Exercise associated to the section of Cohomology of a Noetherian Affine Scheme

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Exercise: (Hartshone's Exercise: II, 5.6.)

Support. Recall the notions of support of a section of a sheaf, support of a sheaf, and sub sheaf with supports from (Ex. II, 1.14) and (Ex. II, 1.20).

(a) Let A be a ring, let M be an A-module, let $X = \text{Spec } A$, and let $\mathcal{F} = \tilde{M}$. For any $m \in M = \Gamma(X, \mathcal{F})$, show that Supp $m = V(\text{Ann } m)$, where Ann m is the *annihilator* of $m = \{a \in A | am = 0\}$.

(b) Now suppose that A is noetherian, and M finitely generated. Show that Supp $\mathcal{F} = V(\text{Ann } M)$.

(c) The support of a coherent sheaf on a noetherian scheme is closed.

(d) For any ideal $\mathfrak{a} \subseteq A$, we define a submodule $\Gamma_{\mathfrak{a}}(M)$ of M by $\Gamma_{\mathfrak{a}}(M) = \{m \in M | \mathfrak{a}^n m = 0 \text{ for some } n > 0\}$. Assume that A is noetherian, and $\mathcal{F} = \tilde{M}$.

(e) Let X be a noetherian scheme, and let Z be a closed subset. If $\mathcal{F}$ is a quasi-coherent(respectively, coherent) $\mathcal{O}_X$-module, then $\mathcal{H}_Z^0(\mathcal{F})$ is also quasi-coherent(respectively, coherent).