Technical Report: Proofs for Reasoning about Preferences in Structured *EAF*s

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1 Proofs for Section 3.1

Lemma 1 If A defeats$_S$ B then $\forall S' \subseteq S$, A defeats$_{S'}$ B.

Proof: Follows straightforwardly from the definition of defeats$_S$.

Lemma 2 Let S be a conflict free subset of $\mathcal{A}$ in the *bh-EAFC* ($\mathcal{A}$, $\mathcal{C}$, $\mathcal{D}$). Let A be an argument that is acceptable w.r.t. S. Then:

1. A does not S-defeat A
2. $\forall C \in S$, C does not S-defeat A

Proof: **(1)** Suppose $A \rightarrow^S A$. By assumption of A acceptable w.r.t. S, $\exists D \in S$ s.t. $D \rightarrow^S A$, $\exists E \in S$ s.t. $E \rightarrow^S D$, contradicting S is conflict free. **(2)** Suppose $\exists C \in S$ s.t. $C \rightarrow^S A$. By assumption of A acceptable w.r.t. S, $\exists D \in S$ s.t. $D \rightarrow^S C$, contradicting S is conflict free.

Lemma 3 Let S and S' be subsets of $\mathcal{A}$ in the *bh-EAFC* ($\mathcal{A}$, $\mathcal{C}$, $\mathcal{D}$). If $S \subseteq S'$ and no argument in S defeats$_S$ an argument in $S' - S$. Then $\mathcal{F}(S) \subseteq \mathcal{F}(S')$.

Proof: Let $A \in \mathcal{F}(S)$. Suppose $\exists B$ s.t. $B \rightarrow^{S'} A$. By lemma 1, $B \rightarrow^S A$, and so $\exists C$, $C \rightarrow^S B$ and there is a reinstatement set R_S for $C \rightarrow^S B$. We show that $C \rightarrow^{S'} B$ and there is a reinstatement set $R_{S'}$ for $C \rightarrow^{S'} B$:
Suppose otherwise. In which case $X \not\rightarrow^{S'} Y$ for some $X \rightarrow^S Y \in R_S$, i.e., $X \rightharpoonup Y$ and $\exists \phi \subseteq S' - S$ s.t. $\phi \twoheadrightarrow (X \rightharpoonup Y)$. But then by assumption of R_S being a reinstatement set, $\exists C' \in S$ s.t. $C' \rightarrow^S B'$, $B' \in \phi$, contradicting the assumption that no argument in S defeats$_S$ an argument in $S' - S$.

Proposition 14-1) is proved in the following proposition 4:

Proposition 4 Let Δ = ($\mathcal{A}$, $\mathcal{C}$, $\mathcal{D}$)($Args$, $\mathcal{R}$, $\mathcal{D}$) be a *bh-EAFC*. If S is an admissible extension of Δ, and A, A' arguments acceptable w.r.t S, then:

1. S' = $S \cup \{A\}$ is admissible.
2. A' is acceptable w.r.t. S'.

Proof:

1) By assumption of A acceptable w.r.t S and lemma 2-2, no argument in S defeats$_S$ an argument in $S' - S$ (= $\{A\}$). Hence, by lemma 3, $\mathcal{F}(S) \subseteq \mathcal{F}(S')$. Since S is admissible and A is acceptable w.r.t. S, then $S \cup \{A\} \subseteq \mathcal{F}(S)$, and so $S' \subseteq \mathcal{F}(S')$, i.e., S' is admissible.

2) Follows from assumption of A' acceptable w.r.t. S and $\mathcal{F}(S) \subseteq \mathcal{F}(S')$

Proposition 5 Let Δ = ($\mathcal{A}$, $\mathcal{C}$, $\mathcal{D}$) be a *bh-EAFC*.

1. The set of all admissible extensions of Δ form a complete partial order w.r.t. set inclusion
2. For each admissible extension E of Δ there exists a preferred extension E' of Δ such that $E \subseteq E'$

Proof: Immediately from proposition 4 and definition of preferred extensions.

