How to fit a Tree in a Box

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PART I

TREES & DRAWINGS

DRAWINGS OF TREES

TREE

ROOTED TREE

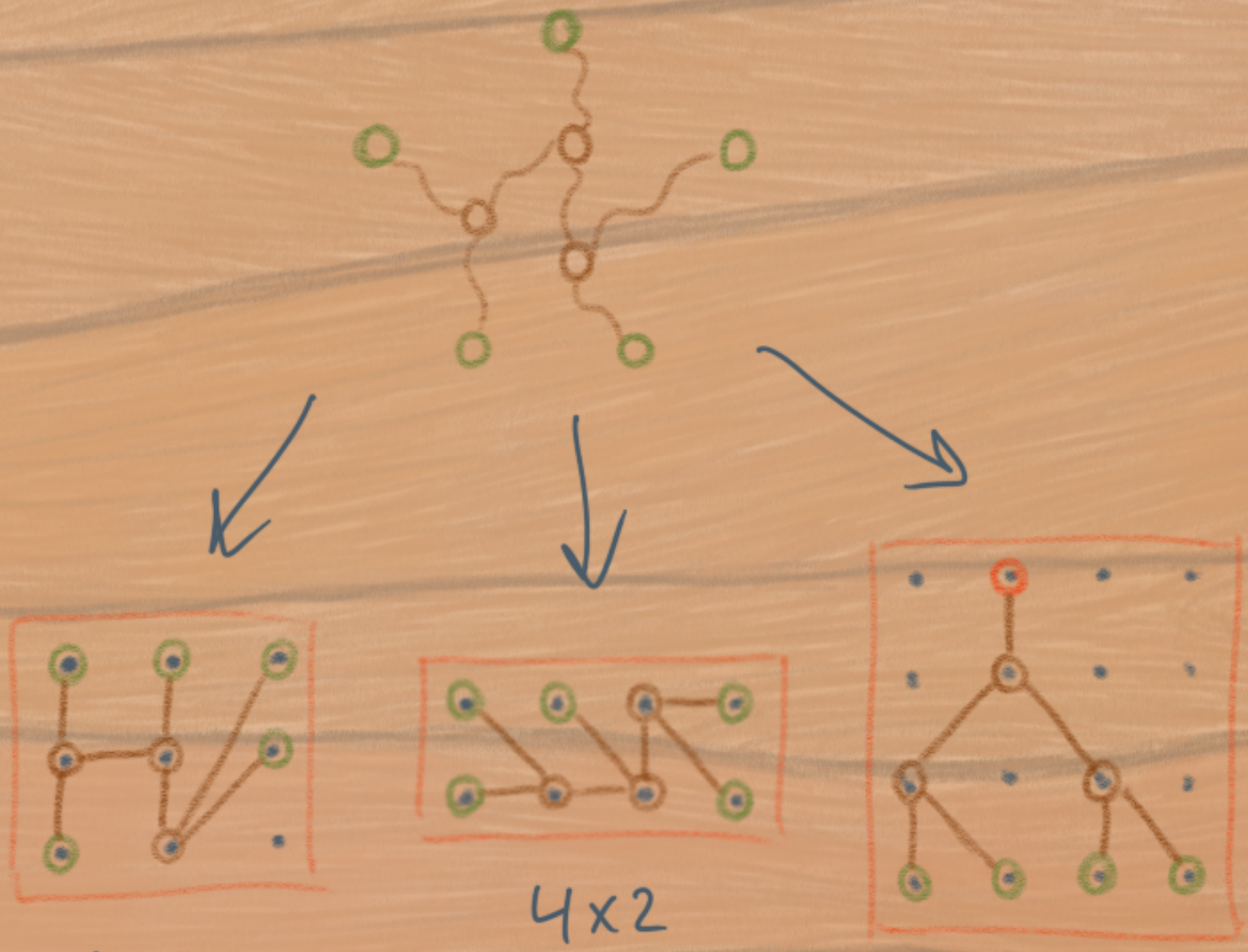
PLANE DRAWING

UPWARD DRAWING

GRID DRAWING

PLANE GRID DRAWING

COMPACT PLANE GRID DRAWINGS



3x3

4x2

4x4
(UPWARD)

KNOWN RESULTS

EVERY TREE HAS A DRAWING WITH

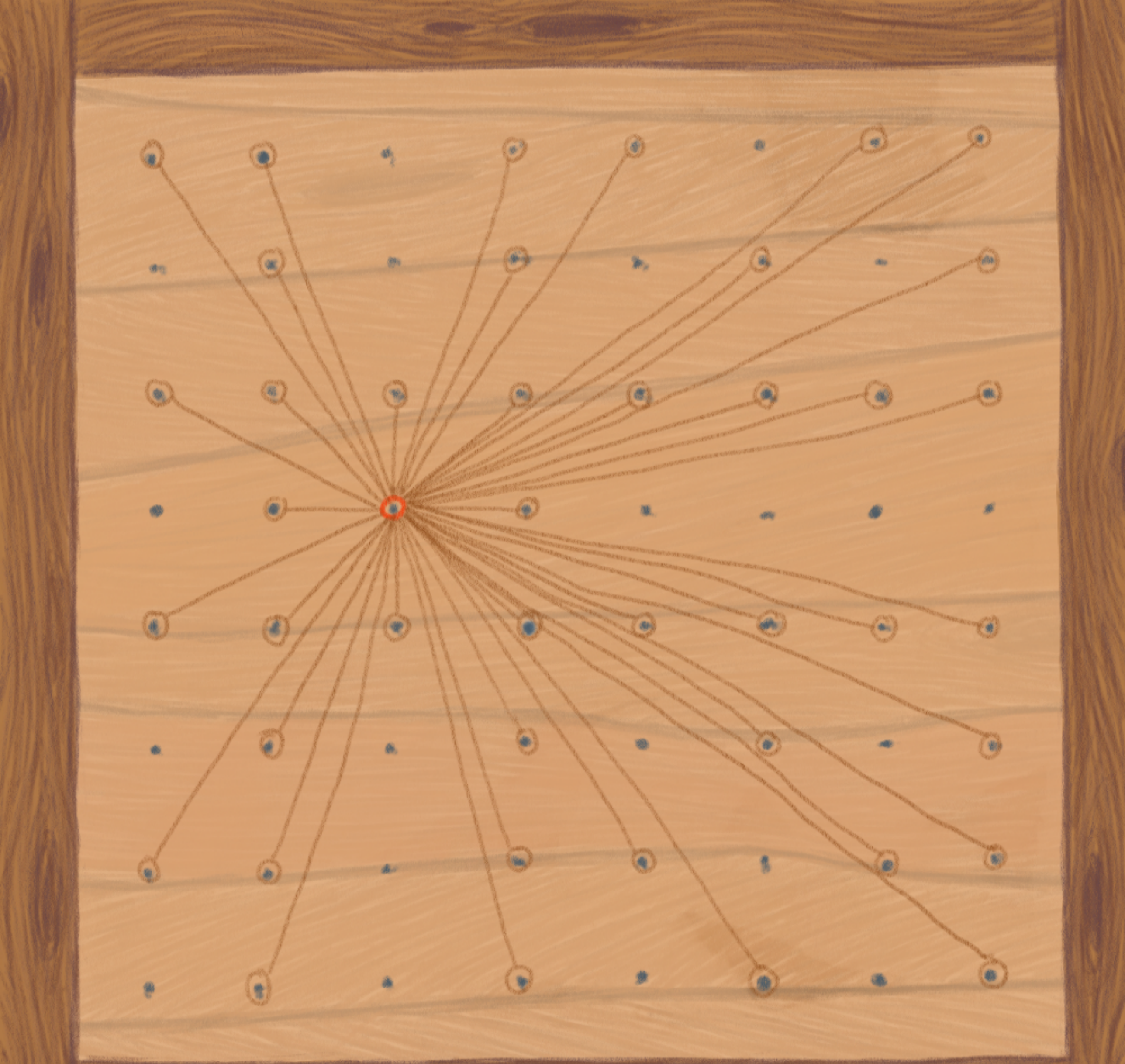
- $O(n \log n)$ AREA [FOLKLORE]
- $n \cdot 2^{O(\sqrt{\log \log n \log \log \log n})}$ [CHAN 2018]

EVERY TREE OF DEGREE $< \sqrt{n}$ HAS DR. WITH

- $O(n)$ AREA [GARG & RUSU 2004]

UPWARD DRAWINGS REQUIRE

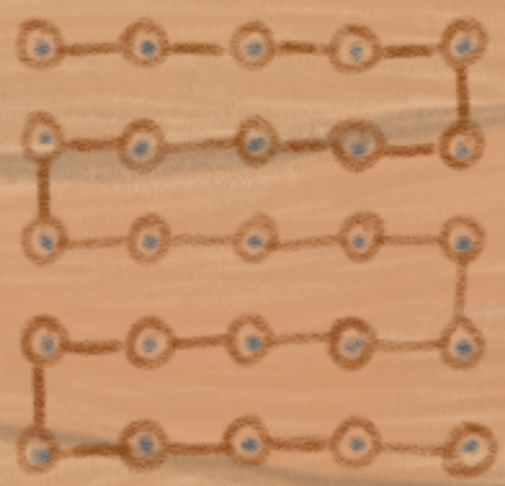
- $\Theta(n \log n)$ AREA [CRES., DI BAT. & PIP. 1992]
- $O(n)$ AREA FOR PERFECTLY BALANCED TREES
- $O(n \sqrt{\log n} (\log \log n)^c)$ AREA FOR WEAK. [CHAN 2018]
- TESTING IS NP-HARD [BIEDL & MONDAL 2017]



Binary Trees

High-degree trees sometimes need a large area

Degree-2 trees can be embedded very compactly



What about degree-3 trees?

