DISCUSSION:

IS ALGEBRAIC LORENTZ-COVARIANT QUANTUM FIELD THEORY STOCHASTIC EINSTEIN LOCAL?*

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The general context of this paper is the locality problem in quantum theory. In a recent issue of this journal, Rédei (1991) offered a proof of the proposition that algebraic Lorentz-covariant quantum field theory is past stochastic Einstein local. We show that Rédei's proof is either spurious or circular, and that it contains two deductive fallacies. Furthermore, we prove that the mentioned theory meets the *stronger* condition of stochastic Haag locality.

1. Introduction. The formal notion of *past stochastic Einstein locality* (SEL^-) was introduced by Hellman (1982b) to close the gap between the physical requirements of special relativity, especially the requirement of no superluminal action, and the idea of *Bell locality*, which is the notion that (the probability of) any measurement outcome does not depend on something outside the *backward* light cone of the measurement event. In discussing the problem of hidden variables for algebraic Lorentz-covariant quantum field theory (AQT) and its relation to a recently proven violation of Bell's inequalities in AQT, Rédei (1991) purported to prove that AQT is a past stochastic Einstein local theory. In our view Rédei failed to do just that.

AQT, founded essentially by R. Haag in the late 1950s and the 1960s, is a heroic attempt to turn Lorentz-covariant quantum field "theory" into

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a mathematically rigorous, axiomatized *theory* like quantum mechanics. Since quantum field "theory" is at present the best we have about the ultimate constituents of the physical world, Haag's and his followers' endeavor is one of the most profound intellectual enterprises of our age. After some introductory remarks to give some idea of what AQT is about (sec. 2), we present an improved formulation of SEL for AQT and explicate Rédei's proof of the Proposition that AQT is past stochastic Einstein local (sec. 3). Our explication is far more detailed than Rédei's original proof, *not* because we love to dwell in an orgy of mathematical and logical "technicalities", but because *only if* the details that Rédei glosses over are put on the stage is it possible to point out the deficiencies of the proof rigorously, hence convincingly. We show that Rédei's proof is either spurious or circular, or else commits a deductive fallacy; that the proof method is incorrect if the spacetime transformation applied in the proof is *active*, which is an obvious reading of the proof; and that it becomes especially spurious if the spacetime transformation is *passive*, which is a less obvious reading of the proof (sec. 4). Subsequently, we offer a simple and straightforward proof that AQT obeys what we call *stochastic Haag locality* (sec. 5). This result yields an affirmative answer to the question in the title of this paper, because this latter locality condition entails SEL.

2. Algebraic Lorentz-Covariant Quantum Field Theory. These introductory remarks on AQT are based on Horuzhy (1990, chap. 1) and Haag (1992). AQT comes in two garden-variety types: an *abstract* one and a *concrete* one. In abstract AQT one starts with the fundamental mapping $O \mapsto \mathfrak{A}(O)$, where *spacetime region* $O \in \mathcal{B}(\mathcal{M})$, where $\mathcal{B}(\mathcal{M})$ is the set of all open-bounded subsets of the Minkowski spacetime manifold $\mathcal{M}$ from special relativity, and where $\mathfrak{A}(O)$ is an abstract C^*-algebra of *local observables on* O. All local algebras are contained in a *quasi-local net* $\mathfrak{A}$, whose properties are specified by the axioms of AQT. Any physical magnitude that is measurable inside spacetime region O is represented by a local observable in the algebra $\mathfrak{A}(O)$. Let $\boldsymbol{\Psi}$ be the set of states; a *state* $\psi \in \boldsymbol{\Psi}$ is defined as a normalized, continuous, positive, linear functional on any local algebra, mapping observables to complex numbers ($\psi : \mathfrak{A}(O) \mapsto \mathbf{C}$). The complex number $\psi(A)$ is the *expectation value* of observable $A \in \mathfrak{A}(O)$ of a system in state ψ. This is the main correspondence rule between the formalism of AQT and measurement results, which is analogous to the Born Rule in Galilean-covariant quantum mechanics. (Given the usual but unnecessary restriction to the self-adjoint part of $\mathfrak{A}$, the expectation values are real.) (In concrete AQT the abstract C^*-algebra $\mathfrak{A}(O)$ is replaced by a von Neumann-algebra $\mathcal{R}(O)$ of operators acting in a Hilbert space and the quasi-local net $\mathfrak{A}$ by a *global net* $\mathcal{R}$.) To present and assess Rédei's proof, and to give our proof, we especially need the following three points of abstract AQT, which is the type considered by Rédei.

First, the probability of finding a value in the Borel set $a \in \mathcal{B}(\mathbf{R})$ when measuring any local observable $A \in \mathfrak{U}(O)$ of a system in state $\psi \in \mathbf{\Psi}$ equals the expectation value of a projector local observable $P_a \in \mathfrak{U}(O)$ in state ψ:

$$\text{Prob}([A]^{\psi} \in a) = \psi(P_a), \tag{1}$$

where $[A]^{\psi}$ denotes the measured value of local observable A of a system in state ψ.

The second point concerns the *axiom of Lorentz-covariance* of AQT. The Poincaré, or Inhomogeneous Lorentz group $\mathcal{P}$ is the spacetime symmetry group of AQT (actually the covering group $\bar{\mathcal{P}}$ in order to deal with spin—we gloss over this point), which is implemented as follows: To each Poincaré transformation $g := (\Lambda, b)$, where Λ denotes a Lorentz boost and b a spacetime translation, there corresponds an automorphism α_g on the net $\mathfrak{U}$ and vice versa:

$$\text{AQT} \vdash \mathfrak{U}(O) \mapsto \mathfrak{U}(g(O)) = \alpha_g \mathfrak{U}(O). \tag{2}$$

This automorphism means that the local algebra $\mathfrak{U}(O)$ of spacetime region O is mapped to the local algebra of the Poincaré transformed spacetime region $O \mapsto g(O)$ in such a way that all algebraic relations between the observables are conserved (form-invariance) (see Horuzhy 1990, 16, and Haag 1992, 110). The states and observables in AQT transform by $g \in \mathcal{P}$ as follows (the existence of inverse transformations is a group property):

$$\psi \mapsto \psi' = \psi \circ \alpha_g^{inv} \text{ and } A \mapsto A' = \alpha_g \circ A \circ g^{inv}. \tag{3}$$

These spacetime transformations (3) of AQT leave scalar quantities invariant, such as expectation values: $\psi(A(O)) = \psi'(A'(g(O)))$.

