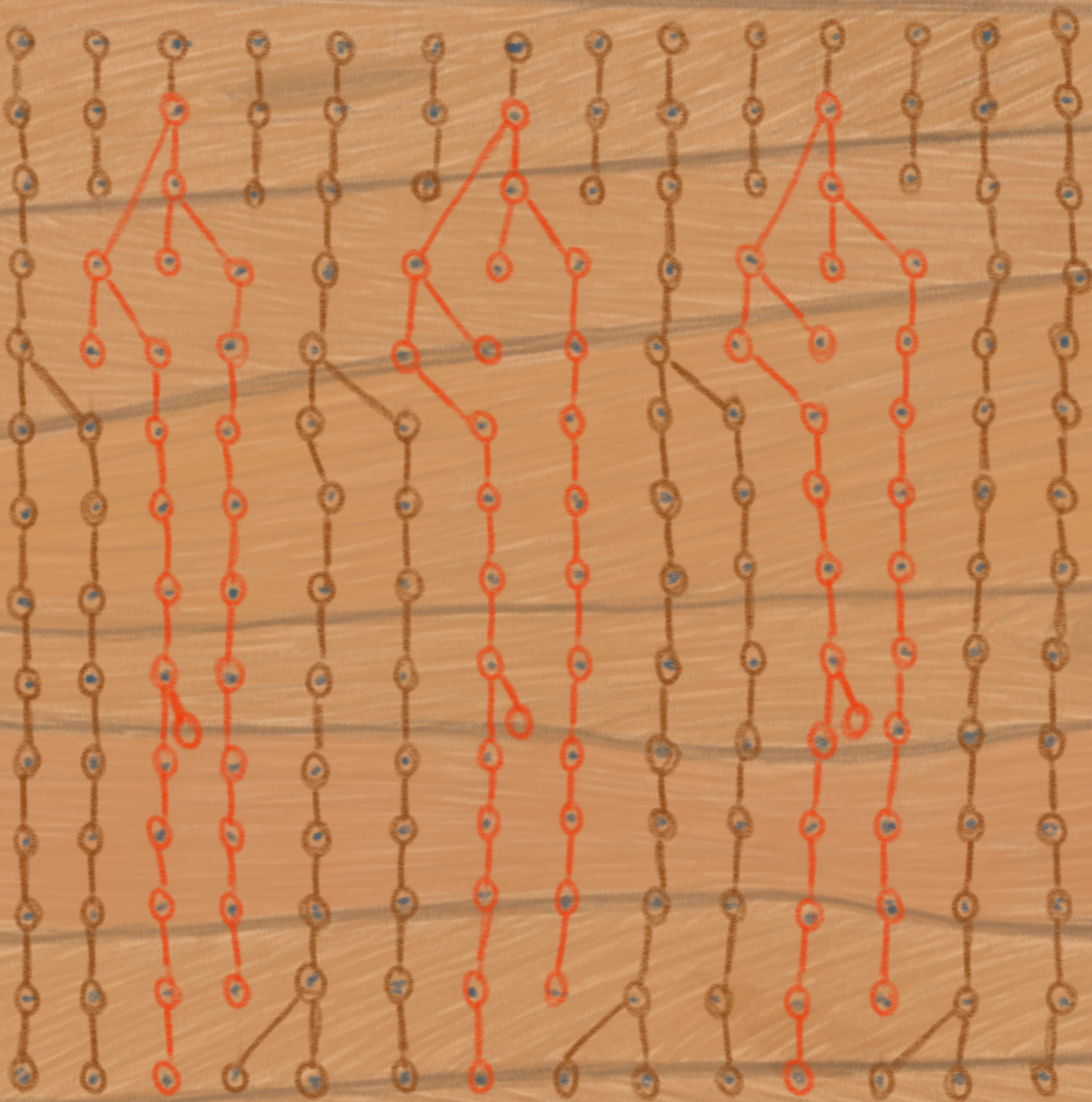


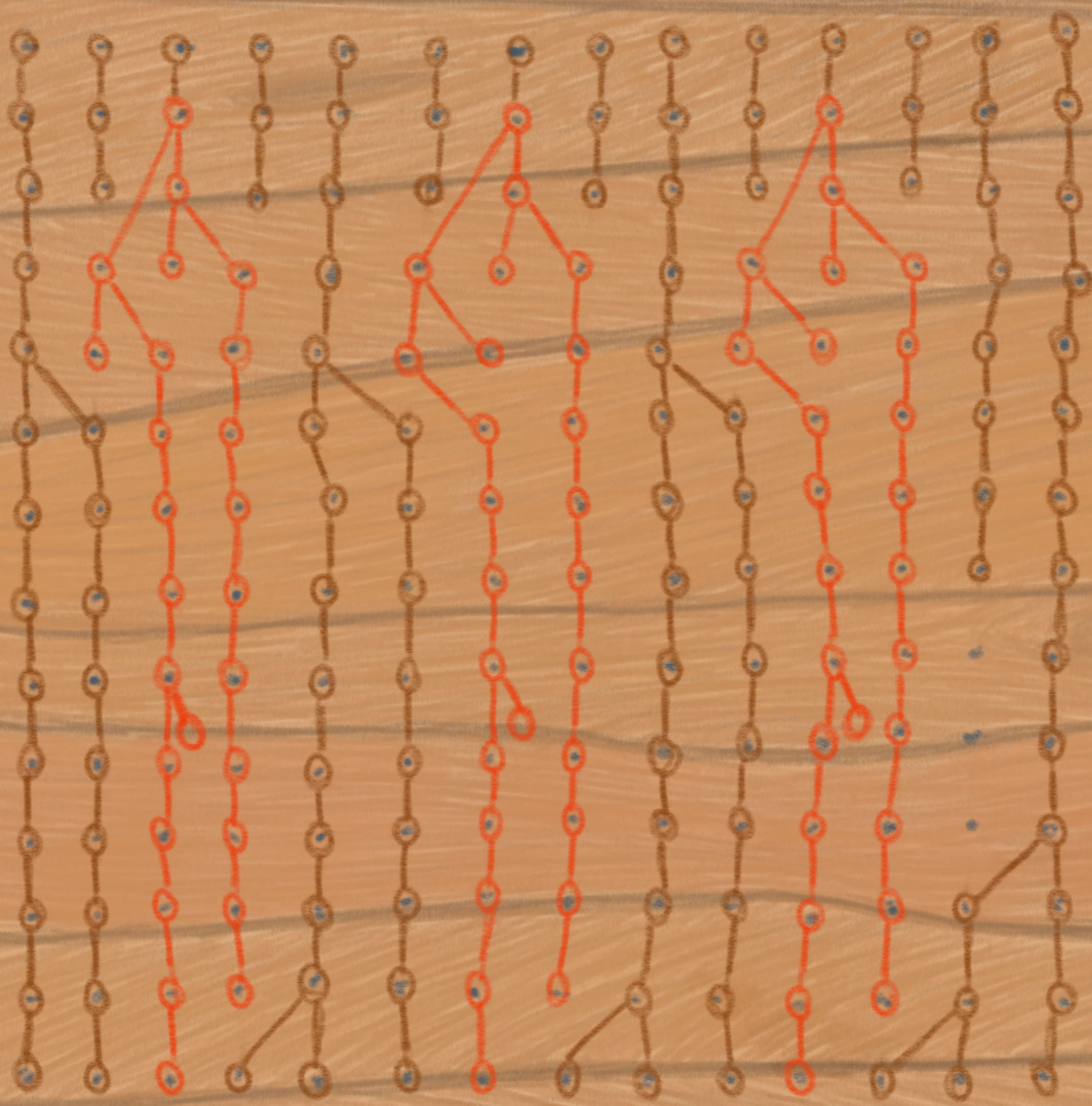


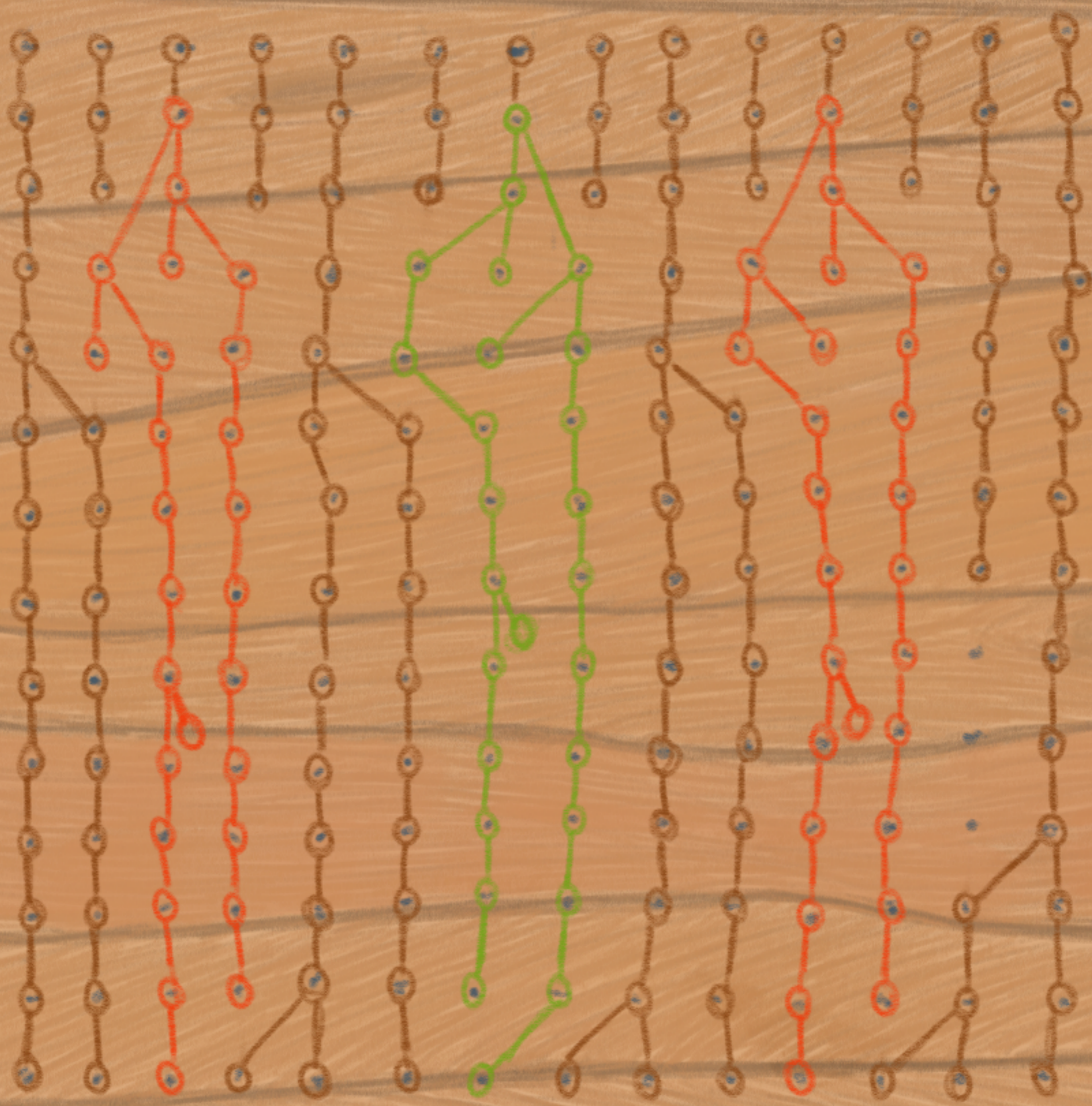


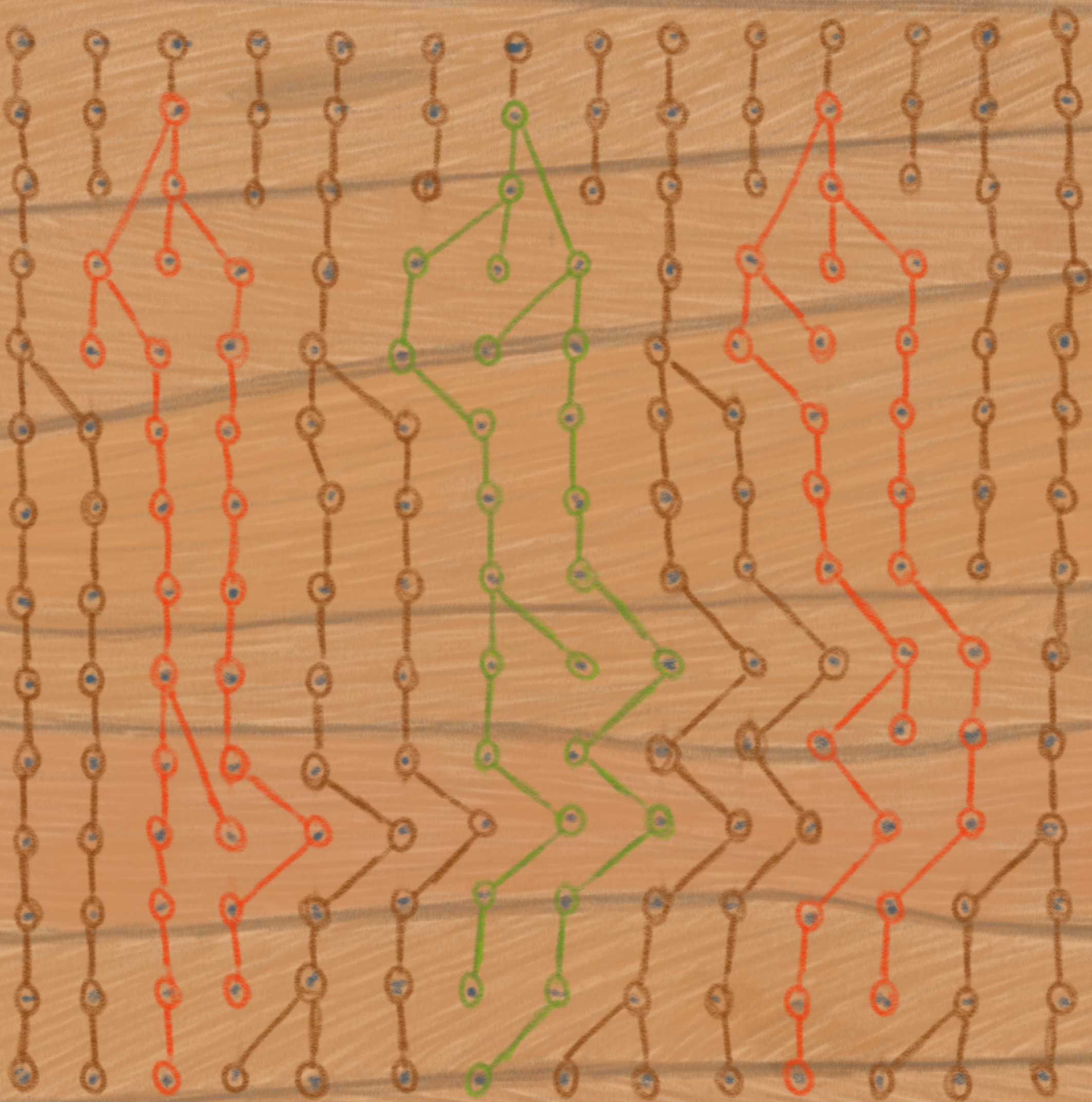