PART II

UPWARD TREES

THEOREM

GIVEN A ROOTED BINARY TREE T

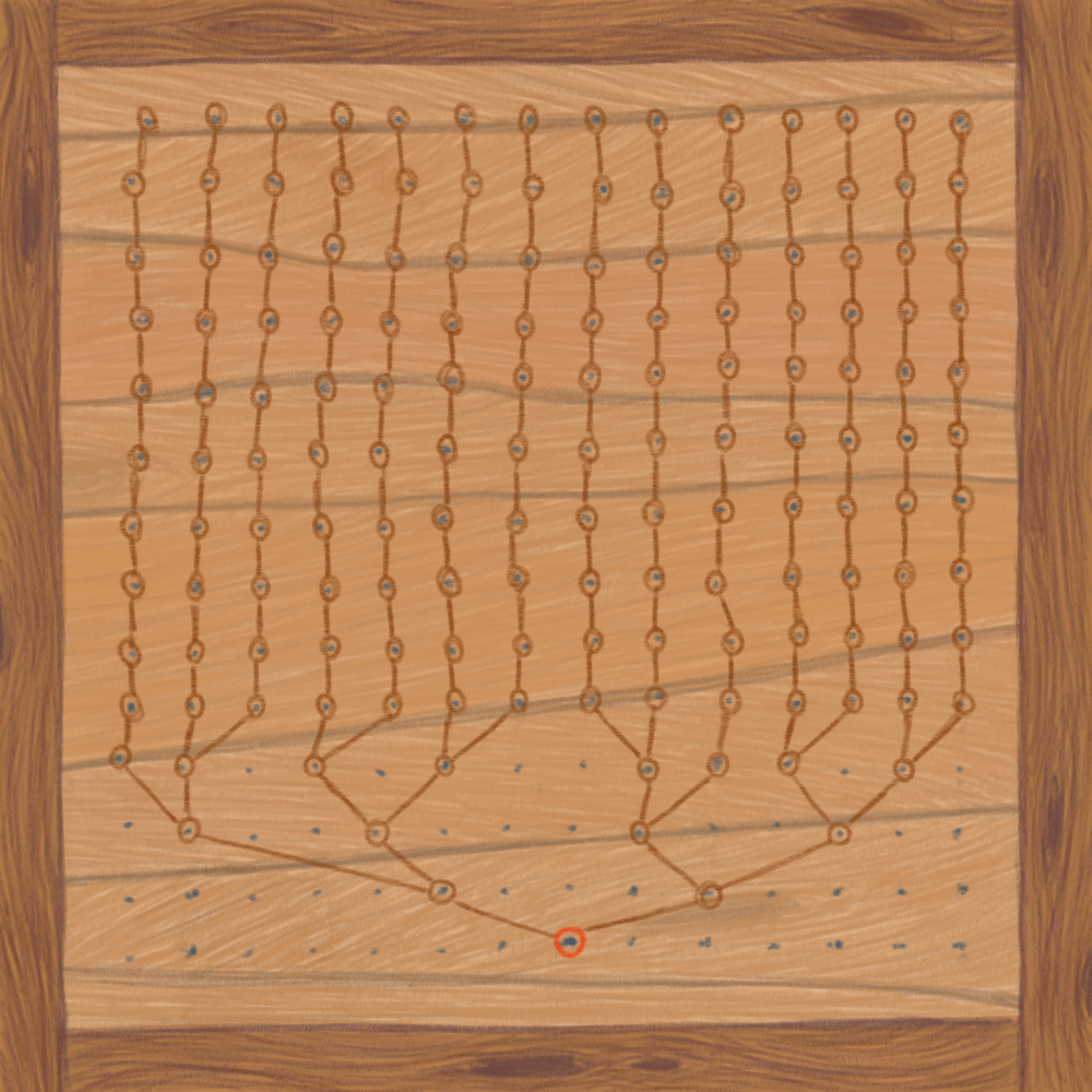
WITH A GIVEN COMBINATORIAL EMBEDDING

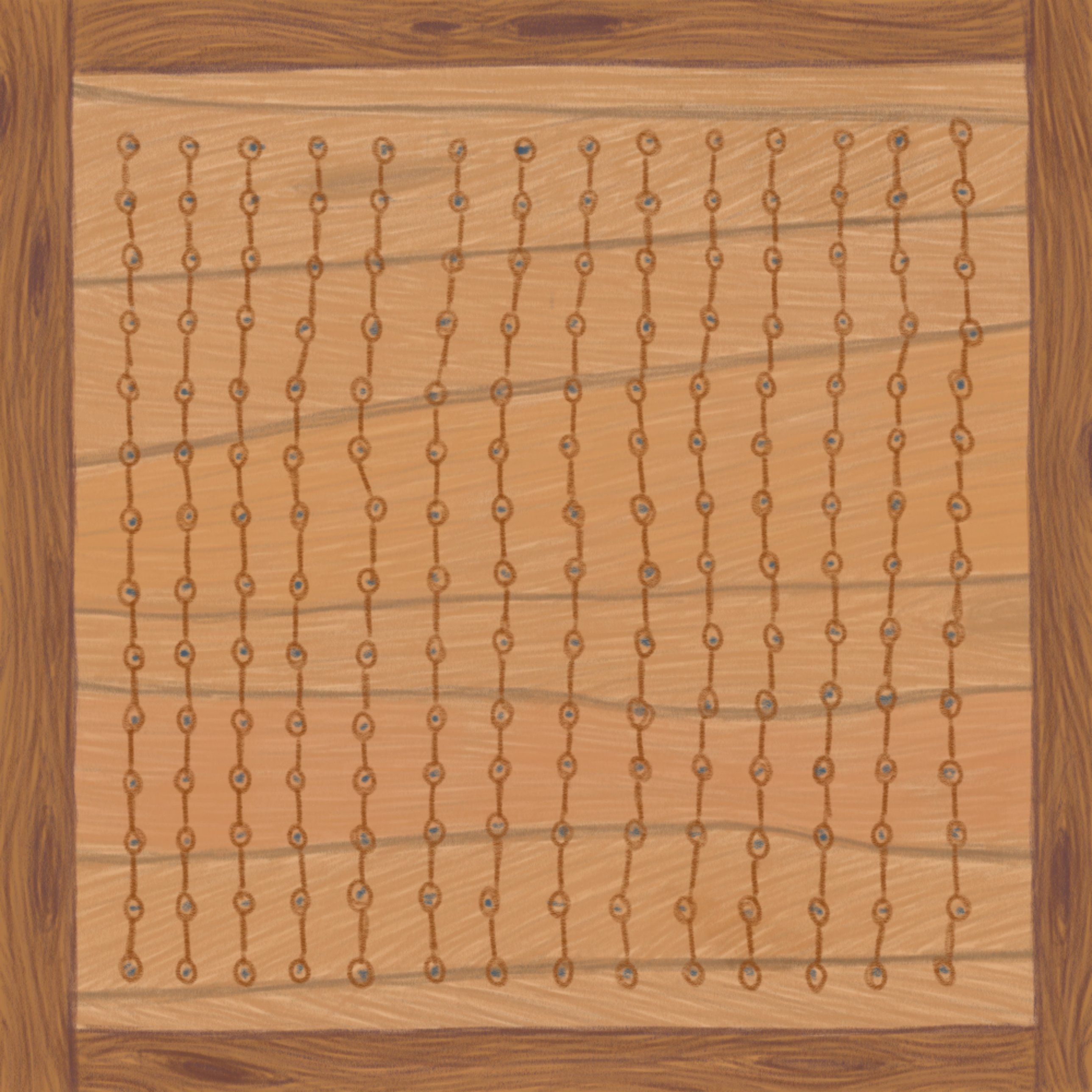
AND A FIXED $(w \times h)$ GRID G,

