Hilbert's Tenth Problem Seminar
Homework set 16

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(Due to Jan. 28th)

Exercise 1. Let K and L be number fields with $K \subset L$. Prove that

a) If R_1 and R_2 are Diophantine relations over O_L then $R_1 \wedge R_2$ and $R_1 \vee R_2$ are too.

b) The relation $x \neq 0$ is Diophantine over $\mathcal{O}_L$.

c) If $\mathbb{Z}$ is Diophantine over $\mathcal{O}_K$ and if $\mathcal{O}_K$ is Diophantine over $\mathcal{O}_L$, then $\mathbb{Z}$ is Diophantine over $\mathcal{O}_L$.

d) If $\mathbb{Z}$ is Diophantine over $\mathcal{O}_L$, then $\mathbb{Z}$ is Diophantine over $\mathcal{O}_K$.

Exercise 2. Let L be a number field and assume $\mathbb{Z}$ is Diophantine over $\mathcal{O}_L$. Prove that a relation is Diophantine over $\mathcal{O}_L$ if and only if is recursively enumerable.