The third point is that we should mention the other axioms of AQT for use in the subsequent sections. The *isotony axiom* states that the local algebra on a spacetime region is a subalgebra of the local algebra on any superregion:

$$\text{AQT} \vdash O_1 \subset O_2 \rightarrow \mathfrak{U}(O_1) \subset \mathfrak{U}(O_2). \tag{4}$$

The *axiom of local commutativity* states that the algebras of two spacelike separated spacetime regions elementwise commute. The *diamond axiom* states that the algebra of observables on the *domain of dependence* of a spacetime region O, denoted by $D(O)$, is identical to the algebra of observables on O:

$$\text{AQT} \vdash \mathfrak{U}(O) = \mathfrak{U}(D(O)). \tag{5}$$

Here the domain of dependence of the spacetime region O is defined as $D(O) := D^-(O) \cup D^+(O)$, where the *future domain of dependence of* O, denoted by $D^+(O)$, is defined as a spacetime region consisting of all points x such that any smooth past inextendible nonspacelike curve that goes through x, intersects O; and *mutatis mutandis* for the *past domain of dependence of* O, denoted by $D^-(O)$. The *additivity axiom* states that the algebra of observables $\mathfrak{A}(O_1 \cup O_2)$ on the union of the two distinct spacetime regions O_1 and O_2 is generated by the algebras $\mathfrak{A}(O_1)$ and $\mathfrak{A}(O_2)$ on these spacetime regions. The final axiom of AQT is the *spectral axiom*, which states that the energy-momentum spectrum is positive.

The axioms of abstract AQT are provably independent and consistent (see Horuzhy 1990, 20). There is little doubt that the field observables of all observationally adequate quantum field theories, like quantum electrodynamics and quantum chromodynamics, obey these axioms (see Haag 1992, sec. 3.1). (The axioms of concrete AQT are completely analogous to the ones stated above.)

3. Stochastic Einstein Locality and Rédei's Proof. The definition of stochastic Einstein locality (SEL) is intended to capture the idea of locality for *stochastic* theories, meaning theories that in general do not ascribe values to physical magnitudes but give probabilities for having values or finding values upon measurement. SEL demands of a stochastic theory that the probabilities for finding the value for any physical magnitude pertaining to a spacetime region G in some Borel set of the reals ($\mathbf{R}$) are determined by everything inside the light cone of G. The definition of SEL in the context of AQT is as follows:

> DEFINITION 3.1. *AQT is stochastic Einstein local (SEL) iff the following holds: For any two models of AQT, for any spacetime region G, if the models agree inside the light cone of G, then they agree on all the probabilities for each local observable on G under two provisos.*

We first have to explain what the *definiens* means formally.

Hellman (1982a, 448–449) and Rédei (1991, 632) adhere to the traditional, so-called *syntactical view* of scientific theories according to which AQT is characterized as a subset of sentences of a formal language $\mathcal{L}$ (predicate logic) closed under derivation. We call $M := \langle \mathcal{M}, \mathfrak{A}, \psi \rangle$ *a model* of AQT, where we have suppressed the interpretation of the terms of the formal language, which is "generated" by the structure (see, e.g., van Dalen 1989, sec. 2.3); the valuation ($\models$) of the sentences, in order to keep notations simple: all sentences derivable from AQT are satisfied ($\models$) by any model of AQT (soundness—see, e.g., van Dalen ibid., 72); and the field of complex numbers. Hellman (1982a) assumes that the theory under consideration "specifies a background ontology of Minkowski space-time: every model M of [the theory] . . . contains a manifold $\mathcal{M}$ of 'events' " (p. 448; our notation). This is true for AQT, though regions replace points. The notion of *matching* between models is construed linguistically: as giving the same valuations to all sentences of the language of the models. This leads to the formal definition of SEL below, where $G_1, \ldots, G_k$ is a k-tuple of open-bounded spacetime regions in $\mathcal{B}(\mathcal{M})$. $\text{Pred}(\mathcal{L}) := \cup_n \text{Pred}(\mathcal{L}, n)$ where the latter is the set of n-ary predicates of $\mathcal{L}$ naming the n-ary relations R^n and the $(n-1)$-ary functions f^{n-1}. Let $LC(O)$ denote the light cone of O: $LC(O) := \cup_{x \in O} LC(x) \setminus O$ where $LC(x) := \{y \in \mathcal{M} | d(x, y) \geq 0\}$, where $d(x, y)$ is the spatiotemporal distance between x and y in $\mathcal{M}$. The backward and forward light cone of a spacetime region O are denoted by $LC^-(O)$ and $LC^+(O)$, respectively; of course $LC(O) = LC^-(O) \cup LC^+(O)$.

FORMAL DEFINITION 3.1. *AQT is past stochastic Einstein local* (SEL^-) *in* $G \in \mathcal{B}(\mathcal{M})$ *iff for any two models* $M_j = \langle \mathcal{M}_j, \mathcal{U}_j, \psi_j \rangle$ $(j = 1, 2)$ *of AQT, if* $\forall\, k \in \mathbf{N}$, $\forall\, G_1, \ldots, G_k \subset LC^-(G) \subset \mathcal{M}_j$, $\forall\, R^k$, $\forall f^k \in Pred(\mathcal{L})$, $\forall r \in \mathbf{R}$:

$$M_1 \models R^k(G_1, \ldots, G_k) \leftrightarrow M_2 \models R^k(G_1, \ldots, G_k) \;\wedge$$
$$M_1 \models f^k(G_1, \ldots, G_k) = r \leftrightarrow M_2 \models f^k(G_1, \ldots, G_k) = r,$$

then $\forall\, A \in \mathcal{U}_j(G)$, $\forall\, a \in \mathcal{B}(\mathbf{R})$, $\forall p \in [0, 1]$:

$$M_1 \models Prob([A]^{\psi_1} \in a) = p \leftrightarrow M_2 \models Prob([A]^{\psi_2} \in a) = p$$

under two provisos.