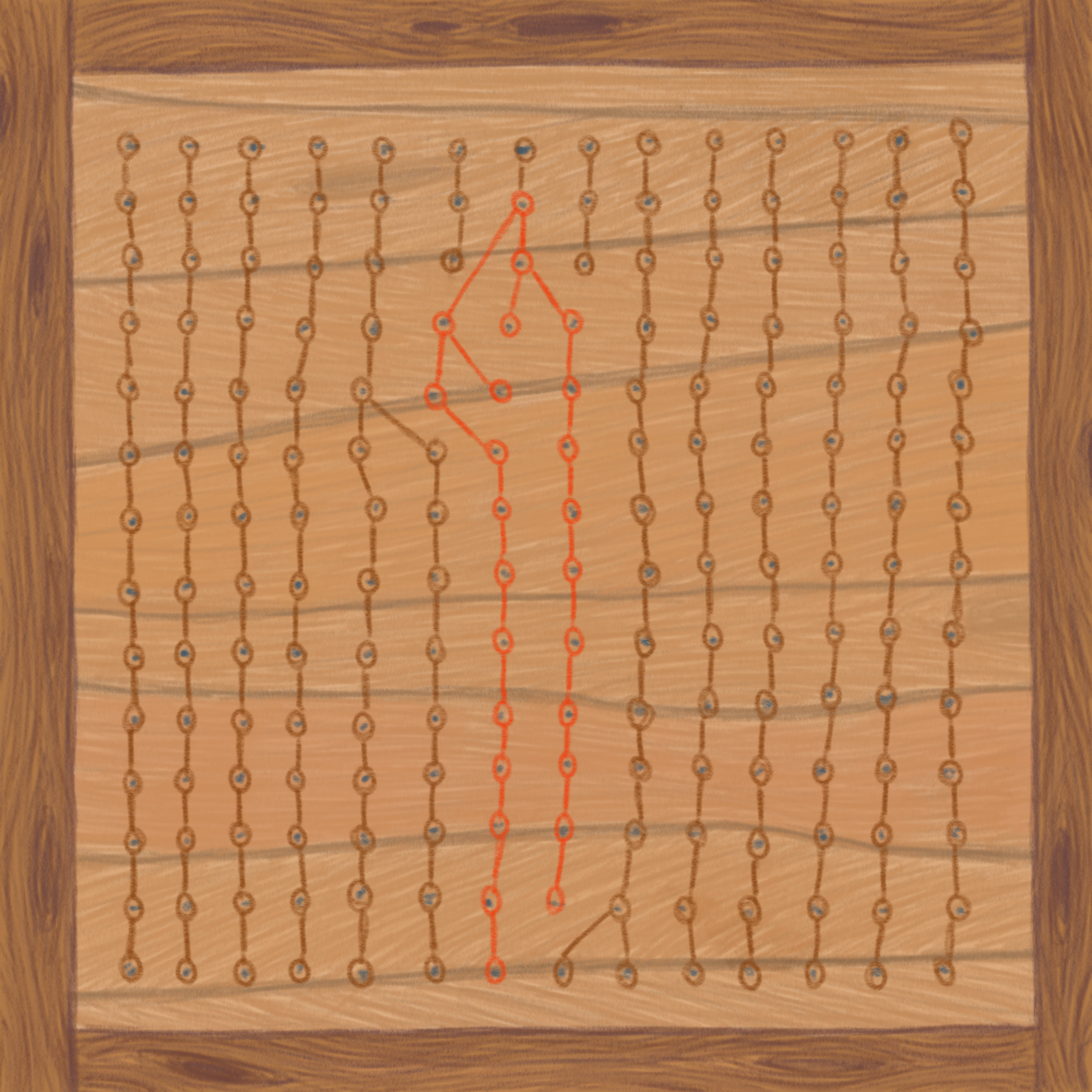
DECIDING WHETHER A CORRESPONDING

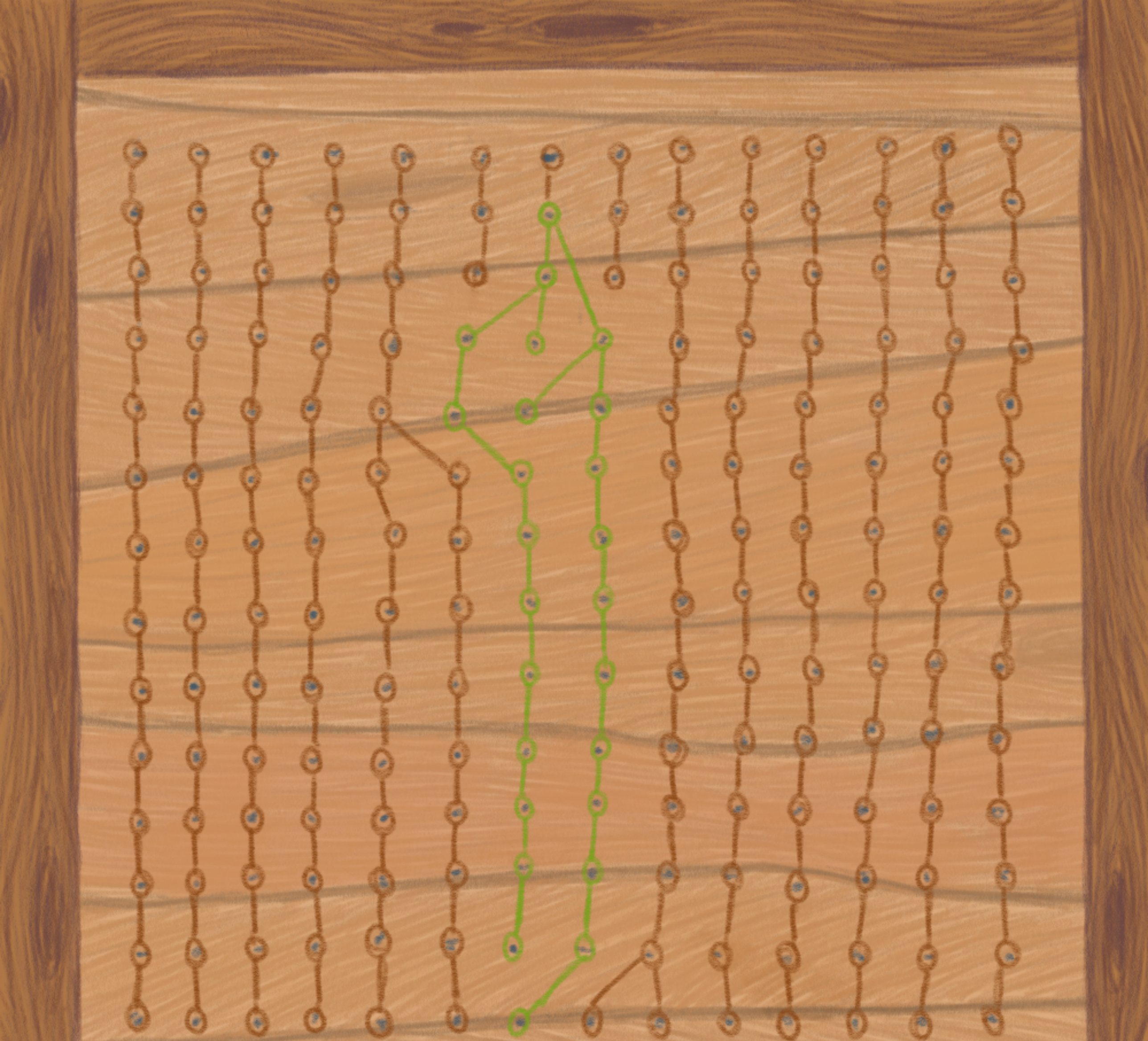
UPWARD PLANAR EMBEDDING OF T ONTO G

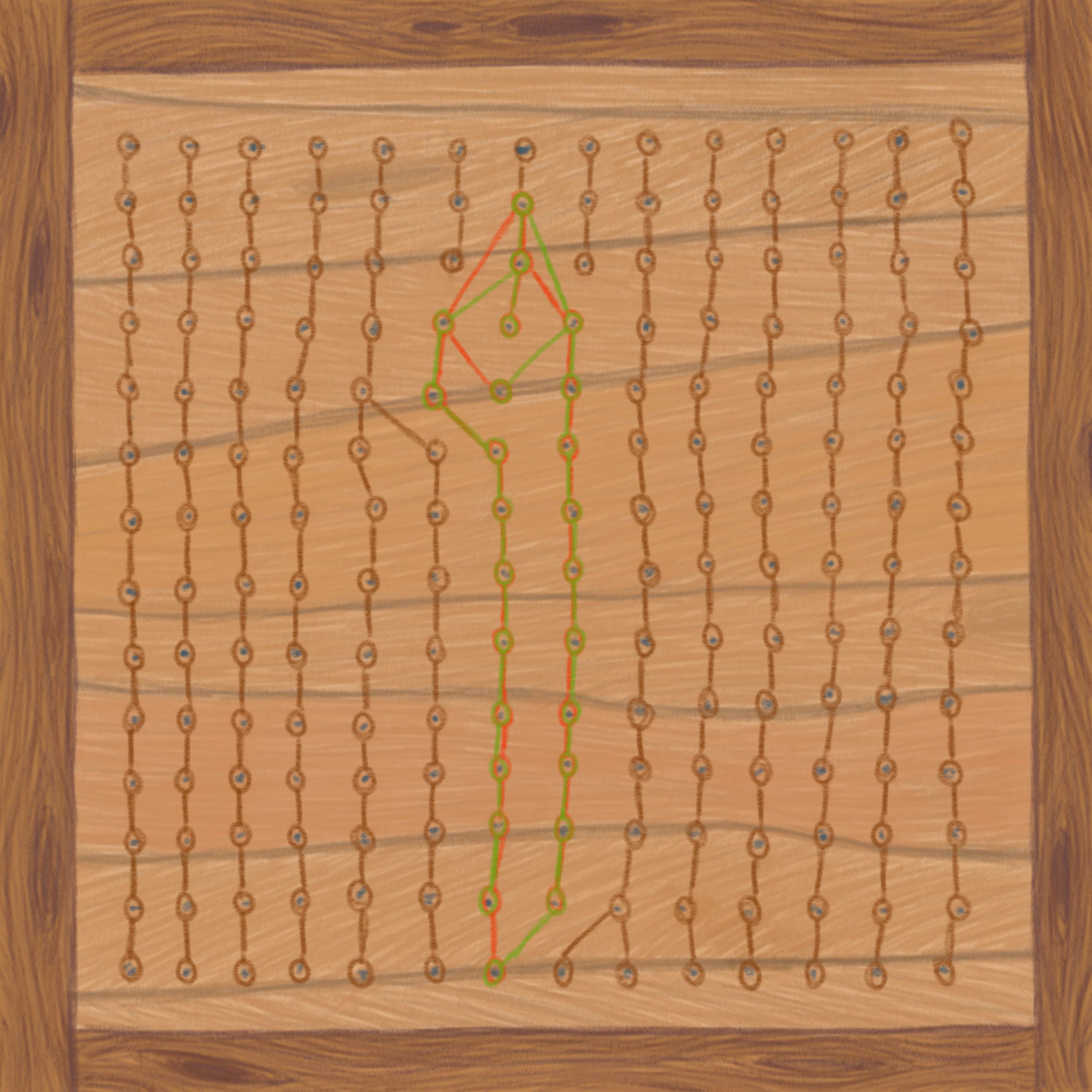
EXISTS IS NP-COMPLETE

