Planarity

1. Show that a bipartite planar graph on n vertices has at most $2n-4$ edges. This implies that $K_{3,3}$ is non-planar.

2. Show how to use the planar separator theorem to find an exact independent set in a planar graph in $2^{O(\sqrt{n})}$ time.

3. Show that the planar separator theorem implies the following. For any $\epsilon > 0$, there is a subset S of at most ϵn vertices than can be removed so that the remaining graph on $V \setminus S$ has components of size at most $O(1/\epsilon^2)$.

4. The Cauchy Schwarz inequality says that for any real numbers $a_1, \ldots, a_n$ and $b_1, \ldots, b_n$

$$\sum_i a_i b_i \leq (\sum_i a_i^2)^{1/2} (\sum_i b_i^2)^{1/2}$$

Use this to show that:

1) For any $a_1, \ldots, a_n$

$$\sum_i a_i \leq \sqrt{n} (\sum_i a_i^2)^{1/2}$$

$$\sum_i a_i \leq (\sum_i |a_i|^{2/3})^{1/2} (\sum_i |a_i|^{4/3})^{1/2}$$

2) For any $x, y, z > 0$

$$x + y + z \leq 2 \left(\frac{x^2}{y+z} + \frac{y^2}{x+z} + \frac{z^2}{x+y} \right)$$