Apart from two additions and one change, this definition of SEL^- in the context of AQT is identical to Rédei's definition. (Rédei has agreed with these changes in personal correspondence; they will have no bearing on the contents of Rédei's proof in the sense that in criticizing his proof we do not commit the fallacy of equivocation concerning SEL^-.) (i) Rédei forgets to mention the two provisos, which should be reformulated in the context of AQT; they are intended to rule out spurious locality violations. An example that apparently turns quantum mechanics into a theory violating SEL^- if provisos are not included is Einstein's remark at the Solvay Conference of 1927 that if *one* free particle is detected somewhere, then the probability of finding it somewhere else drops instantaneously to zero. However, as Hellman submits, "there is no basis . . . for inferring that some energy or force has propagated faster than light" (1982b, 467). So one of his two provisos renders spurious a violation of SEL due to the instantaneous collapse of the wave function in quantum mechanics. Because the two provisos do not come into play in Rédei's proof, we do not give a formal account of them; we refer to Hellman (ibid., 467–478) and Butterfield (1994, secs. 6, 7) for a discussion of these provisos. (ii)

We changed $f^k(G_1, \ldots, G_k)$ into $f^k(G_1, \ldots, G_k) = r$, which is more appropriate for functions. (iii) Rédei omits the states $\psi_1, \psi_2 \in \mathbf{\Psi}$. But they must be added because the probability of finding a value for an observable depends on the state.

AQT is SEL$^-$ iff it is SEL$^-$ in all $G \in \mathcal{B}(\mathcal{M})$. The formal definition of *future stochastic Einstein locality* (SEL$^+$) is obtained by replacing the backward light cone of G by the forward light cone of G in the formal definition of SEL$^-$; and the definition of *stochastic Einstein locality* (SEL) is obtained by replacing the backward light cone of G by the entire light cone of G. The logical relation between these three locality conditions is that each of SEL$^-$ and SEL$^+$ is a sufficient condition for SEL because both are stronger locality conditions (the same is determined by less), but neither SEL$^-$ nor SEL$^+$ is a necessary one. (Thus we distinguish between past and future versions of SEL, though Hellman and Rédei do not. We are motivated to do so by our preference for time symmetry: So long as the spacetime region determining the probabilities in G lies inside, or coincides with, the light cone of G, we refuse to speak of a violation of locality *simpliciter* (SEL). Specifically, if in addition to the backward light cone of G some spacetime region in the *forward* light cone of G contributes to the determination of the probabilities in G, but no spacetime region outside the light cone of G does so, we speak of a violation of *past* locality (SEL$^-$), not of a violation of locality *simpliciter* (SEL), as Hellman and Rédei are prepared to do. For us, to speak of a violation of locality *simpliciter* (SEL), some region *spacelike separated from G* must contribute to the determination of the probabilities in G. This difference from Hellman and Rédei will not prejudge any issues between us and Rédei; our critique could be presented wholly in terms of SEL$^-$.)

We now turn to Rédei's claim to prove his paper's central proposition:

PROPOSITION. *Algebraic Lorentz-covariant quantum field theory is a past stochastic Einstein local theory.*

Rédei's proof (1991, 633–634) is a *reductio ad absurdum* argument. Assume AQT and that AQT violates SEL$^-$. Then there exists a pair of models M_1 and M_2 of AQT and a spacetime region G for which the antecedent of SEL$^-$ is true and the consequent false. We start by clarifying the latter. Rédei implicitly adopts what we call *algebra-matching* on G, which means that the local algebras on G in M_1 and M_2 are the same:

$$\mathcal{U}_1(G) = \mathcal{U}_2(G). \tag{6}$$

Rédei even implicitly adopts the stronger matching of the two entire nets of observables of the two models. But that supposition, to be referred to as *net-matching*, is unnecessarily strong. In section 5 we *deduce* algebra-matching on G; for now we treat Rédei charitably by granting him algebra-matching on G. The consequent of SEL^- is assumed to be false. Since Rédei ignored the provisos, we interpret him as assuming, for *reductio*, that a genuine violation of SEL^- occurs. In other words, again we interpret Rédei charitably so that merely spurious violations are not at issue in his proof. We continue with the following *reductio* assumption: There is a local observable $A(G) \in \mathcal{U}(G)$ and a Borel set $a \in \mathcal{B}(\mathbf{R})$ such that the consequent of SEL^- is false for some $p \in [0, 1]$. Rédei implicitly supposes, presumably for simplicity, that the observable $A \in \mathcal{U}(G)$ pertains to the entire spacetime region G (like the total energy in G) in order to justify writing $A(G)$. To see that this supposition is not compulsory, notice that every observable pertaining to a subregion of G (like the total energy of some region $O \subset G$), is also contained in $\mathcal{U}(G)$ due to the isotony axiom (4), in which case writing $A(G)$ would be misleading—one would write $A(O)$. So by giving Rédei this implicit supposition, we again interpret him charitably. The negation of the consequent of SEL^- is true *ex hypothesi*, which yields in combination with (1) that for some projector $P_a \in \mathcal{U}(G)$ there is a $p \in [0, 1]$ such that:

$$M_1 \vDash \psi_1(P_a(G)) = p \wedge M_2 \vDash \psi_2(P_a(G)) \neq p. \tag{7}$$

Define the real scalar function $h_j := \psi_j \circ P_a$ $(j = 1, 2)$, so that $h_j(G) = \psi_j(P_a(G))$; rewriting (7) in these terms yields (a implicit in h_j):

$$M_1 \vDash h_1(G) = p \wedge M_2 \vDash h_2(G) \neq p. \tag{8}$$

This is the final form of the *reductio* assumption. Now the proof can really commence.

