Seminar on Set Theory

Hand-in Lecture 4

October 12, 2015

Note: The footnote was previously incorrect.

Assignment 1. *Let B be a Boolean algebra and F be a principal, $\mathcal{P}(B)$-complete ultrafilter on B.*[1] *Define*

$$\pi(a) = \{\pi(x) : x \in dom(a), a(x) \in F\}.$$

Prove that for any $a, b \in V^{(B)}$,

$$[\![a \in b]\!]^B \in F \text{ iff } \pi(a) \in \pi(b) \tag{1}$$

$$[\![a = b]\!]^B \in F \text{ iff } \pi(a) = \pi(b). \tag{2}$$

Formulating a (working) induction hypothesis is worth 1 point. Each direction of statements 1 and 2 is worth 1.5 points.

Assignment 2. *Let α be an ordinal and for every $\xi < \alpha$, let $b_\xi \in B$, where B is a complete Boolean algebra. For each $\xi < \alpha$, we define*

$$a_\xi = b_\xi \wedge \left(\bigvee_{\eta < \xi} b_\eta \right)^* .$$

Show that

$$\bigvee_{\xi < \alpha} a_\xi = \bigvee_{\xi < \alpha} b_\xi .$$

This assignment is worth a total of 3 points.

[1] That is, for any $U \subset B$, if $\bigvee U \in F$ then $u \in F$ for some $u \in U$.