Seminar on Set Theory

Hand-in exercise 12

December 11, 2015

$\mathcal{P}\omega \cap L$ can be countable

(a) *(6 points)* Let λ be a cardinal with $\lambda \geq \aleph_0$ and let B be the the collapsing $(\aleph_0, 2^\lambda)$-algebra. Show that

$$V^{(B)} \models \mathcal{P}\hat{\lambda} \cap L \textit{ is countable.}$$

Hints:

(i) First prove that $V^{(B)} \models \mathcal{P}\hat{\lambda} \cap L \subseteq \widehat{\mathcal{P}\lambda}$.

(ii) Recall Theorem 1.46.

(b) *(4 points)* Let M be a countable transitive model of ZFC + $2^{\aleph_0} = \aleph_1$, put $B = (\mathrm{RO}(\omega_1^\omega))^{(M)}$ and let U be an M-generic ultrafilter in B. Show that

$$M[U] \models \mathcal{P}\omega \cap L \textit{ is countable.}$$