Exercises for Cech Cohomology

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1. X is a topological space, $\mathscr{F}$ is a sheaf of abelian groups on X, $\mathfrak{U}$ is an open covering of X, section $\alpha \in \Gamma(V, \mathscr{C}^p(\mathfrak{U}, \mathscr{F}))$, where $V \subset U_j$ for some $U_j \in \mathfrak{U}$. Now for any p-turple $i_0 < \ldots < i_{p-1}$, define $k : C^p \to C^{p-1}$ as

$$(k\alpha)_{i_0,\ldots,i_{p-1}} = \alpha_{j,i_0,\ldots,i_{p-1}}$$

Check that for any $p \geqslant 1$, we have

$$(kd + dk)(\alpha) = \alpha \ ,$$

where d is the coboundary map.

2. Let $X = \mathbb{A}_k^2 = \mathrm{Spec}(k[x, y])$, and let $U = X - \{(0, 0)\}$. Using a suitable cover of U by open affine subsets, show that $H^1(U, \mathcal{O}_U)$ is isomorphic to the k-vector space spanned by $\{x^i y^j |\ i, j < 0\}$. In particular, it is infinite-dimensional.