Let $b^\mu \in \mathbf{R}^4$ be a timelike vector pointing backwards; choose $\|b^\mu\|$ sufficiently large so that it points from G to a similar disjoint spacetime region bG in its own backward light cone. The second biconditional of the antecedent of SEL^- holds *ex hypothesi* for the special case $G_1 := bG \subset LC^-(G)$ $(k = 1)$ for all real scalar functions f of the theory $(r \in \mathbf{R})$:

$$M_1 \vDash f(bG) = r \leftrightarrow M_2 \vDash f(bG) = r. \tag{9}$$

Now Rédei performs the (active) spacetime translation $b \in \mathcal{P}$ to the situation. Spacetime region G is shifted (actively) onto the spacetime region $bG \subset LC^-(G)$; so the image region of G under the Poincaré map b, denoted by $b(G)$, and the spacetime region bG of the same form as G lying inside $LC^-(G)$ are identified (see figure 3.1a):

$$b(G) = bG. \tag{10}$$

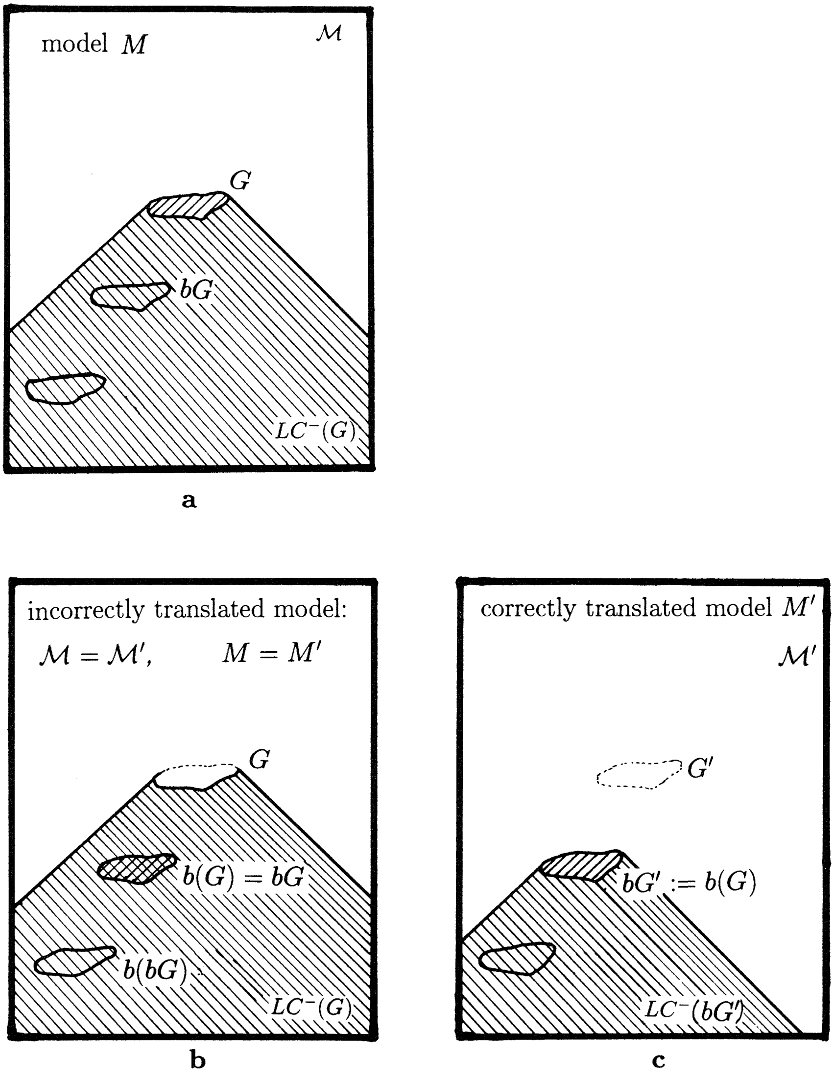

Figure 3.1

At least, that is how we interpret the role of b; that the transformation b is active in contrast to passive (coordinate transformation) is strongly suggested by Rédei's coordinate-free description—for further elaboration of this point, see section 4, parts (c) and (d). Observable

$P_a(G)$ and state ψ_j transform according to (3) with $g = b$. As can be easily checked by using (3), h_j transforms to $h'_j := \psi'_j \circ P'_a = h_j \circ b^{inv}$ where $P'_a \in \mathcal{U}(b(G))$. So we obtain from (8) and (10):

$$M_1 \models h'_1(bG) = p \wedge M_2 \models h'_2(bG) \neq p. \tag{11}$$

Next Rédei implicitly adopts what we call *state-matching on b(G)*, which means that $\forall B \in \mathcal{U}(b(G))$: $\psi'_1(B) = \psi'_2(B)$. Such state-matching on $b(G)$ is usually denoted by:

$$\psi'_1|\mathcal{U}(b(G)) = \psi'_2|\mathcal{U}(b(G)). \tag{12}$$

This yields for (11) because $P'_a \in \mathcal{U}(b(G))$:

$$M_1 \models h'(bG) = p \wedge M_2 \models h'(bG) \neq p \tag{13}$$

where $h' := h'_1 = h'_2$ and for which we have used (10) in (12). On the other hand, since (9) holds for all real scalar functions, one is allowed to choose the special case $f = h'$; so $\forall r \in \mathbf{R}$:

$$M_1 \models h'(bG) = r \wedge M_2 \models h'(bG) = r. \tag{14}$$

Whatever the value of p—introduced by existential quantification in the negation of the consequent of SEL$^-$: (7)—that is, whether $p = r$ or $p \neq r$, statement (13) contradicts (14). To avoid this absurdity, Rédei concludes that AQT obeys SEL$^-$. QED

Rédei's own version of this proof is far more concise than ours; for instance, Rédei does not explicitly mention algebra-matching on G, net-matching, or state-matching on $b(G)$. The more detailed exposition above will pay off considerably in section 4 where the ramifications of these matching assumptions will be discussed without mercy and the deductive fallacies will be exposed.

4. Refutation of Rédei's Proof. First, Rédei's proof uses only the symmetry of the group of spacetime *translations*, which is a proper subgroup of the Poincaré group $\mathcal{P}$. So the proof holds too for an algebraic field theory having the Galilei group as its spacetime symmetry group. But surely a Galilean-covariant field theory need not obey SEL$^-$. Crucial ingredients one expects to appear in the proof are the axiom of local commutativity or the diamond axiom (5). But these axioms are absent from the proof. This indicates an error within the proof.