Corollary 6 Every *bh-EAFC* possesses at least one preferred extension.

Proof: From proposition 5 and the fact that $\emptyset$ is an admissible extension of any *bh-EAFC*.

Proposition 14-2) is proved in the following proposition 7:

Proposition 7 Let $\mathcal{F}$ be the characteristic function of a *bh-EAFC* Δ = ($\mathcal{A}$, $\mathcal{C}$, $\mathcal{D}$). Let S and S' be conflict free subsets of $\mathcal{A}$ such that $S \subseteq S'$. Then $\mathcal{F}(S) \subseteq \mathcal{F}(S')$.

Proof: Assume A is acceptable w.r.t. S. We show that A is acceptable w.r.t. S'

Suppose $\exists B1$ s.t. $B1 \rightarrow^{S'} A$. By lemma 1, $B1 \rightarrow^{S} A$, and by assumption of A is acceptable w.r.t. S, $\exists C1 \in S$ s.t. $C1 \rightarrow^{S} B1$ and there is a reinstatement set R_S for $C1 \rightarrow^{S} B1$.

Since $S \subseteq S'$, then $C1 \in S'$, and so to show A is acceptable w.r.t. S', it suffices to show there is a reinstatement set $R_{S'}$ for $C1 \rightarrow^{S'} B1$.

Since Δ is hierarchical, we can identify a sequenced partition of the reinstatement R_S for $C1 \rightarrow^{S} B1$. Let Δ_H = ($((\mathcal{A}_1, \mathcal{C}_1), \mathcal{D}_1)$, ..., $((\mathcal{A}_n, \mathcal{C}_n), \mathcal{D}_n)$) be the partition of Δ. Since Δ is bounded ($\mathcal{D}_n$ = $\emptyset$) one can represent R_S by the finite:

$$R_{S_i} = \{C1 \rightarrow^{S} B1\} \cup R_{S_{i+1}} \cup \ldots \cup R_{S_k}, \text{ where:}$$

1) For each defeat in R_{S_j}, $j < k$, any attack on the defeat originates from a set ϕ of arguments, at least one of which is itself defeated in $R_{S_{j+1}}$. That is, for $j = i \ldots k-1$: $R_{S_j} = \{C_1^j \rightarrow^S B_1^j, C_2^j \rightarrow^S B_2^j \ldots\}$ such that for $m = 1, 2, \ldots$, $(C_m^j, B_m^j) \in \mathcal{C}_j$, and if $\phi \twoheadrightarrow (C_m^j \rightharpoonup B_m^j)$ then $\exists B \in \phi$ s.t. B is some $B^{j+1} \in \mathcal{A}_{j+1}$, and $C^{j+1} \rightarrow^S B^{j+1}$ $\in R_{S_{j+1}}$

2) Since Δ is bounded, R_{S_k} is a set of defeats $\{C_1^k \rightarrow^S B_1^k, C_2^k \rightarrow^S B_2^k, \ldots\}$ such that for $m = 1, 2, \ldots$, $\neg\exists\, \phi \subseteq \mathcal{A}$ s.t. $\phi \twoheadrightarrow (C_m^k \rightharpoonup B_m^k)$.

We show the existence of $R_{S'}$ by showing that if $C \rightarrow^S B \in R_S$ then $C \rightarrow^{S'} B$. Proof is by induction on the above sequenced partition:

Base case: $C \rightarrow^S B \in R_{S_k}$. Suppose $C \not\rightarrow^{S'} B$. Then $\exists \phi \subseteq S'$, $\phi \twoheadrightarrow (C \rightharpoonup B)$, contradicting **2)**.

Inductive hypothesis: For $l > j$, $C \rightarrow^S B \in R_{S_l}$ implies $C \rightarrow^{S'} B$.