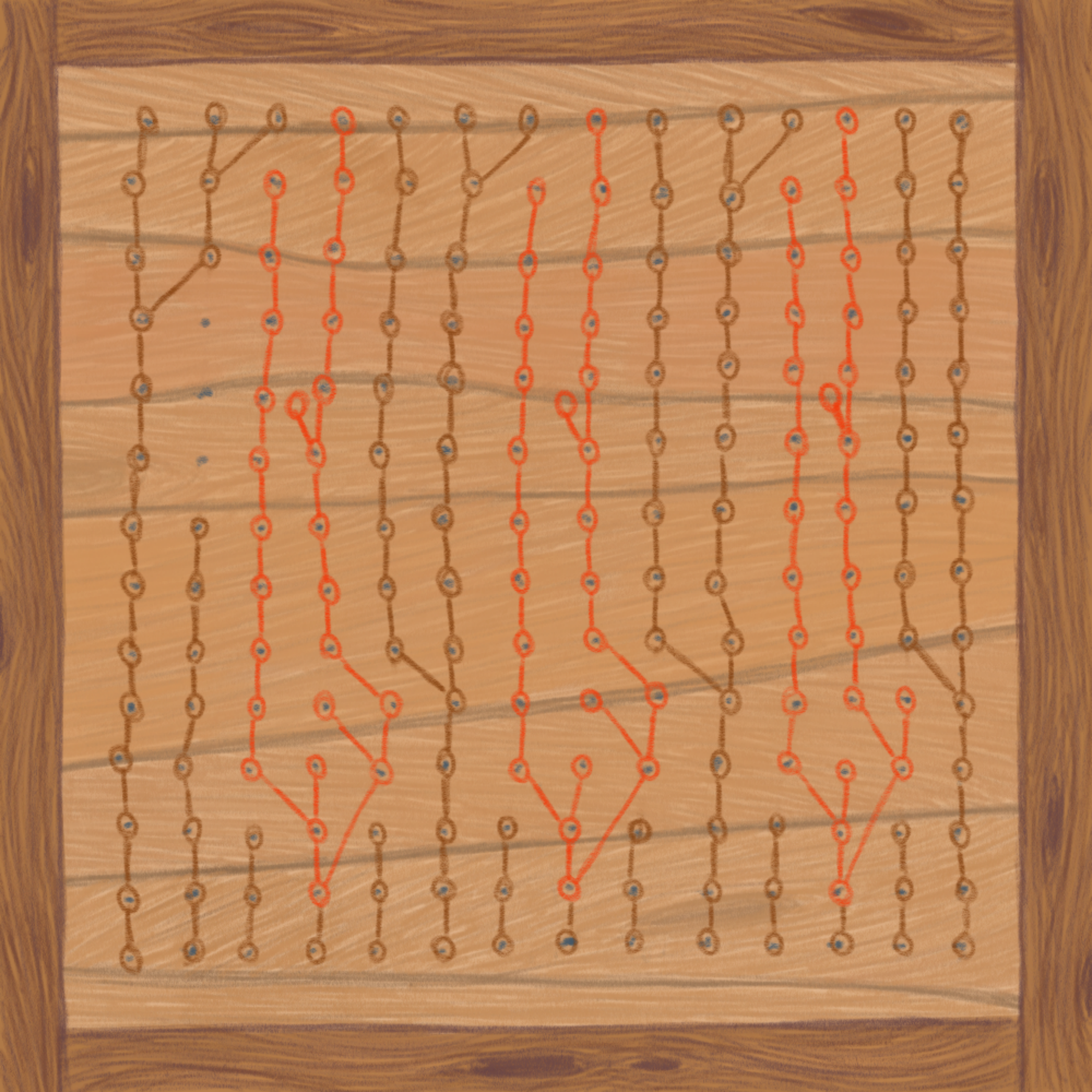


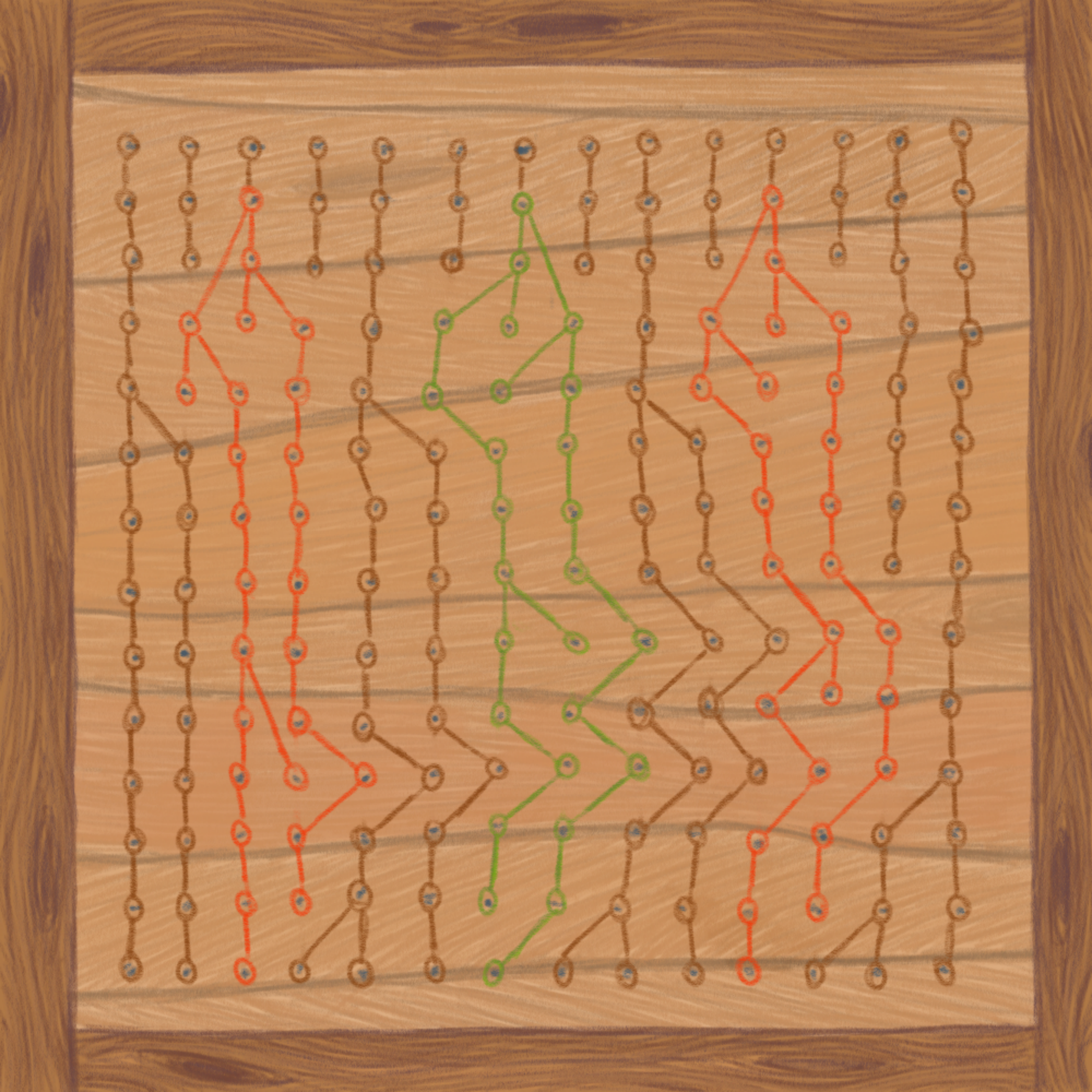










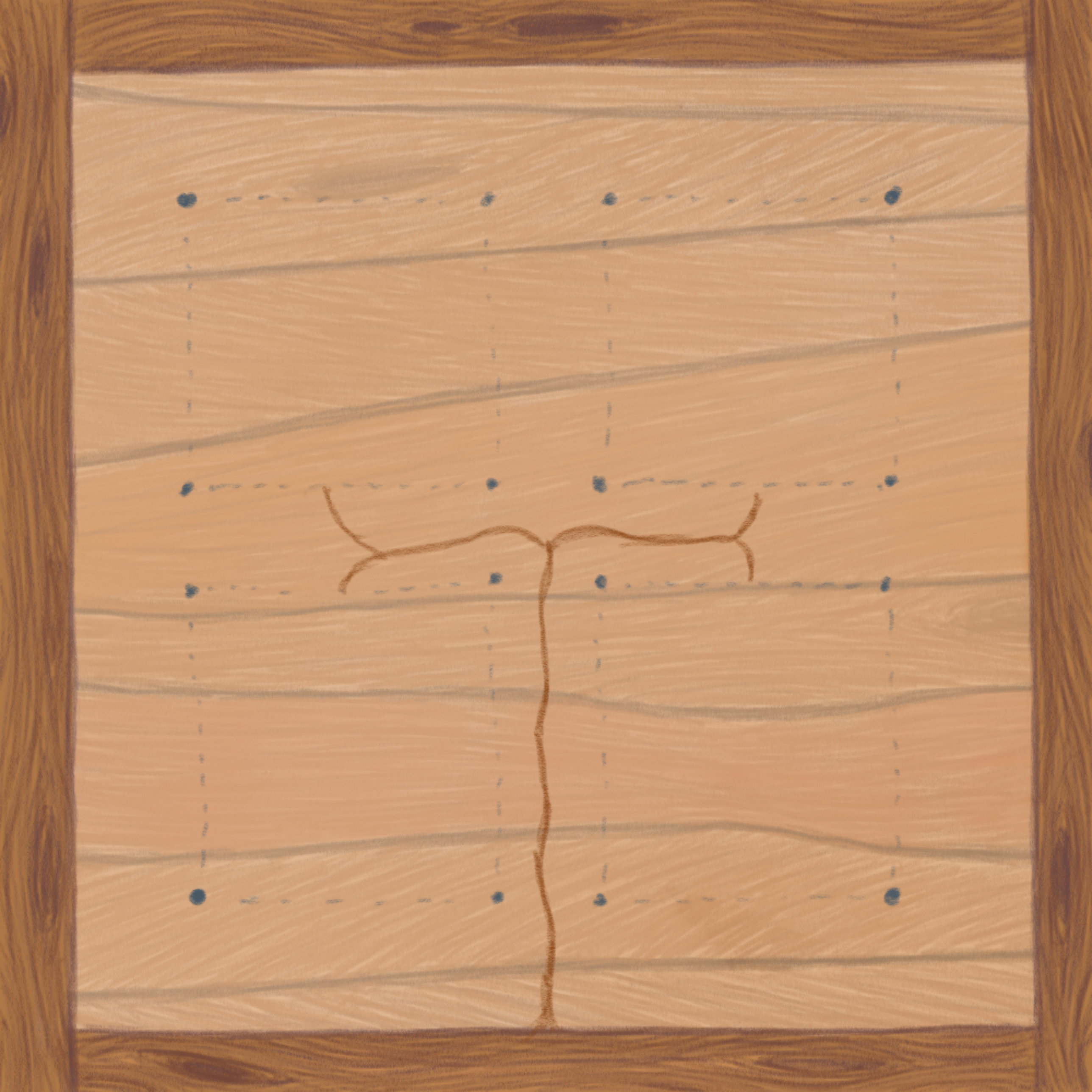


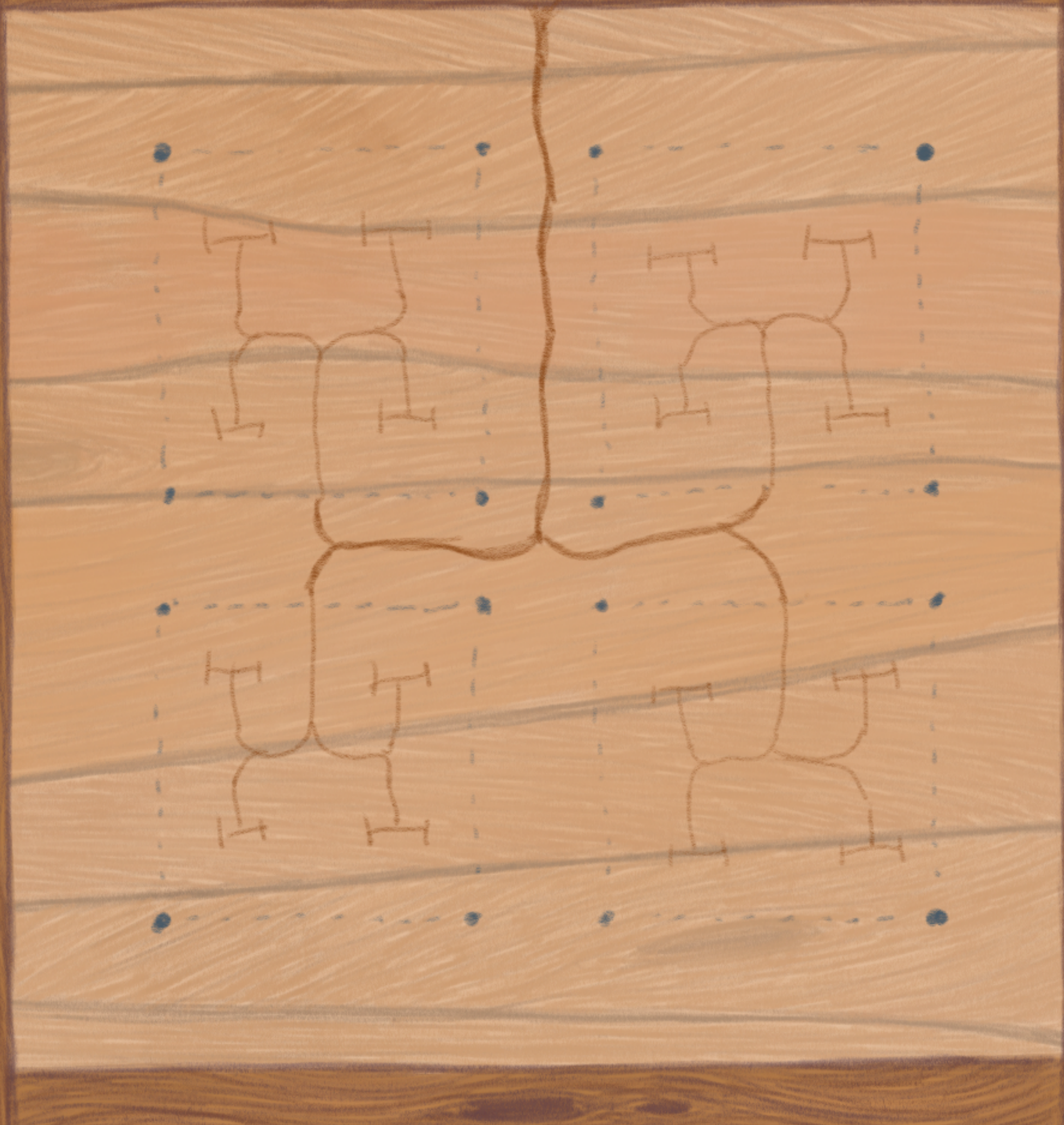
PART III

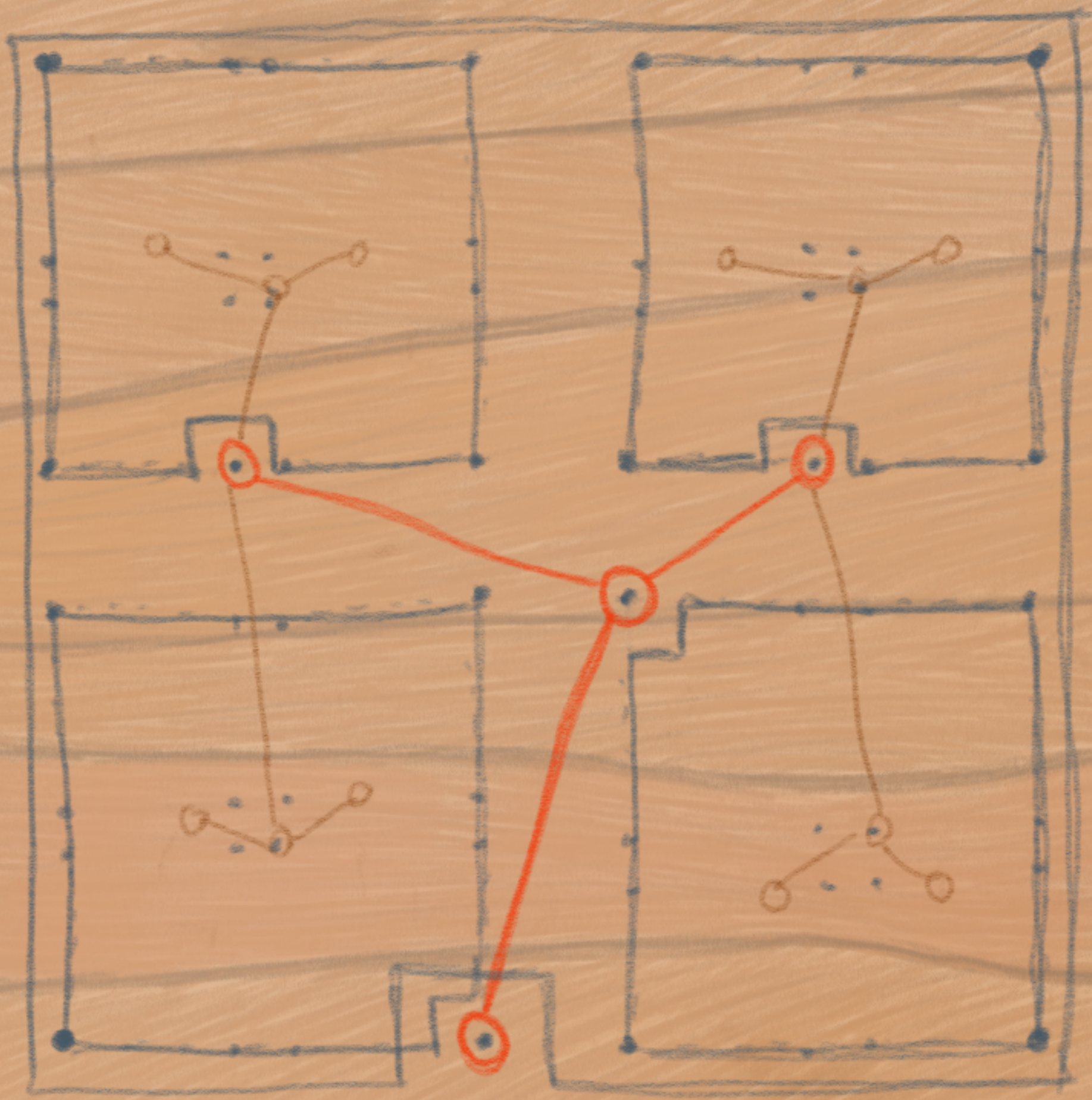