Second, from a logical point of view, Rédei's proof would be conclusive iff algebra-matching (6) on G and state-matching (12) on $b(G)$ were derived from the premise AQT or from the *reductio* assumption $\neg$SEL$^-$, because the *reductio* argument establishes only that AQT, $\neg$SEL$^-$ and the two matching assumptions (6) and (12) form an inconsistent set of premises. Since algebra-matching on G and state-matching on $b(G)$ are not deduced from the other two premises but tacitly assumed, the proof is inconclusive—even *if* the reasoning were valid. In section 5 we deduce algebra-matching (6) on G; in what follows we therefore embrace (6). We treat Rédei even more charitably by giving him net-matching, too. Rédei offers no argument in favor of net-matching, though there is a good physical motivation for adopting it. The net of observables supposedly characterizes a physical system. Without net-matching, one would be comparing models which pertain to different physical systems; in other words, one would be comparing eggs and apples. So even if such a comparison were to give rise to a violation of SEL, it would not be genuine. This consideration would lead to an extra proviso in SEL, restricting the choice of a pair of models to pairs having identical nets. Despite this physical plausibility argument, in section 5 we do not need such restrictions in our theorem.

In this section we endeavor to show that (a) without state-matching on bG, the proof contains a *non sequitur*; (b) with state-matching on bG, the proof is either circular or spurious; (c) the proof method is fallacious if we take the employed spacetime transformation to be active; and (d) the proof is especially spurious if we take the employed spacetime transformation to be passive.

(a) If one does not assume algebra-matching (6) on G and state-matching (12) on $b(G)$, then there is no reason to identify the functions $h'_1 := \psi'_1 \circ P'_a$ and $h'_2 := \psi'_2 \circ P'_a$. Whence the inference of statement (13) from (11) in Rédei's proof is a *non sequitur*.

(b) Recall that we granted Rédei algebra-matching (6) on G. On the other hand, if we also assume state-matching (12) on $b(G)$, then statement (13) does indeed follow from (11), given the identification (10) of spacetime regions $b(G)$ and bG; therefore, conditional on the *reductio* assumption (8), we endorse (13). So on these assumptions is Rédei home?

Rédei never left home, really. Let us assume algebra-matching (6) on G, which is, due to (2), equivalent to assuming algebra-matching on $b(G)$. Let us recall that due to (3) and to the Lorentz covariance axiom (2):

$$\text{AQT} \vdash \psi'_1|\mathfrak{U}(b(G)) = \psi'_2|\mathfrak{U}(b(G)) \leftrightarrow \psi_1|\mathfrak{U}(G) = \psi_2|\mathfrak{U}(G). \tag{15}$$

Assuming also state-matching (12) on $b(G)$ is, due to (15), equivalent to assuming algebra-matching on G, that is, $\forall A \in \mathfrak{U}(G)$, $\forall z \in \mathbf{R}$:

$$M_1 \models \psi_1(A) = z \leftrightarrow M_2 \models \psi_2(A) = z. \tag{16}$$

By invoking (1) we see that on the matching assumptions (6) and (12), one is assuming in one majestic sweep that the models also match on their probabilities of finding any value for any local observable on G, which is precisely the consequent of SEL^-. If on these matching assumptions one tries to deduce the consequent of SEL^- directly, then the proof is bound to be a *petitio principii* because the consequent of SEL^- is already assumed. In this case Rédei's coming home with a conclusion is a hoax, for he stuck the conclusion in his pocket when he left home.

(One might reply that if

$$(6), (12) \vdash \text{consequent of SEL}^-$$

is proven, it follows trivially by the principle of weakening the premises that

$$(6), (12), \text{antecedent of SEL}^- \vdash \text{consequent of SEL}^-.$$

That is of course perfectly true. However, our point is not that the proof based on the matching assumptions (6) and (12) is fallacious, but that it is spurious because *the antecedent of* SEL^- *is on this account nowhere used in the proof.* Put differently, it also follows by the principle of weakening the premises that

$$(6), (12), \neg(\text{antecedent of SEL}^-) \vdash \text{consequent of SEL}^-.$$

That is, if a proof based on the matching assumptions (6) and (12) were accepted as a genuine proof, one would also have "proven" the following: Two models that do *not* match on the backward light cone of G ($\neg$(antecedent of SEL^-)) do, nonetheless, match on all their probabilities in G. This is, we feel, somewhat contrary to what Rédei intends to prove.)

If, on the other hand, one tries to prove SEL^- by a *reductio* argument starting from the matching assumptions (6) and (12), that is, by assuming matching on $LC^-(G) \cup G$, then the *reductio* assumption (7), asserting a difference in valuation of some sentence referring to the subset $G \subset LC^-(G) \cup G$, is already in blatant contradiction to the assumed state- and algebra-matching on $LC^-(G) \cup G$. There is no *proof* in this case, only the blunt assertion of premises that are virtually each other's logical negations. Having hardly opened door (8) to leave home, Rédei stumbles upon an absurdity in the doorway. So there is no point in dancing around the house with the spacetime translation $b \in \mathcal{P}$, *which is the purported content of the proof.*

Obviously state-matching on $b(G)$ has to be rejected as a premise. One should try to *deduce* (12), or equivalently, (16), from the antecedent of SEL^-. We conclude that Rédei's proof either (a) contains a fallacious move or else (b) is either circular or spurious.

(c) A global coordinate frame of a spacetime manifold is a smooth one-to-one mapping $F : \mathcal{M} \mapsto \mathbf{R}^4$. A *passive* spacetime transformation is a smooth one-to-one mapping from one coordinate frame to another of the same manifold: $F[\mathcal{M}] \mapsto F'[\mathcal{M}]$. An *active* spacetime transformation is a smooth one-to-one mapping from one manifold to itself or to another manifold: $g : \mathcal{M} \mapsto \mathcal{M}'$. To every active transformation from one manifold to itself ($\mathcal{M} = \mathcal{M}'$) corresponds a passive one, and vice versa, by means of $x^{\mu}(g(p)) = x'^{\mu}(p)$—the coordinates of the (actively) transformed point equal the (passively) transformed coordinates of the original point. A passive transformation is just a relabeling of the spacetime points of the manifold, whereas an active transformation means going to another manifold or "moving around" in one manifold. Manifolds $\mathcal{M}$ and $\mathcal{M}'$ do not necessarily coincide; they may even have disjoint base sets, in which case the manifolds have no spacetime points in common. The Poincaré symmetry of AQT can be expressed beautifully in active language as follows: If $M = \langle \mathcal{M}, \mathcal{U}, \psi \rangle$ is a model of AQT, then every actively Poincaré transformed model $M' = \langle \mathcal{M}', \alpha_g \mathcal{U}, \psi' \rangle$ is also a model of AQT.

