

(all use of the exchange rule is implied to be inductive)

$$\begin{array}{c}
 1d \quad \frac{\Phi \vdash \psi \Rightarrow \psi'}{\Phi, \psi \vdash \psi'} \quad (\Rightarrow\text{-Adj}) \quad \frac{\Phi' \vdash \psi}{\Phi, \psi \vdash \psi'} \quad (\text{cut}) \\
 \hline
 \frac{\Phi', \Phi \vdash \psi'}{\Phi, \Phi' \vdash \psi'} \quad (\text{exchange})
 \end{array}$$

$$\begin{array}{c}
 6 \quad \frac{\frac{\Phi, \psi \vdash \theta}{\bar{\Phi}, \psi, \bar{\psi}' \vdash \theta} \quad (\text{weaken}) \quad \frac{\bar{\psi}', \psi' \vdash \theta}{\bar{\psi}', \psi', \bar{\psi}' \vdash \theta} \quad (\text{weaken})}{\frac{\bar{\psi}', \psi', \bar{\psi}' \vdash \theta}{\bar{\psi}', \bar{\psi}', \psi' \vdash \theta} \quad (\text{exchange})} \quad (\text{exchange}) \\
 \hline
 \frac{\bar{\psi}', \bar{\psi}', \psi' \vdash \theta}{\bar{\psi}', \bar{\psi}', \psi' \vdash \theta} \quad (\vee\text{-adj}) \\
 \hline
 \frac{\frac{\bar{\psi}', \bar{\psi}', \psi' \vdash \theta}{\bar{\psi}', \bar{\psi}', \psi' \vee \psi' \vdash \theta} \quad (\Rightarrow\text{-adj}) \quad \frac{\bar{\psi}' \vdash \psi \vee \psi'}{\bar{\psi}' \vdash \psi \vee \psi'}}{\bar{\psi}', \bar{\psi}', \bar{\psi}'' \vdash \theta} \quad (\text{cut } a)
 \end{array}$$

2 Consider the discrete one object category  $\mathcal{C}_0 = \{C\}$ .  
 Let  $\text{prop}_{\mathcal{C}_0}(C)$  be the three element linear order  $x \leq y \leq z$ .  
 Since the only morphism is the identity, all operations are conserved under pullback.  
 Define  $\wedge$  and  $\vee$  as usual. Notice that since  $\wedge$  preserves the order, by linearity,  $\wedge$  with  $L=x$  and  $T=z$ .  
 Define  $A \rightarrow B$  as  $\begin{cases} \text{true} & \text{if } A \leq B \\ B & \text{if otherwise} \end{cases}$   
 [in the first case  $C \wedge A \leq B$  always holds so all elements must be  $\leq$  than  $A \rightarrow B$   
 in the second  $C \wedge A \leq B$  holds iff  $C \leq B$  so  $A \rightarrow B = B$ .]  
 Hence  $\mathcal{C}$  is a prop category with the desired properties, but  
 $((y \rightarrow x) \rightarrow x) = (x \rightarrow x) = z > y$ . The result follows by 5.4.7. 