BALANCED TREES

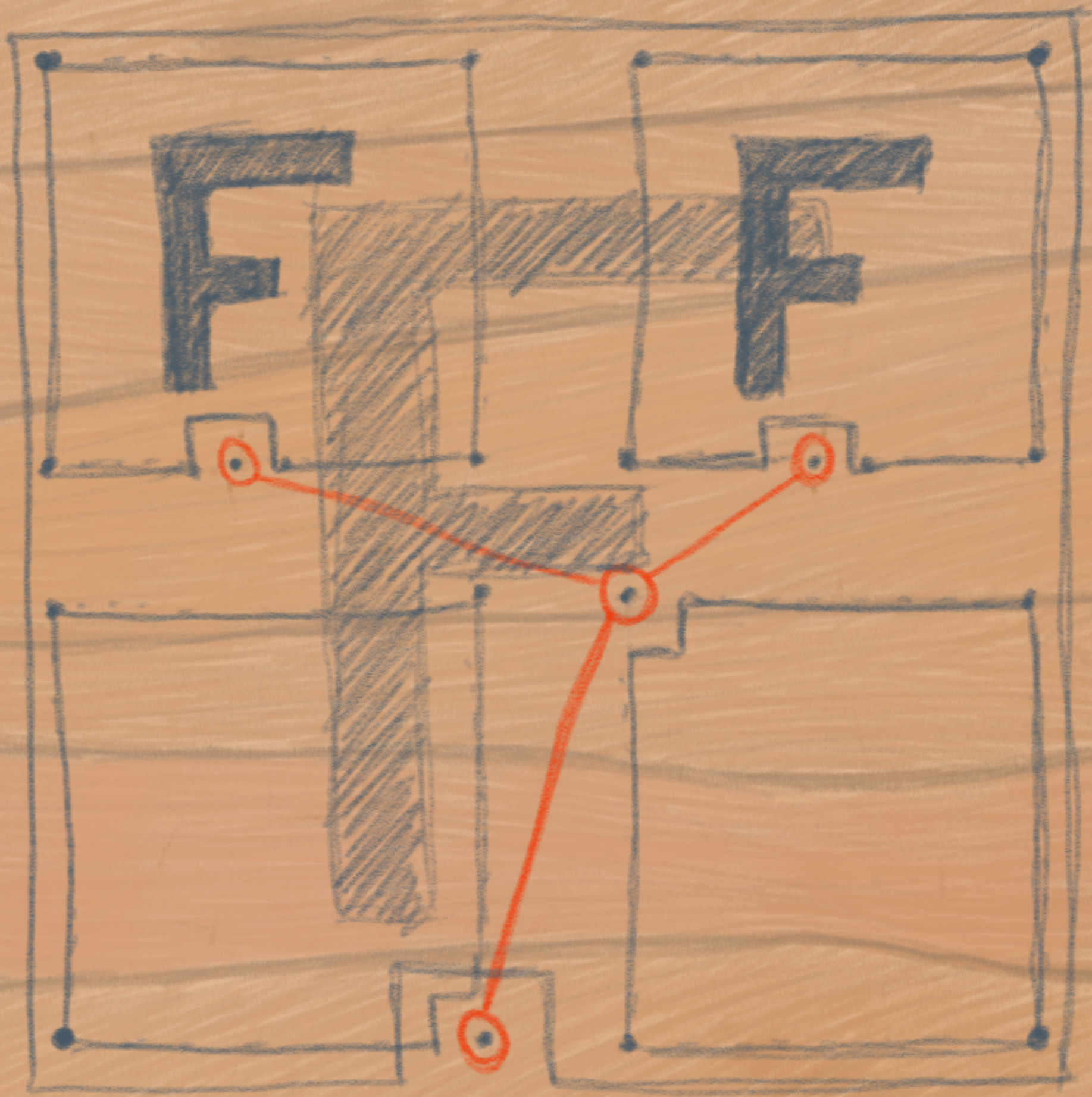
THEOREM

THE PERFECT BALANCED BINARY TREE WITH $n = 2^k$ VERTICES (FOR EVEN k) HAS A PLANAR STRAIGHT-LINE EMBEDDING ONTO THE $\sqrt{n} \times \sqrt{n}$ GRID