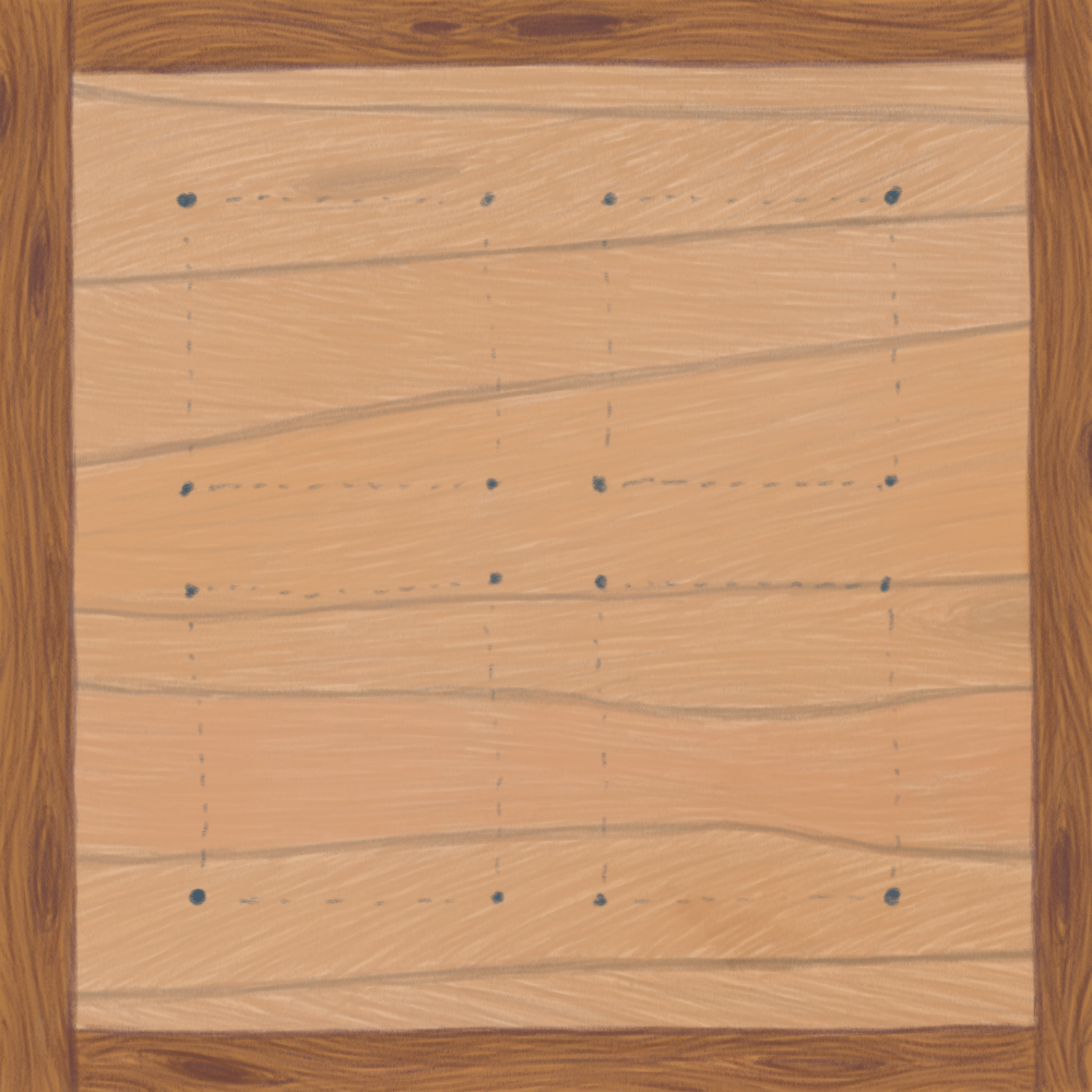


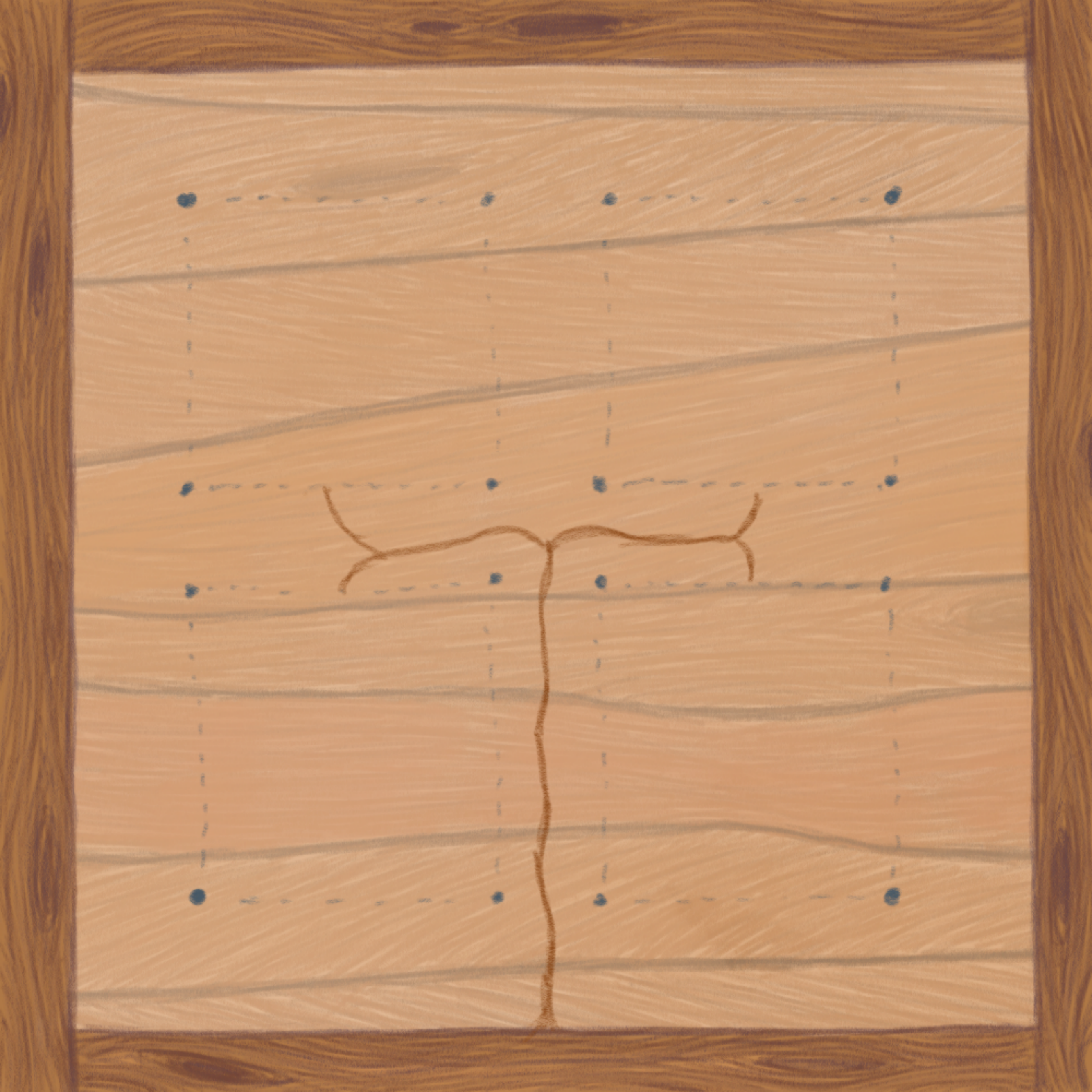
The background of the image is a wood-grain texture, consisting of horizontal, wavy lines in various shades of brown and tan. The texture is most prominent in the top and bottom sections, where the lines are more defined and darker. In the center, there is a lighter, more uniform area where the text is located.

