AG Homework March 22

Exercise. Let $0 \to A \to B \to C \to 0$ be an exact sequence in $\mathfrak{C}$ and let $I^\cdot$ and $J^\cdot$ be injective resolutions of A and C respectively. Then there exists an exact sequence

$$0 \to B \to I^0 \oplus J^0 \to I^1 \oplus J^1 \to \dots$$

such that the following diagram commutes:

$$\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & A & \longrightarrow & B & \longrightarrow & C & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & I^0 & \longrightarrow & I^0 \oplus J^0 & \longrightarrow & J^0 & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & I^1 & \longrightarrow & I^1 \oplus J^1 & \longrightarrow & J^1 & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & \vdots & & \vdots & & \vdots & &
\end{array}$$

where the morphisms $I^\cdot \to I^\cdot \oplus J^\cdot$ and $I^\cdot \oplus J^\cdot \to J^\cdot$ are the canonical ones.