To avoid muddles, one must distinguish the original model M from an actively transformed model M'; one should also distinguish the manifold $\mathcal{M}$ of M from the manifold $\mathcal{M}'$ of M', which may be called each other's *counterparts*. The first distinction is compulsory; the latter is not, but we recommend it for clarity.

The method of Rédei's proof is the following. Assume matching between the two models inside the backward light cone of spacetime region G, but no-matching on G. Transform spacetime region G by translating it onto spacetime region bG inside its own backward light cone. Due to the Poincaré symmetry, spacetime region $b(G) = bG$ is a no-matching spacetime region inside the backward light cone of G (see figures 3.1a and 3.1b). But inside the light cone of G, matching was assumed. Contradiction.

This method is hopelessly wrong because *after* the translation $b \in \mathcal{P}$ has been performed, the matching region is no longer the backward light cone of G: $b(LC^{-}(G)) \neq LC^{-}(G)$, but the backward light cone of $b(G)$: $b(LC^{-}(G)) = LC^{-}(b(G))$—see figure 3.1c. Rédei translates only the no-matching region G *but forgets to translate the matching region* $LC^{-}(G)$. In other words, Rédei forgets to translate *everything relevant to the situation under consideration*, which consists according to the antecedent of SEL^{-} of all functions and relations in the entire backward light cone of G. Forgetting to translate the backward light cone of G leads Rédei to contradictory statements about whether bG is a matching region or not *after* the translation (*no-matching* according to (13)) and *before* the translation (*matching* according to (14)), whereas according to logic there is before *and* after either matching or no-matching between the models in a spacetime region bG. *After* the translation has been performed, there *should not* be matching inside the backward light cone of the original spacetime region G, but inside the backward light cone of the translated spacetime region $b(G)$. So if cognizance is taken of everything relevant to the situation under consideration, no contradictions arise.

We can actually pinpoint the deductive fallacy by distinguishing the models ($M_j \neq M_j'$) and, just for clarity, their manifolds too. Then $b(G) \neq bG$, which invalidates the identification (10) of these spacetime regions because they are now counterpart regions belonging to distinct manifolds: G, $bG \subset \mathcal{M}_j$ whereas $bG' := b(G) \subset \mathcal{M}_j'$ ($j = 1, 2$; see figure 3.1). Applying the translation $b \in \mathcal{P}$ on the *reductio* assumption (8) yields:

$$M_1' \vDash h_1'(bG') = p \wedge M_2' \vDash h_2'(bG') \neq p. \tag{17}$$

Rédei's inference of (11) from (8) by applying $b \in \mathcal{P}$ comes from the implicit but illicit identification of the models: $M_j = M_j'$, and of the manifolds: $\mathcal{M}_j = \mathcal{M}_j'$, and in particular of the counterpart regions: $bG' = bG$, which is (10). Not (11), but (17) follows from (8). And from (17) and (14) clearly no absurdities emerge.

(d) In our discussion of Rédei's proof in section 3 onwards, we have interpreted Rédei as applying an active spacetime translation to the pair of models. On that natural interpretation, the inference of (11) from (8) is fallacious, as we have just shown in part (c). But perhaps we have misread Rédei's proof; perhaps Rédei never intended to perform an active translation. After all, he never *says* that b is playing the role of an active translation. On the other hand, Rédei often does not mention suppositions and assumptions that are germane to his proof; so his silence is hardly evidence for a misreading. Nonetheless, in this section we now investigate briefly the possibility of interpreting the proof without any appeal to active transformations and, in particular, thereby retaining only *one* pair of models instead of two. We point out that such an investigation can at most circumvent the deductive fallacy (11) from (8), but cannot circumvent the criticisms of parts (a) and (b), which are independent of whether or not active translations are involved in the proof. So a vindication of the entire proof is ruled out beforehand.

We proceed as in our exposition of Rédei's *reductio* proof in section 3: We assume algebra-matching (6) on G and equivocate between bG and $b(G)$ since these expressions are now taken to refer to *one* spacetime region in *one* manifold; the identification (10) is on this account a triviality. The observables on G and bG are then related by an automorphism α_b according to (2), where $b \in \mathcal{P}$ is the transformation linking G to bG. This transformation is now taken to "compare situations in different spacetime regions of one manifold". From the antecedent of SEL$^-$ one has algebra-matching on $bG \subset LC^-(G)$: $\mathcal{U}_1(bG) = \mathcal{U}_2(bG)$. (For an explication of why this follows from the antecedent of SEL$^-$, we refer to the proof in sec. 5.) Consider the restrictions of the state of the models to the algebra $\mathfrak{A}(bG) := \mathfrak{A}_1(bG)$; their matching is also a consequence of the antecedent of SEL^-:

$$\psi_1|\mathfrak{A}(bG) = \psi_2|\mathfrak{A}(bG). \tag{18}$$

Next consider the restrictions of the state to the algebra $\mathfrak{A}(G)$, which are, according to the *reductio* assumption (7), different:

$$\psi_1|\mathfrak{A}(G) \neq \psi_2|\mathfrak{A}(G).$$

Invoking (15) it follows immediately that

$$\psi_1'|\mathfrak{A}(bG) \neq \psi_2'|\mathfrak{A}(bG). \tag{19}$$

To obtain a contradiction between (18) and (19) in a logically impeccable way, one is forced to assume that ($j = 1, 2$):

$$\psi_j|\mathfrak{A}(bG) = \psi_j'|\mathfrak{A}(bG).$$

Since spacetime region G and translation b are arbitrary, this assumption means that ψ_j is a *translational invariant state*; that is, ψ_j is the vacuum state $\mathbf{\Omega} \in \mathbf{\Psi}$. But a proof of SEL^- for AQT which is restricted to pairs of models *both* in the vacuum state yields, we submit, a vacuous proof. Furthermore, if both models have the same state $\mathbf{\Omega}$, one assumes state matching (12) on $b(G) = bG$, which yields, as we have shown in part (b), a spurious proof.