PART III  
BALANCED TREES

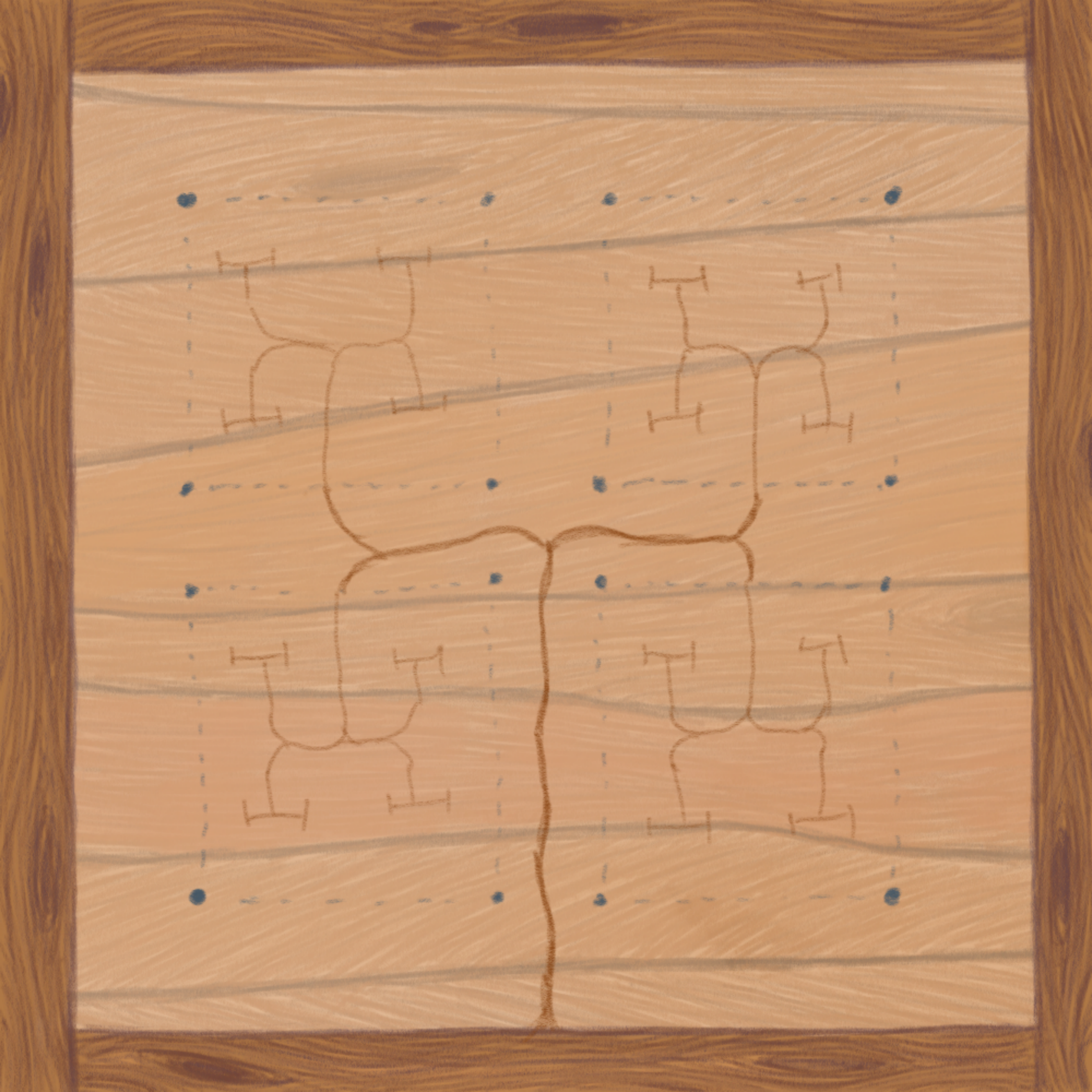
## THEOREM

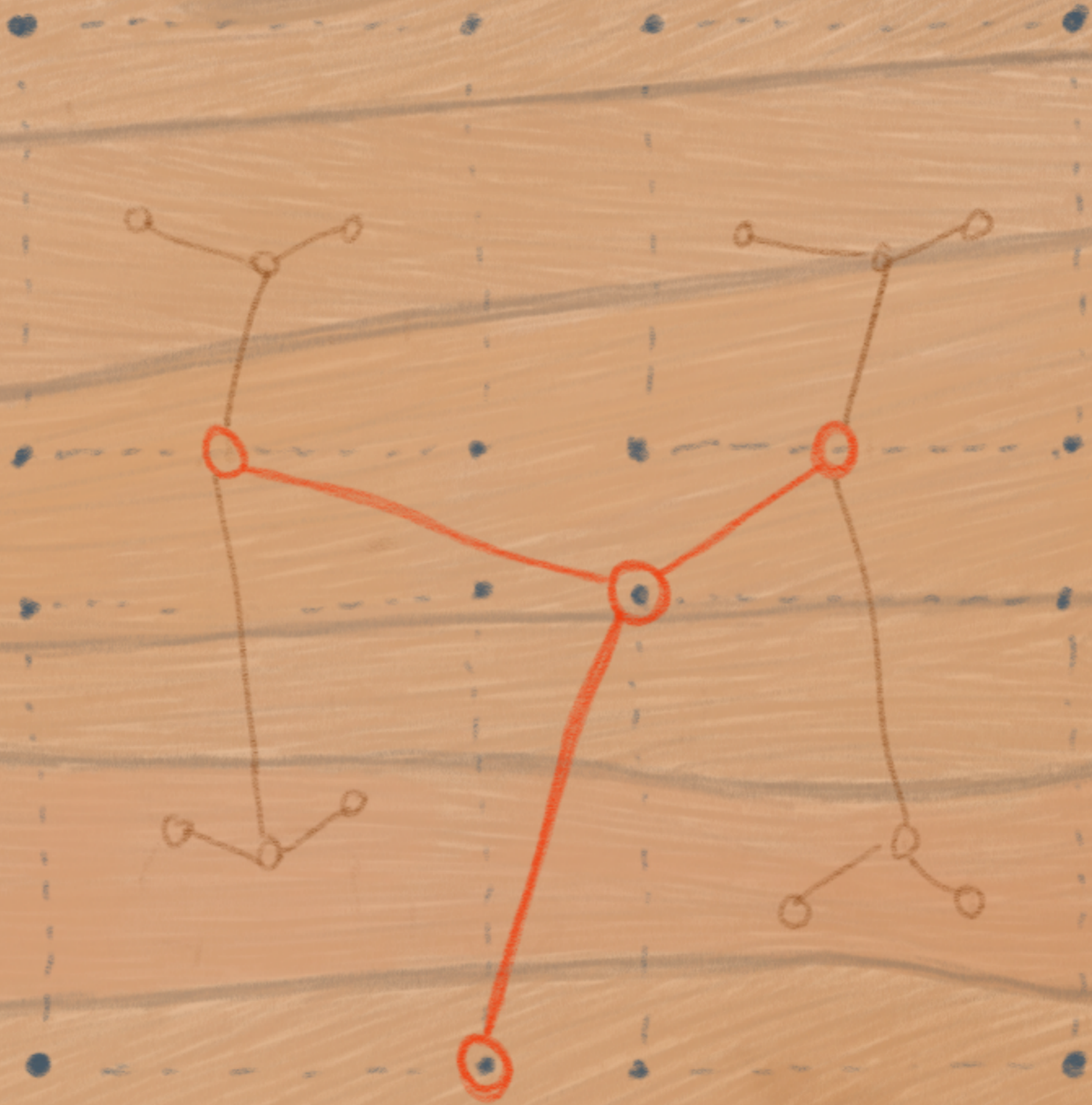
THE PERFECT BALANCED BINARY TREE  
WITH  $n = 2^k$  VERTICES (FOR EVEN  $k$ )  
HAS A PLANAR STRAIGHT-LINE EMBEDDING  
ONTO THE  $\sqrt{n} \times \sqrt{n}$  GRID