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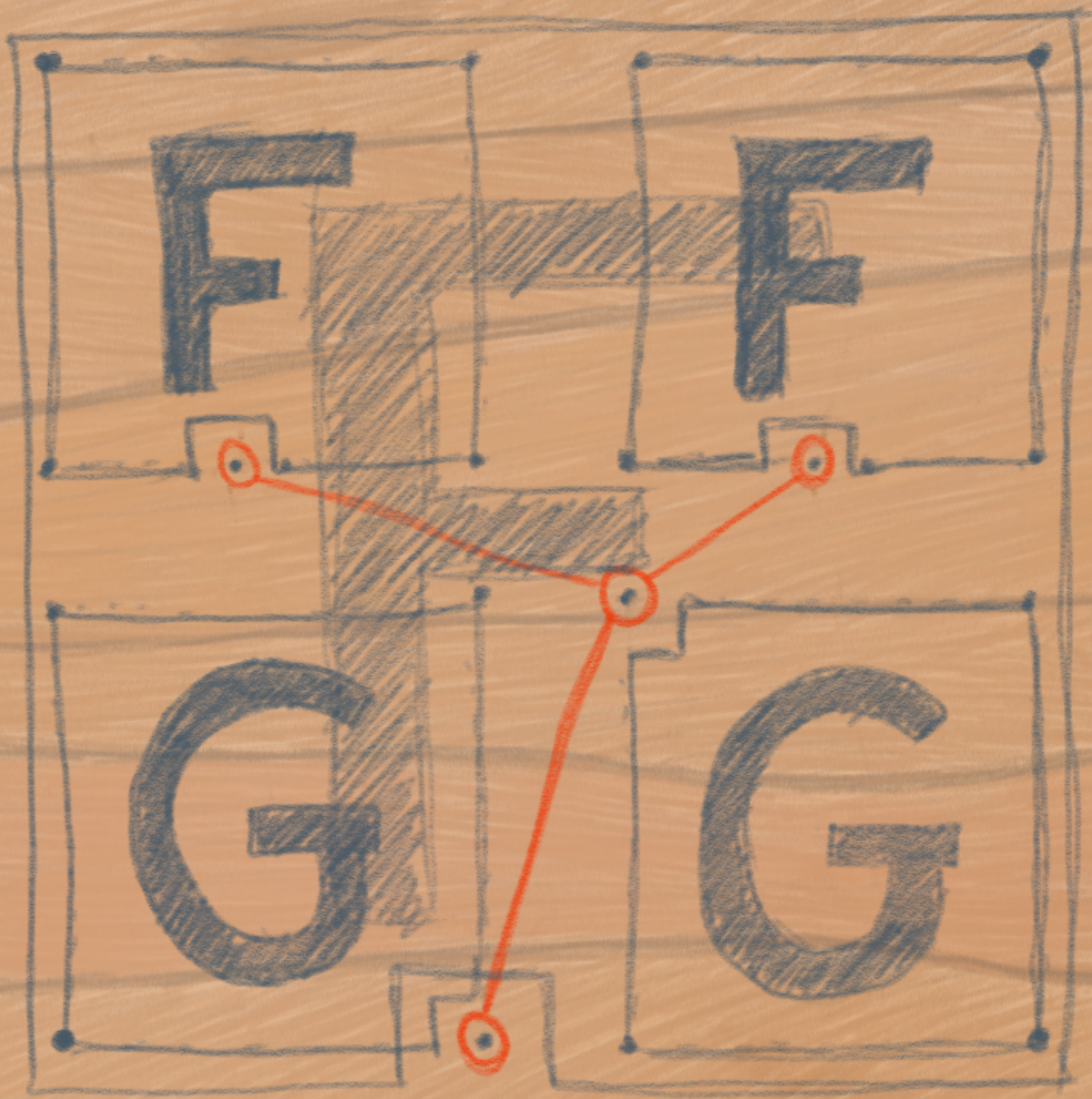
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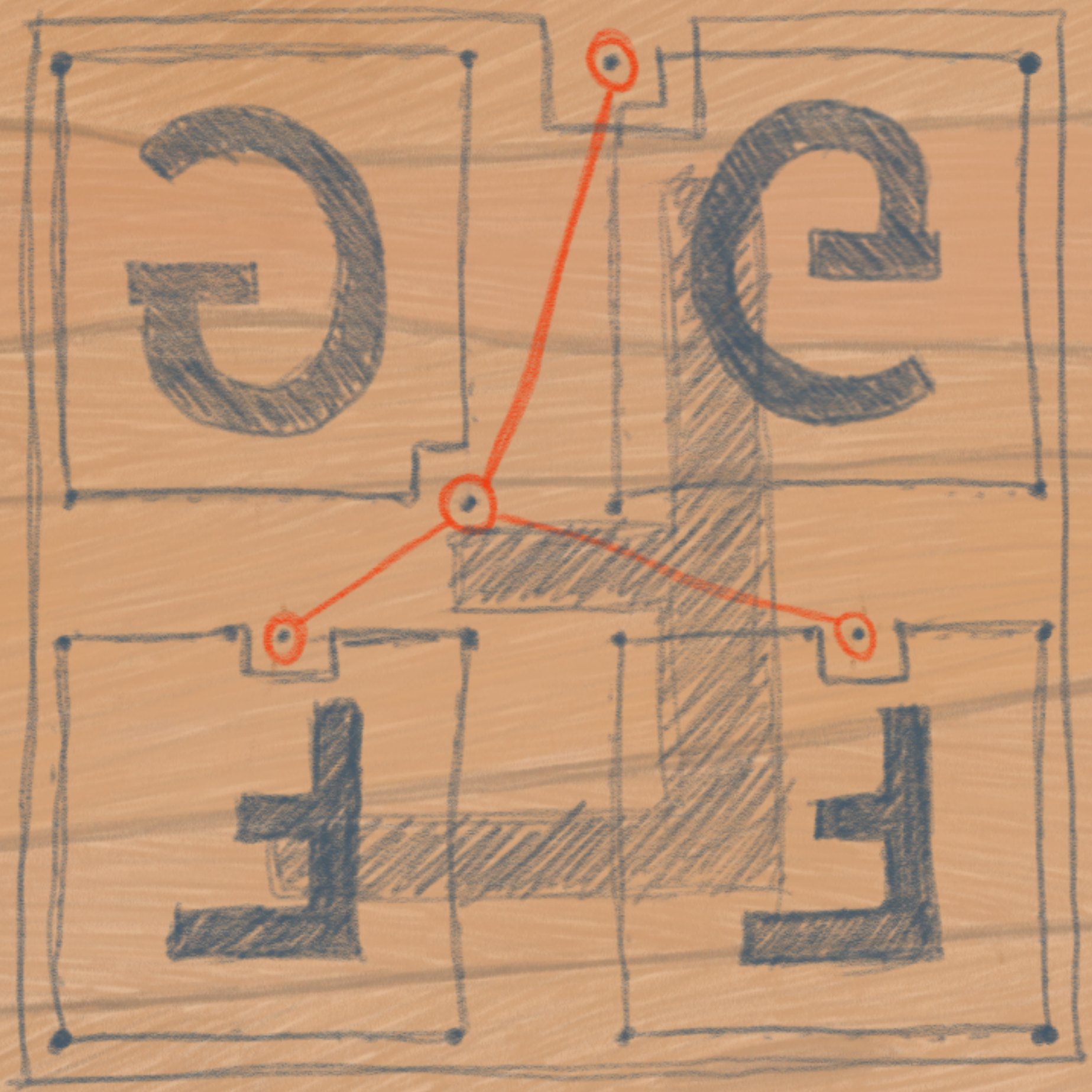