Hence we conclude that if we interpret Rédei's proof as not performing an active translation so as to retain one pair of models, we end up with an especially spurious proof.

5. Proof of Stochastic Haag Locality. Having in previous sections rebutted Rédei's proof, we turn to the more constructive task of trying to correctly prove SEL for AQT. Before doing so, we must note the various mathematically precise locality concepts in the AQT literature (diamond axiom, local commutativity, split property, Schlieder property, duality, extended locality, C^*-independence, strict locality, and so on; see Horuzhy 1990, 21). There are also various locality concepts in the philosophical and foundations of physics literature about the threat of a locality violation, or even of superluminal causation, arising from the violation of Bell's inequalities, to which Rédei (1991) also refers. Hellman (1982b) introduced SEL as such a locality concept. In this section we prove that AQT satisfies SEL and an even stronger locality notion we call *stochastic Haag locality* (SHL).

We first define past stochastic Haag locality.

DEFINITION 5.1. *AQT is past stochastic Haag local (SHL^-) iff the following holds: For any two models of AQT, for any spacetime region*

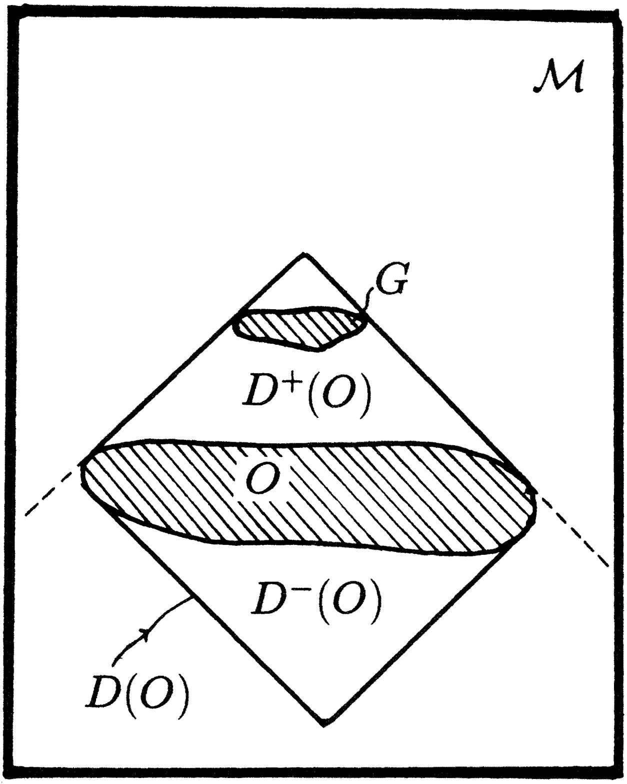

Figure 5.1

> *G, if the models match on some slab O lying fully across the backward light cone of G, then they agree on the expectation values of each local observable on G.*

SHL$^-$ differs from SEL$^-$ in two respects. The first is that the determining spacetime region O is bounded (see figure 5.1), whereas the determining spacetime region in SEL$^-$, namely, the backward light cone of G, is unbounded. So SHL$^-$ is stronger than SEL$^-$: The same is determined by less. The second difference is the deletion of the two provisos. The reason is that the consequent of SHL is limited to expectation values, so the probabilities conditional on measurement results obtained elsewhere-when, particularly at spacelike separation, are explicitly not included, which removes the need for provisos; for a discussion of this point, we refer again to Hellman (1982b, 467–468) and Butterfield (1994, secs. 6, 7).

Let us call spacetime region O a *backward light cone slab of* G iff $O \subset LC^-(G)$ and $G \subset D^+(O)$. See figure 5.1. Let $\mathcal{S}^-(G) \subset \mathfrak{B}(\mathcal{M})$ denote the set of all backward light cone slabs of spacetime region G.

FORMAL DEFINITION 5.1. *AQT is past stochastic Haag local (SHL⁻) in* $G \in \mathfrak{B}(\mathcal{M})$ *iff for any two models* $M_j = \langle \mathcal{M}_j, \mathfrak{A}_j, \psi_j \rangle$ $(j = 1, 2)$ *of AQT,* $\forall\, O \in \mathcal{S}^-(G)$:
If $\forall\, k \in \mathbf{N}$, $\forall\, O_1, \ldots, O_k \subseteq O$, $\forall\, R^k$, $\forall\, f^k \in Pred(\mathcal{L})$, $\forall r \in \mathbf{R}$:

$$M_1 \vDash R^k(O_1, \ldots, O_k) \quad \leftrightarrow M_2 \vDash R^k(O_1, \ldots, O_k)$$

$$M_1 \vDash f^k(O_1, \ldots, O_k) = r \leftrightarrow M_2 \vDash f^k(O_1, \ldots, O_k) = r,$$

then $\forall\, A \in \mathfrak{A}(G)$, $\forall\, z \in \mathbf{R}$:

$$M_1 \vDash \psi_1(A) = z \leftrightarrow M_2 \vDash \psi_2(A) = z.$$

AQT is SHL⁻ iff it is SHL⁻ in all $G \in \mathfrak{B}(\mathcal{M})$. The definition of *future stochastic Haag locality* (SHL⁺) is obtained by replacing $\mathcal{S}^-(G)$ by $\mathcal{S}^+(G)$ in the definition of SHL⁻; and the definition of *stochastic Haag locality* (SHL) is obtained by replacing $\mathcal{S}^+(G)$ by $\mathcal{S}(G)$, which is defined as the set of all backward and forward light cone slabs of G: $\mathcal{S}(G) := \mathcal{S}^-(G) \cup \mathcal{S}^+(G)$. Hence AQT is stochastic Haag local (SHL) iff it is both SHL⁻ and SHL⁺.