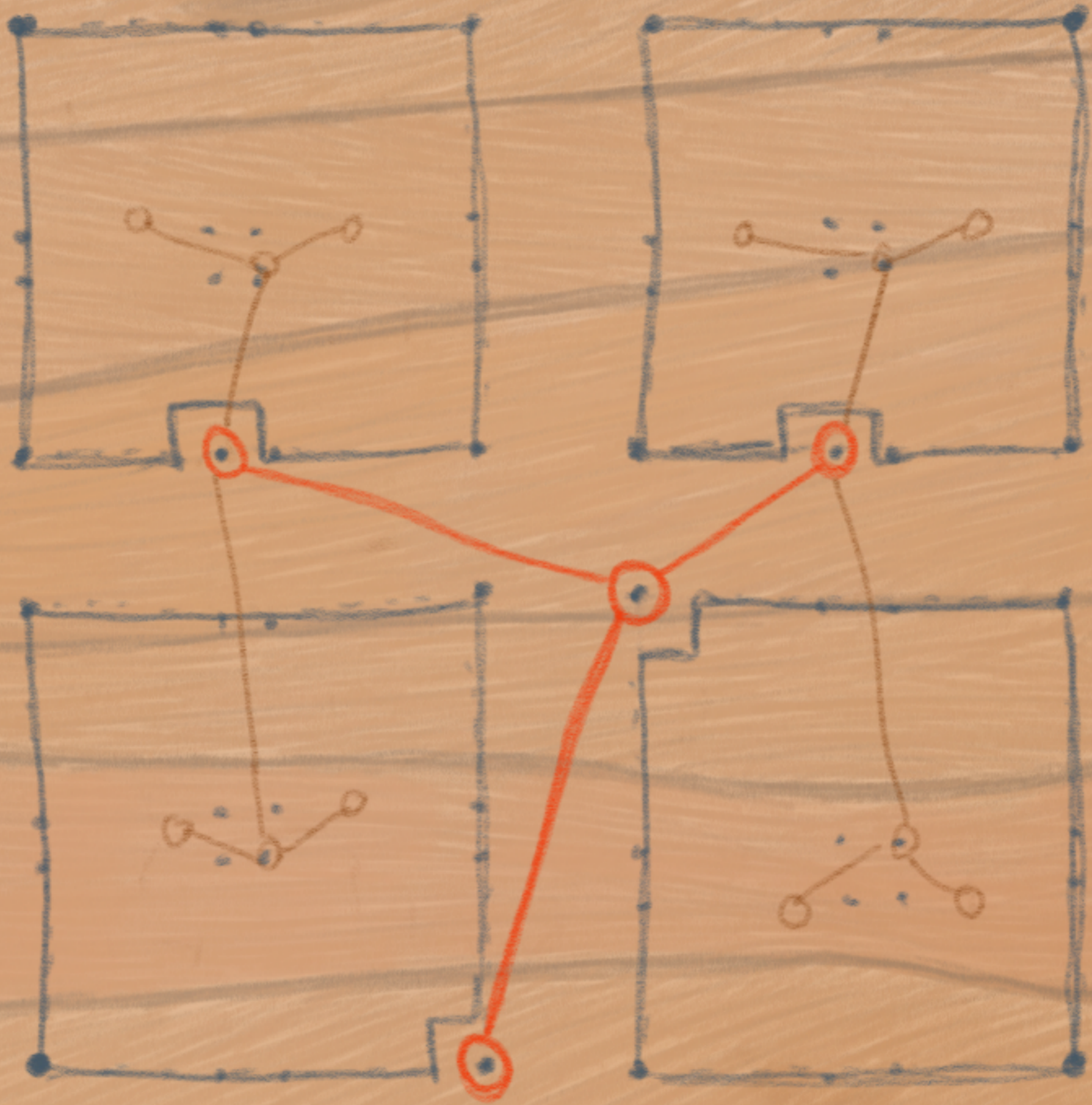


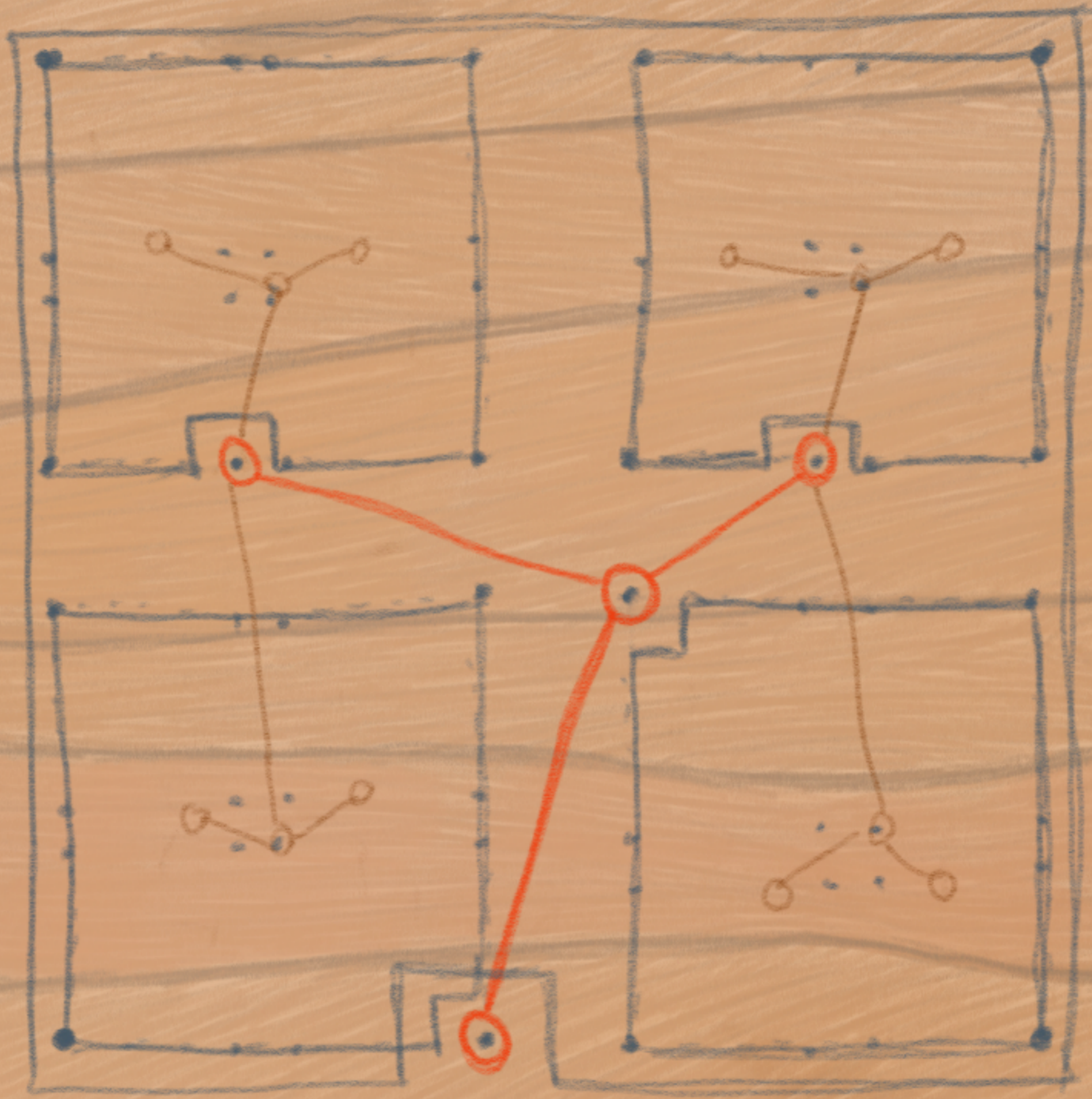


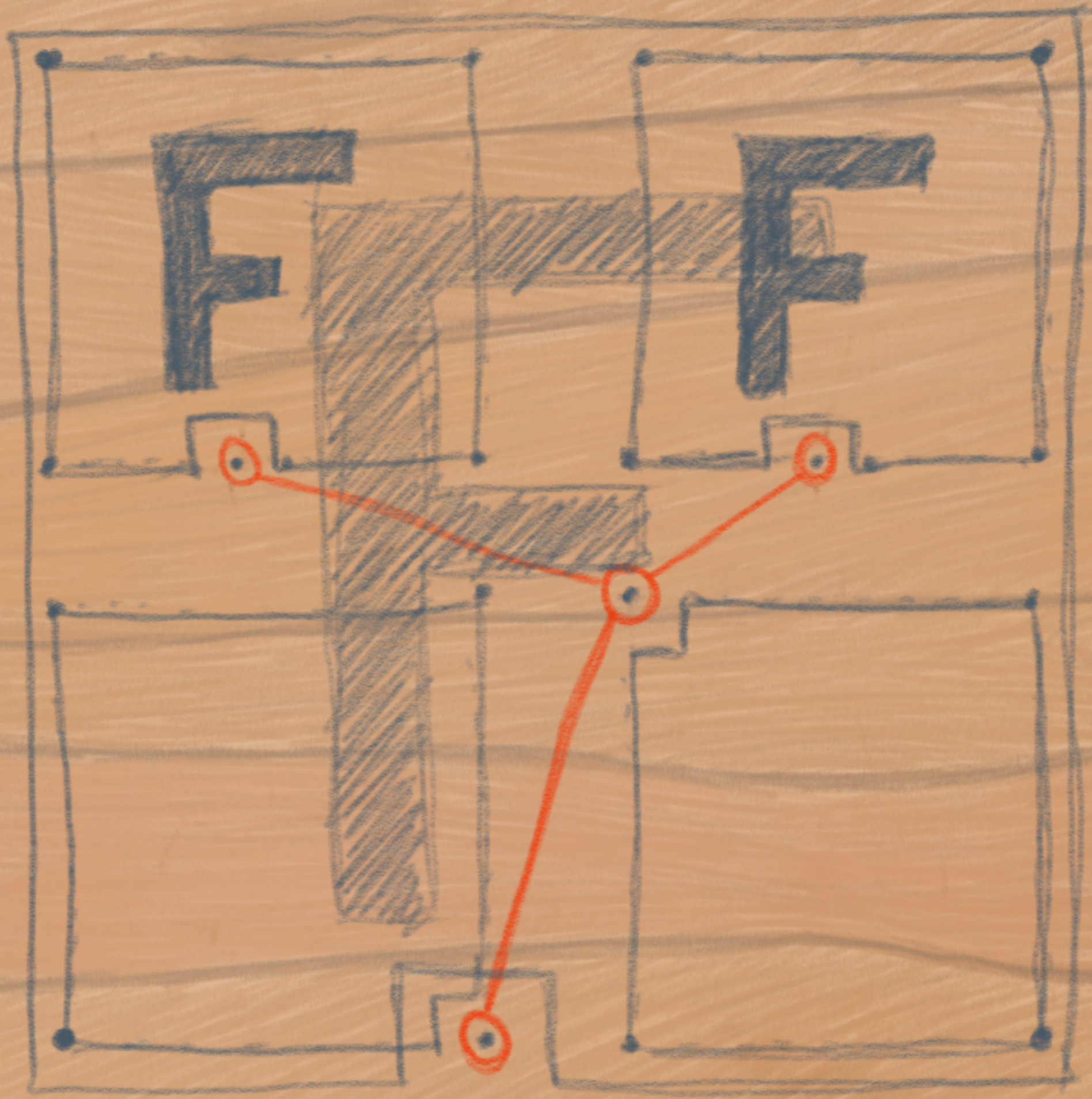