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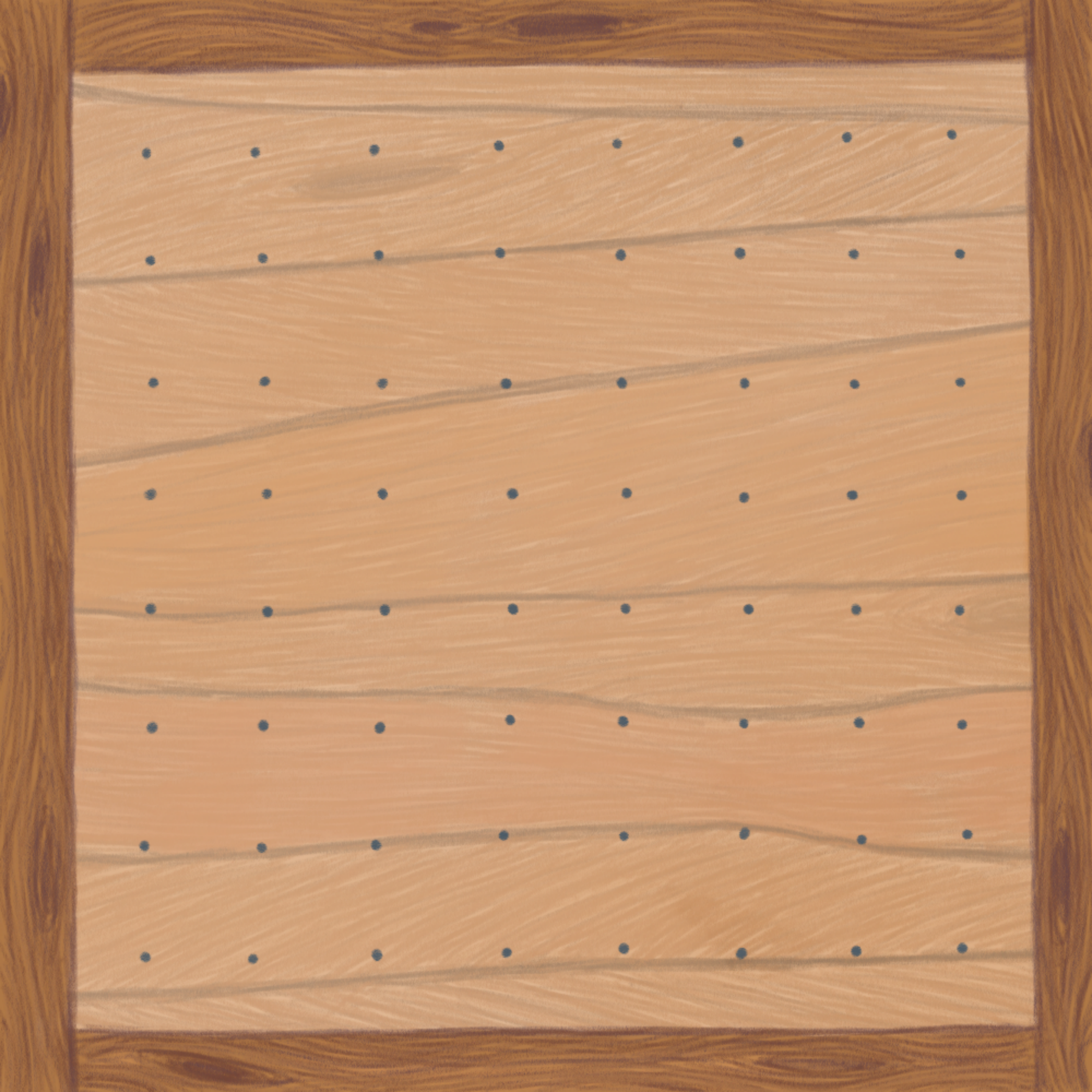
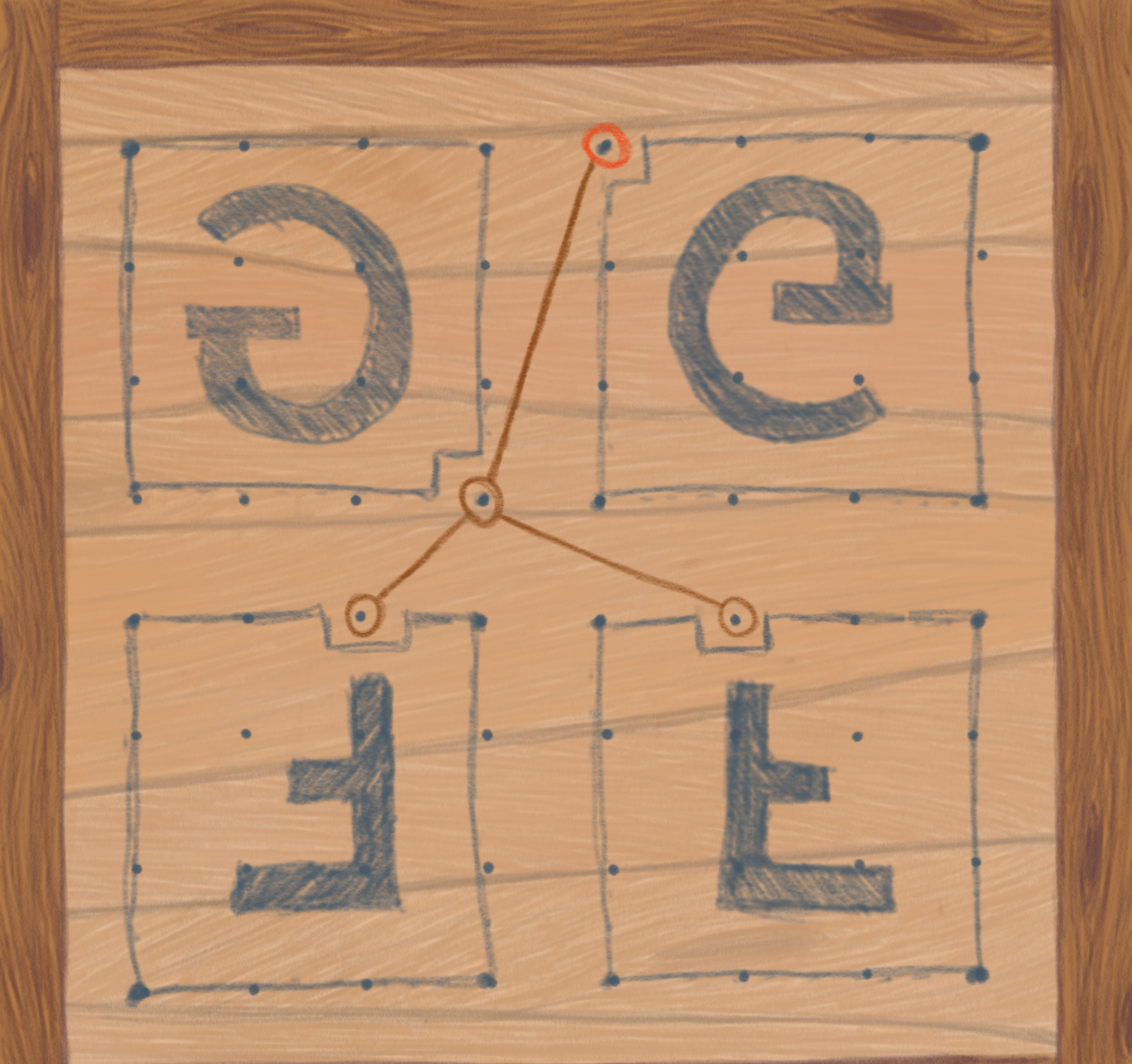
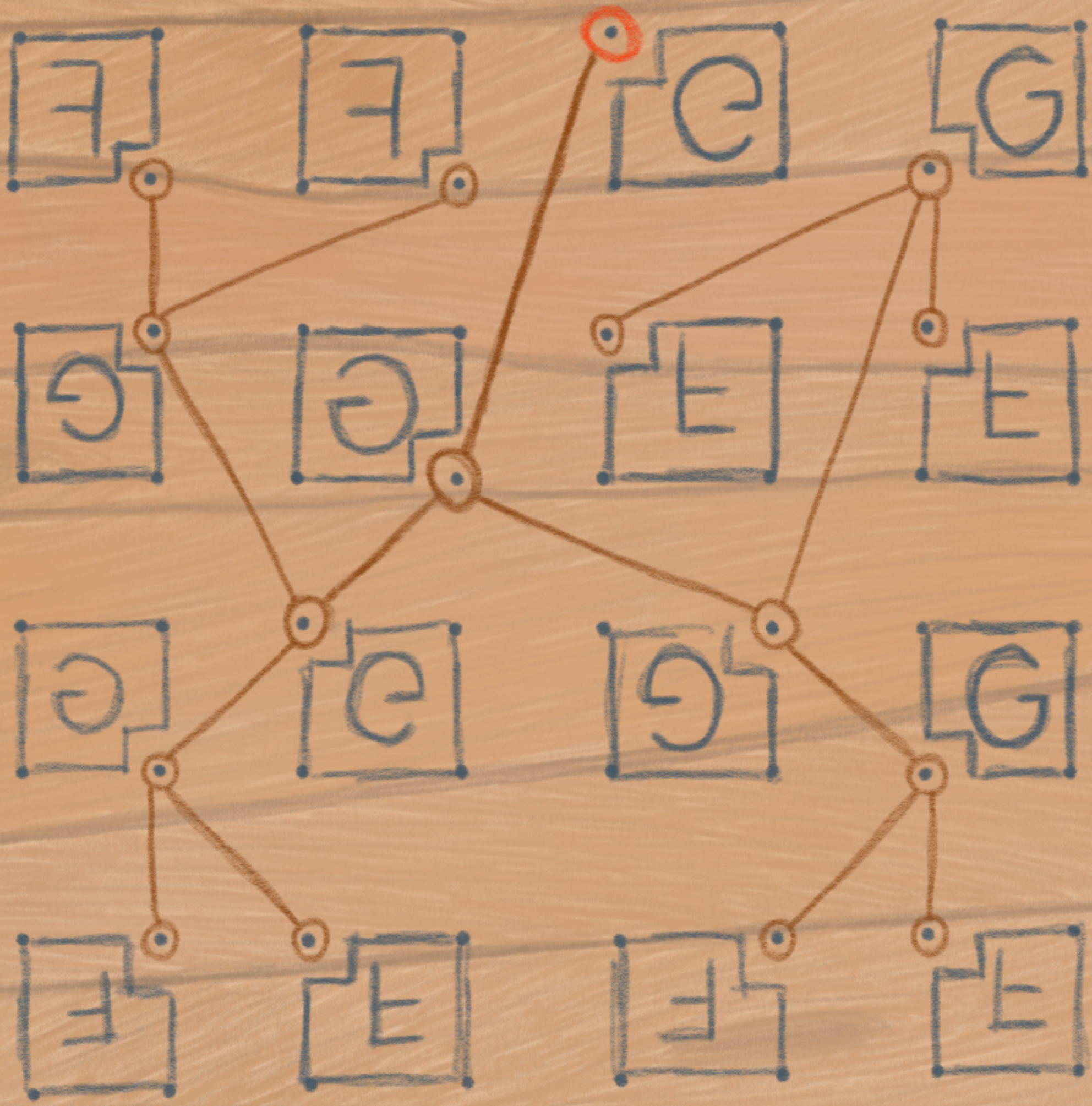
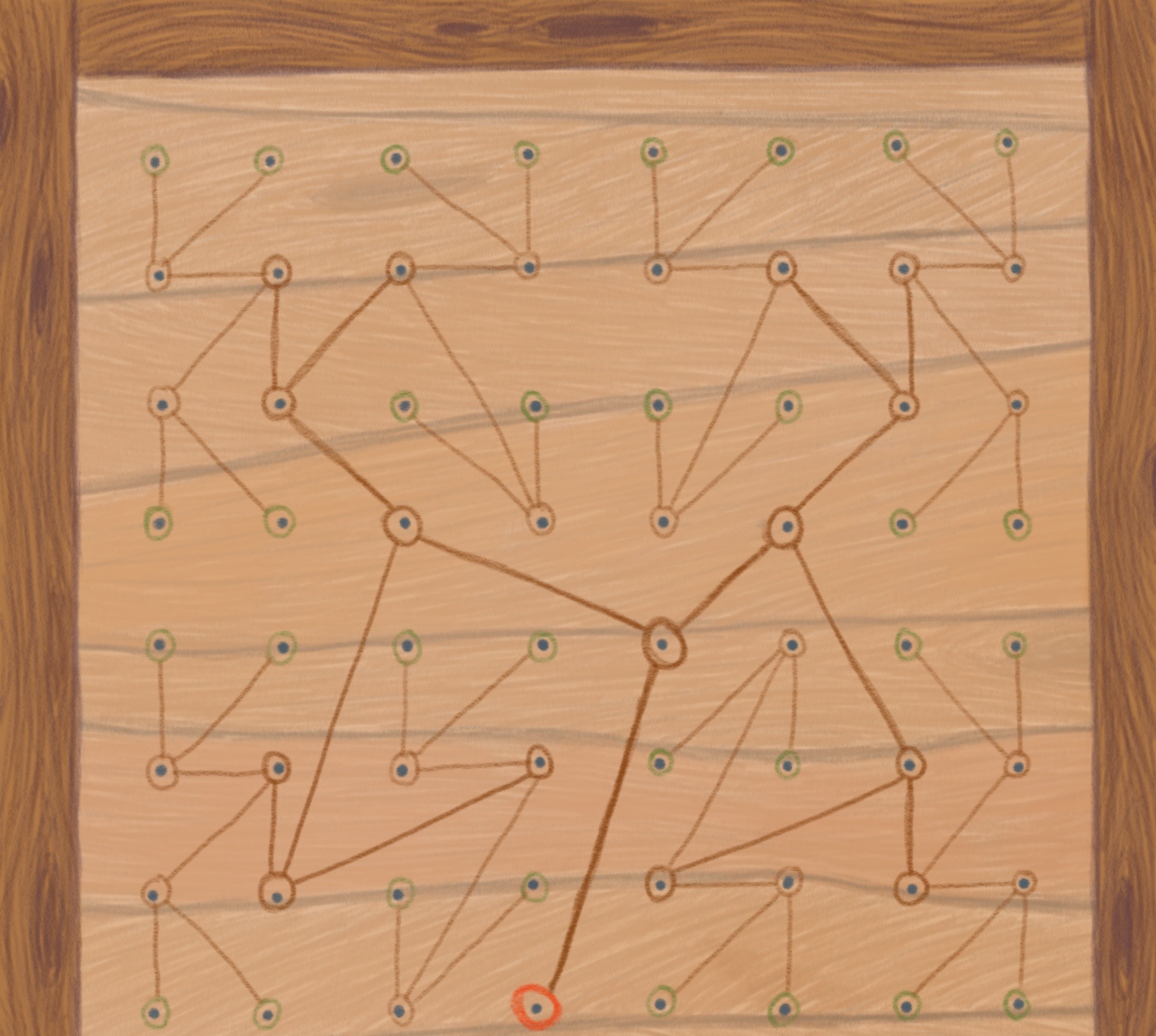



DIG
IT





Conjecture

Every binary tree (degree ≤ 3)
with $n = 2^k$ vertices (for even k)
has a planar straight-line embedding
onto the $\sqrt{n} \times \sqrt{n}$ grid.


THANK YOU!