Seminar on Set Theory

Hand-in exercise 6

October 23, 2015

Constructible Universe

Using theorem 1.46, show that if $V \neq L$, then $[\![V \neq L]\!] = 1$.
(*This part is worth 3 points*)

Sigma 1 formula's

Show that the formula $\mathrm{Card}(\alpha)$ is not Σ_1.
(*This part is worth 4 points*)

Example of Boolean Algebra with the ccc

Show that the Boolean algebra $RO(2^I)$, where 2 is given the discrete topology, and 2^I is given the product topology satisfies the ccc.
(*This part is worth 3 points*)

Hints

The following hints are cryptographed to make reading more difficult. Make sure to seriously attempt solving the problems using only the first n-1 hints before reading hint n. The key is bobbyfischer and it only applies to the text, not the formulas. Have fun!

1. Qostc ypw hvvdvzb $Card(\alpha)$ bli napk xyf dspzqmecamt qosu. lte xkuh ro swfl xqersii gcsnsqi. Zqd gro mpv nwwng px zt bpu Σ_1?

2. Uc qsmam ljhx $\phi(x)$ zt bpu Σ_1 gy amhmmtfg up dnvv c tsufz M pg ZF ysl sp lpvnsou $x \in M$ qfbaumczou $\phi(x)$ jo rmm eqkic, xvfscfa ap yirmwuz $\neg\phi(x)$.

3. Dcotrwcuv h qfesm pd ZF hwfupwkjbh pd tvda jslohbcjj awvz.

4. Tvf gmqtgyprx goduq riq dl lvmdgvj: Qwwylryfwn-Tittwo Alvpffn, Ktalqdwbj'g Dpjqihuprx Msnny, fvv Ehrkpf't Ufjwjgt, xybh tuyymk $2^\kappa > \kappa$.

1. Hwwfl fv spamtiojo, nwwng alru wu dys jw yymkuso bq f bjgl ayfff fyhp fqki ybg gjlnbwnf qrom csyskzgz.

2. Uc dplxbjwjx kis uscj itqci, rtgpdgfbw vv irdv tfr U_λ nv ljl eeuwdiynv s rhmi pt gjlnbw ulxj $\langle A_\lambda, B_\lambda \rangle$ tidi rmil hvv rmz λ, $A_\lambda \subset B_\lambda$, bob kwj csp $\lambda \neq \mu$, $B_\lambda \cap B_\mu \neq \emptyset$ ror $A_\lambda \neq A_\mu$.

   To construct the tree above, associate to each set U_λ in the antichain a pair of finite sets $\langle A_\lambda, B_\lambda \rangle$ such that for all λ, $A_\lambda \subset B_\lambda$, and for all $\lambda \neq \mu$, $B_\lambda \cap B_\mu \neq \emptyset$ and $A_\lambda \neq A_\mu$.

3. Uc gjli ameo trjft, dmsaaflv kis qsmomuvpset cwfp kqfkai jvptfrx wx I.