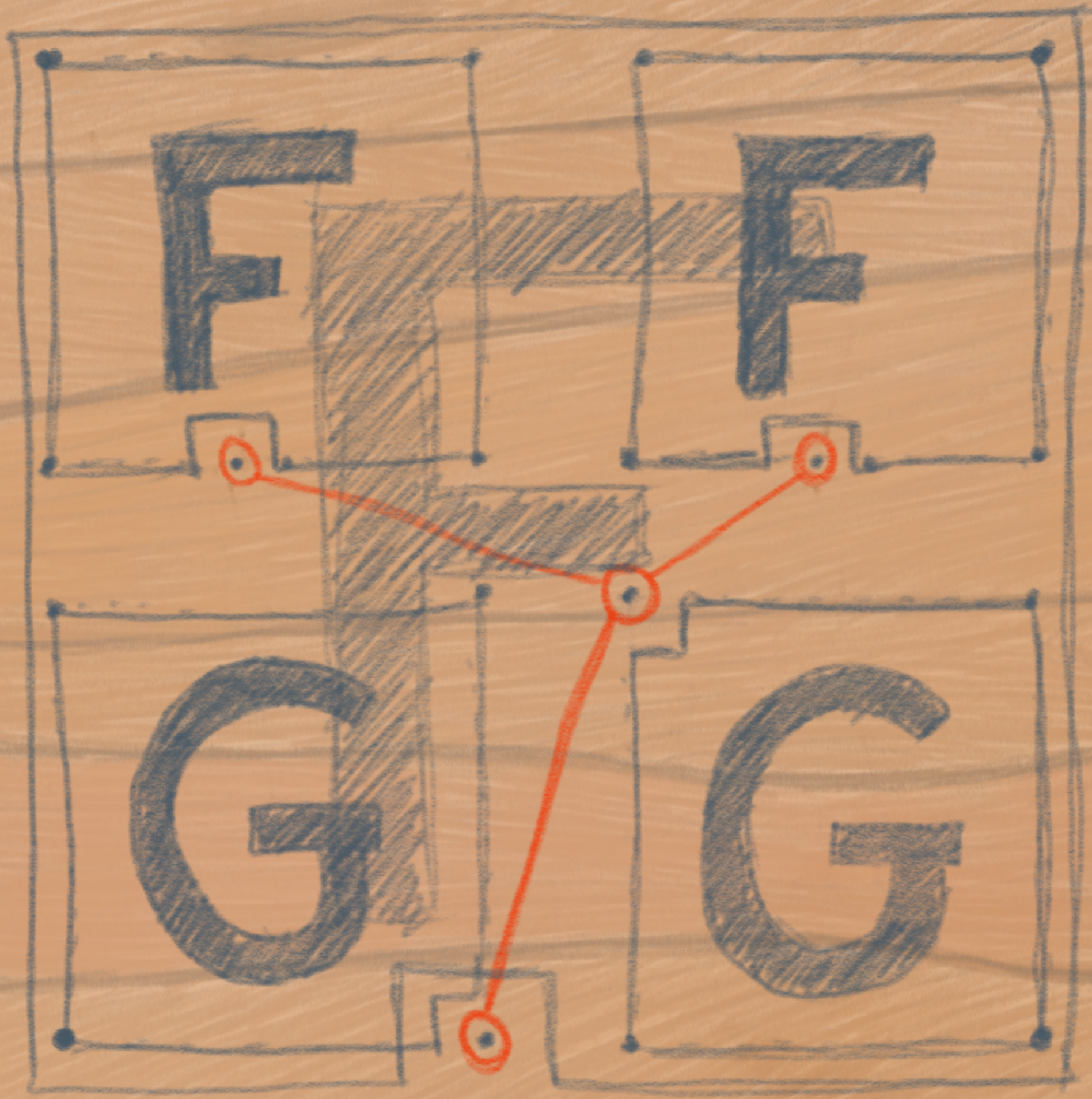


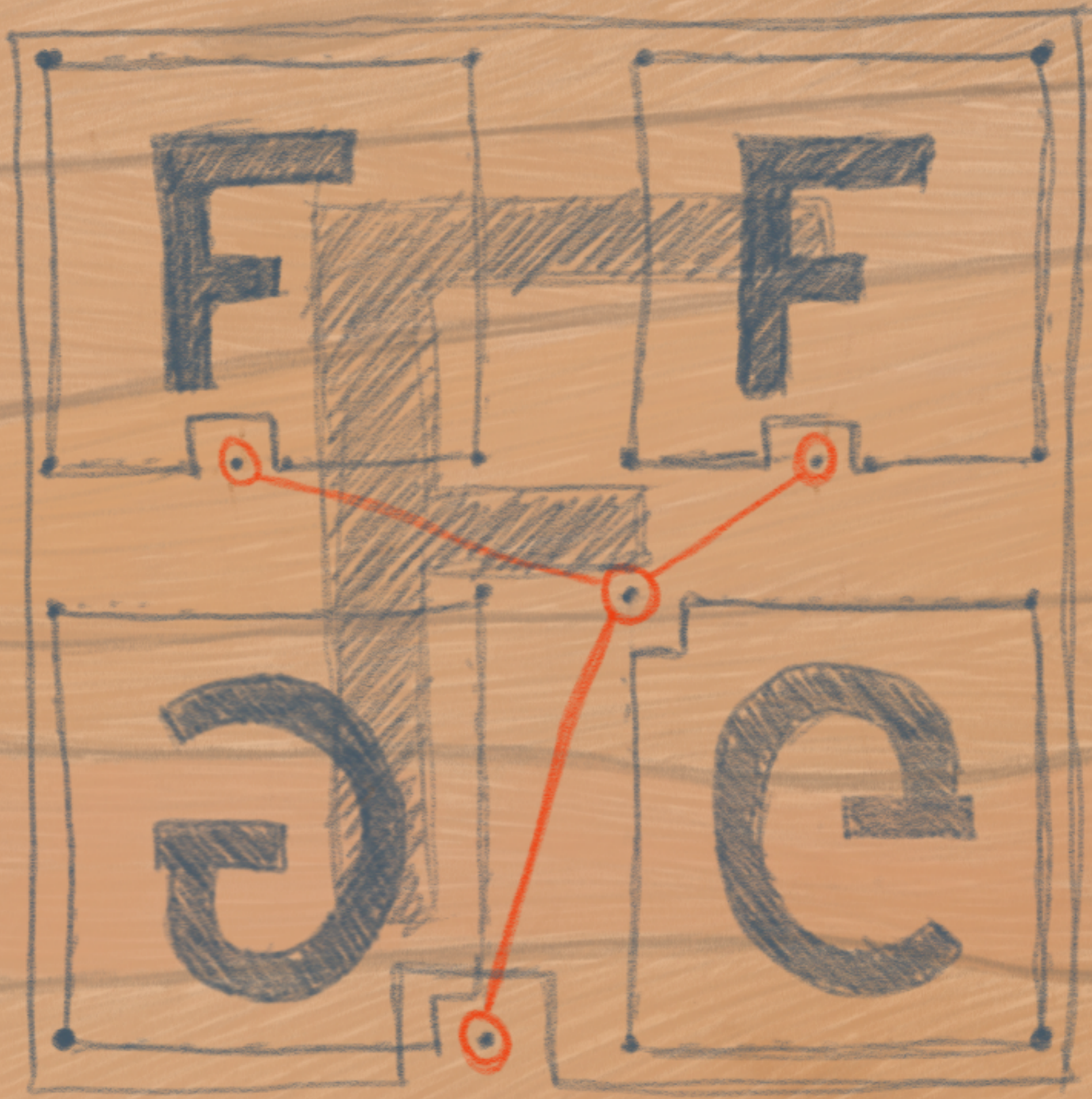










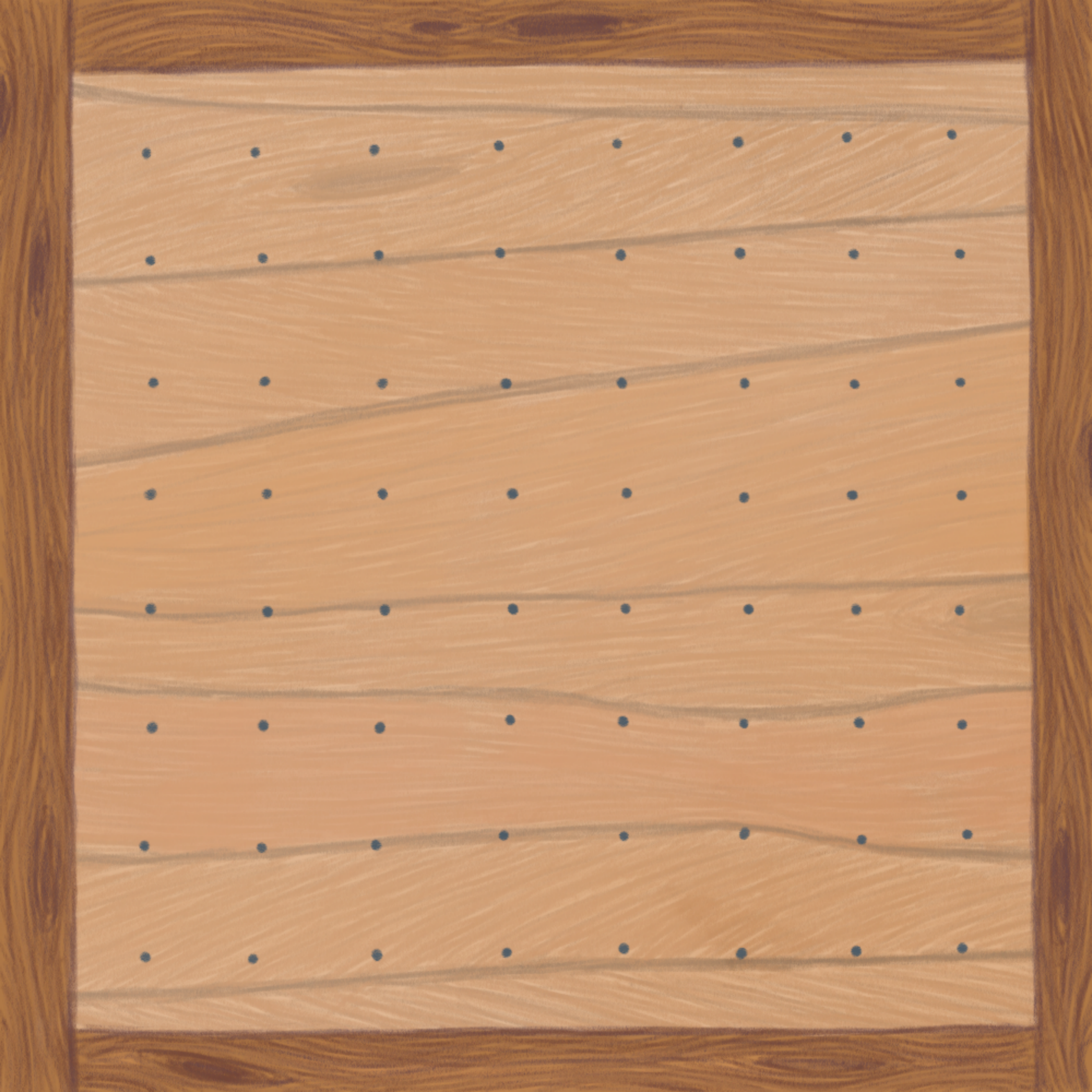