We now state our central theorem:

THEOREM. *Algebraic Lorentz-covariant quantum field theory is a stochastic Haag local theory.*

Proof. In our proof we assume AQT and the antecedent of SHL⁻ for an arbitrary spacetime region $G \in \mathfrak{B}(\mathcal{M})$ and then deduce the consequent of SHL⁻.

Take two models $M_1 = \langle \mathcal{M}_1, \mathfrak{A}_1, \psi_1 \rangle$ and $M_2 = \langle \mathcal{M}_2, \mathfrak{A}_2, \psi_2 \rangle$ of AQT. We assume the models match on a backward light cone slab $O \in \mathcal{S}^-(G)$ (figure 5.1). Any local observable $A \in \mathfrak{A}(O)$ is, logically speaking, a predicate of a spacetime region because of the fundamental mapping $O \mapsto \mathfrak{A}(O)$ of AQT (sec. 2). Spacetime region G lies by definition in the slab's future domain of dependence: $G \subset D^+(O)$ (figure 5.1). Then it follows from the antecedent of SHL⁻ that the observable algebras on O of the two models match: $\mathfrak{A}_1(O) = \mathfrak{A}_2(O)$. Then according to the diamond axiom (5) we have:

$$\mathfrak{A}_1(D(O)) = \mathfrak{A}_2(D(O)). \tag{20}$$

The isotony axiom (4) yields from (20) that $\mathfrak{A}_1(G) \subset \mathfrak{A}(D(O))$ and $\mathfrak{A}_2(G) \subset \mathfrak{A}(D(O))$. Now let us assume, in order to prove by means of a little *reductio ad absurdum* argument that these subalgebras coincide, that they do not coincide: $(*)$ $\mathfrak{A}_1(G) \neq \mathfrak{A}_2(G)$. That is, there is at least one observable $A \in \mathcal{U}(D(O))$ such that $A \in \mathcal{U}_1(G)$ but $A \notin \mathcal{U}_2(G)$. Then the local observable A pertains in model M_1 to some spacetime region $G' \subseteq G$ (A is, say, the total momentum in G'): $A = A(G') \in \mathcal{U}_1(G') \subseteq \mathcal{U}_1(G)$ and, since $A \in \mathcal{U}_2(D(O)\backslash G)$, A pertains in model M_2 to some spacetime region $G'' \subseteq D(O)\backslash G$ (A is, say, the total momentum in G''): $A = A(G'') \in \mathcal{U}_2(G'') \subseteq \mathcal{U}_2(D(O)\backslash G)$. From $A = A(G')$ and $A = A(G'')$, it follows that $A(G') = A(G'')$. Since spacetime regions individuate observables that pertain to the whole spacetime region, it must be the case that $G' = G''$. But by construction $G' \cap G'' = \varnothing$. Contradiction. Hence the *reductio* assumption (*) is false. So we have proven algebra-matching on G, which is (6): $\mathcal{U}_1(G) = \mathcal{U}_2(G)$.

Next we derive state-matching on G. It follows from the antecedent of SHL$^-$ that the expectation values for each local observable $A \in \mathcal{U}(O)$ of the two models also match, because the expression for the expectation value $\psi_j(A)$ is, logically speaking, a function predicate ($z \in \mathbf{R}$):

$$M_1 \vDash \psi_1(A) = z \leftrightarrow M_2 \vDash \psi_2(A) = z. \tag{21}$$

By virtue of the diamond axiom (5), the variable A in (21) ranges over $\mathcal{U}(D(O))$. Because $G \subset D(O)$, it follows from isotony (4) that the variable A ranges over $\mathcal{U}(G) \subset \mathcal{U}(D(O))$. This means that (21) also holds for each $A \in \mathcal{U}(G)$. Since the state of a system in spacetime region G is uniquely specified in AQT by the expectation values of all its local observables on G, we have deduced state-matching (16) on G—as promised in section 4, end of part (b). More importantly, statement (21) is the consequent of SHL$^-$. Spacetime region G is arbitrary, so AQT is SHL$^-$.

The proof for the statement that AQT is SHL$^+$ is virtually identical; AQT is SHL$^+$ as well as SHL$^-$. Hence AQT is SHL. QED

From the proof that AQT is SHL$^-$ we immediately arrive at the Proposition that Rédei purported to prove, namely, AQT is SEL$^-$ (sec. 3). (The proof for this Proposition establishes the connection between SEL and the diamond axiom, to which Rédei has alluded in personal correspondence.) For if two models match in the backward light cone $LC^-(G)$ of some spacetime region G, then they match on any subset of $LC^-(G)$, in particular on a backward light cone slab $O \in \mathcal{S}^-(G)$ (see figure 5.1). This is precisely the antecedent of SHL$^-$. (Notice that SEL$^-$ *without provisos* is a corollary of SHL$^-$, so surely with provisos it remains a corollary of SHL$^-$.) Hence AQT is SEL$^-$, which is the desired Proposition, and therefore SEL. Similarly one derives that AQT is SEL$^+$ from the fact that AQT is SHL$^+$. Thus we end with the following:

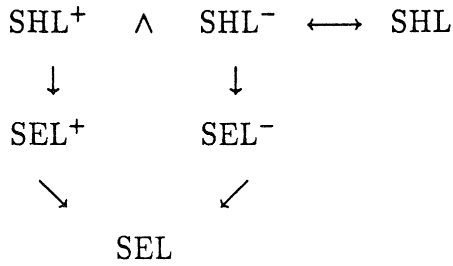

Figure 5.2

COROLLARY. *Algebraic Lorentz-covariant quantum field theory is a past and future stochastic Einstein local theory.*

The logical relations between the different stochastic Einstein and stochastic Haag locality conditions are summarized in the logical diagram in figure 5.2.

Finally we remark that the light cone and the domain of dependence of a spacetime region (diamonds) are Lorentz-invariant spacetime regions; therefore, our notion of stochastic Haag locality, and the notion of stochastic Einstein locality for that matter, are Lorentz invariant notions. (So the proof of our Theorem holds for every pair of models that is a Poincaré transformed pair of the arbitrary pair in the proof above.) Hence by using the diamond axiom, our proof contains an extra, distinctively Lorentz-covariant element that Rédei's proof lacked.

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