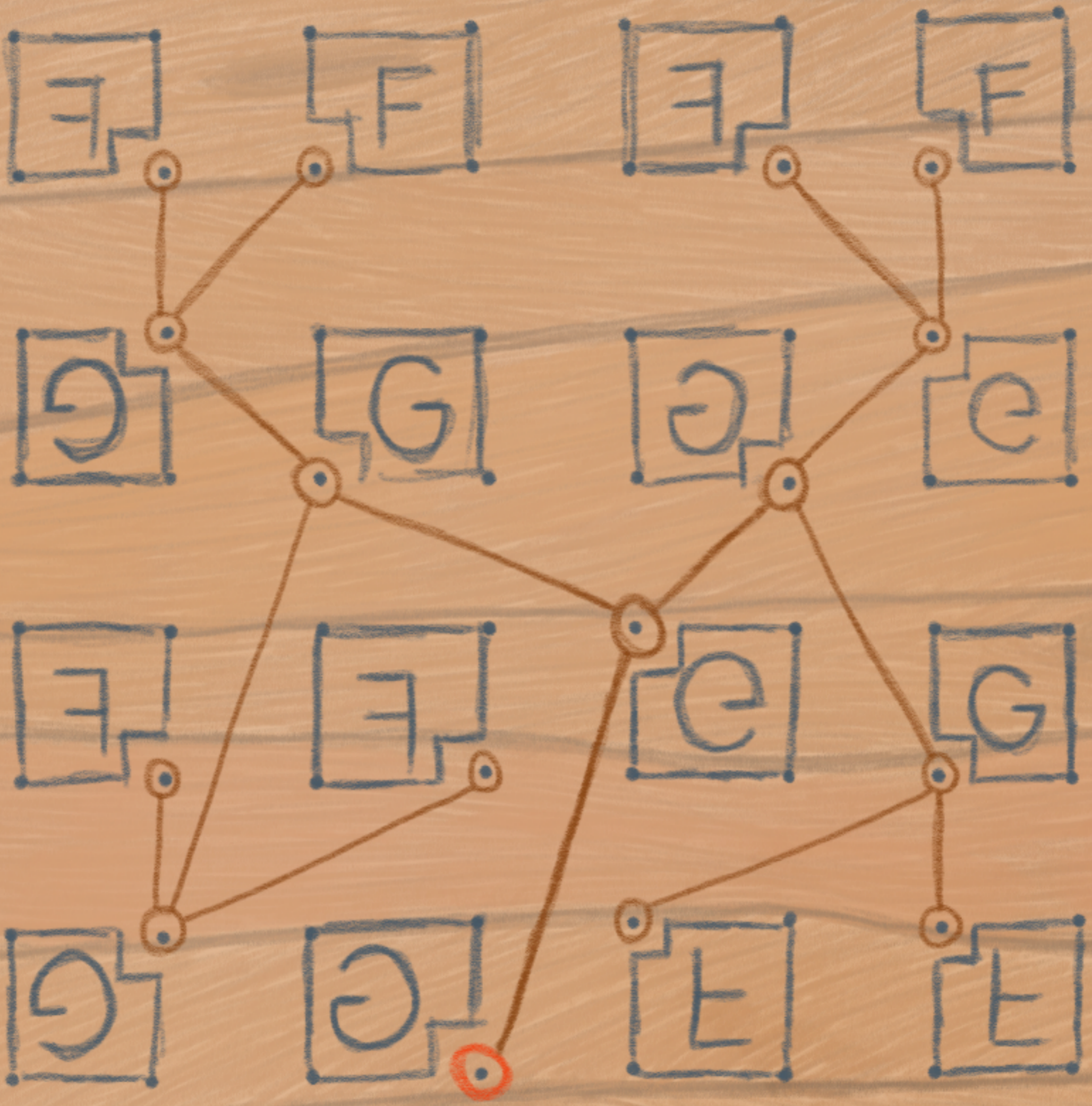


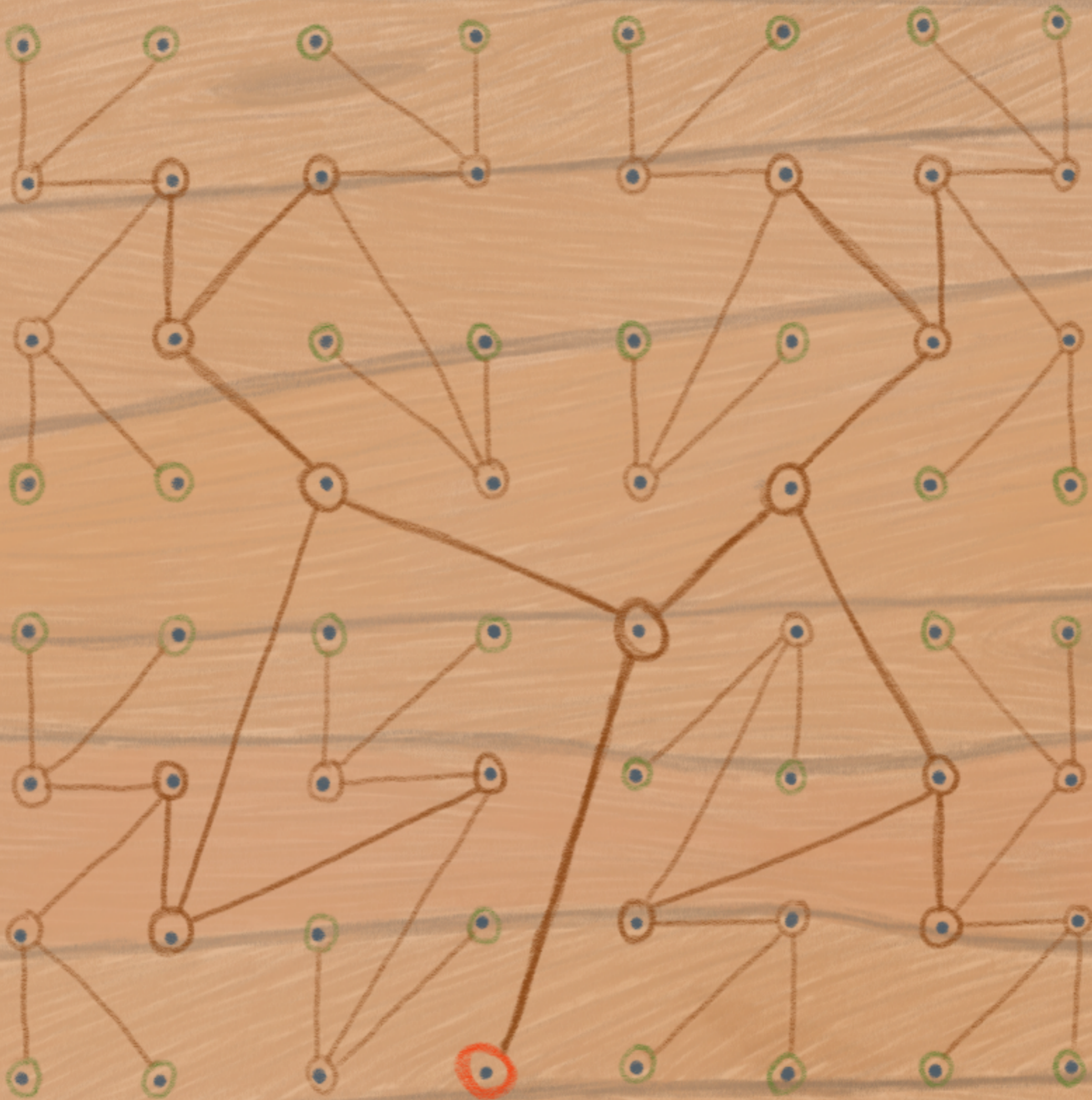
















## CONJECTURE

EVERY BINARY TREE (DEGREE  $\leq 3$ )  
WITH  $n = 2^k$  VERTICES (FOR EVEN  $k$ )  
HAS A PLANAR STRAIGHT-LINE EMBEDDING  
ONTO THE  $\sqrt{n} \times \sqrt{n}$  GRID



THANK YOU!