General case: $C \rightarrow^S B \in R_{S_j}$. Suppose $C \nrightarrow^{S'} B$. Then $\exists \phi \subseteq S'$, $\phi \twoheadrightarrow (C \rightharpoonup B)$. Since R_S is a reinstatement set, $\exists C' \in S$, $C' \in \mathcal{A}_{j+1}$, $C' \rightarrow^S B' \in R_{S_{j+1}}$, $B' \in \phi$. By inductive hypothesis, $C' \rightarrow^{S'} B'$. But then since $C', B' \in S'$, this contradicts S' is conflict free.

2 Proofs for Section 3.2

Lemma 8 Let $(\mathcal{A}, \mathcal{C}, \mathcal{D})$ be a *bh-EAFC*$_{EAT}$, $E \subseteq \mathcal{A}$. If $X \rightarrow^E Y'$, Y' is a sub-argument of Y, then $X \rightarrow^E Y$.

Proof: Suppose $X \nrightarrow^E Y$ (note that by assumption of $(X, Y') \in \mathcal{C}$ then $(X, Y) \in \mathcal{C}$). Then $\exists \phi \subseteq E$, $(\phi, (X, Y)) \in \mathcal{D}$, and so for all sub-arguments Z of Y on which X rebuts or undermines Z, $\exists \phi' \subseteq \phi$ s.t. $X \prec Z$. Since Y' is a sub-argument of Y, then it follows that for all sub-arguments Z' of Y on which X rebuts or undermines Y', $\exists \phi' \subseteq \phi$ s.t. $X \prec Z'$. Hence $\exists \psi \subseteq \phi$ s.t. $(\psi, (X, Y')) \in \mathcal{D}$, contradicting $X \rightarrow^E Y'$.

Lemma 9 If X is acceptable w.r.t. an admissible set E, then $E \cup \{X\}$ is conflict free.

Proof Suppose $E \cup \{X\}$ is not conflict free. Then either:
1) $\exists Y \neq X$ s.t. $Y \in E \cup \{X\}$ and $Y \rightarrow^{E \cup \{X\}} X$, and so $Y \rightarrow^E X$.
By assumption of the acceptability of X w.r.t. E, $\exists Z \in E$ s.t. $Z \rightarrow^E Y$, contradicting E is conflict free; or
2) $\exists Y \in E \cup \{X\}$ s.t. $X \rightarrow^{E \cup \{X\}} Y$ (this takes care of case 1 where $Y = X$), hence $X \rightarrow^E Y$.
By assumption of the admissibility of E, $\exists Z \in E$ s.t. $Z \rightarrow^E X$, and by assumption of the acceptability of X, $\exists V \in E$ s.t. $V \rightarrow^E Z$, contradicting E is conflict free.

Theorem 10 Let $(\mathcal{A}, \mathcal{C}, \mathcal{D})$ be a *bh-EAFC*$_{EAT}$ and E any of its extensions under a given semantics subsumed by complete semantics. Then for all $A \in E$: if $A' \in \mathtt{Sub}(A)$ then $A' \in E$.

Proof Suppose $\exists B$, $B \rightarrow^E A'$. By lemma 8, $B \rightarrow^E A$. Since A is acceptable w.r.t. E, there is a reinstatement set for $C \rightarrow^E B$. Hence A' is acceptable w.r.t. E. By lemma 9, $E \cup \{A'\}$ is conflict free. Hence, since E is complete, $A' \in E$.

Theorem 11 [Closure under strict rules] Let $(\mathcal{A}, \mathcal{C}, \mathcal{D})$ be a *bh-EAFC*$_{EAT}$ and E any of its extensions under a given semantics subsumed by complete semantics. Then $\{\mathtt{Conc}(A) | A \in E\} = Cl_{\mathcal{R}_s}(\{\mathtt{Conc}(A) | A \in E\})$ [1].

Proof It suffices to show that for any $\varphi_1, \ldots, \varphi_n \rightarrow \varphi \in \mathcal{R}_s$, if for $i = 1 \ldots n$, $A_i \in E$ s.t. $\mathtt{Conc}(A_i) = \varphi_i$, then $A = A_1, \ldots, A_n \rightarrow \varphi \in E$:

Suppose $B \rightarrow^E A$. By definition of attacks, it must be that for some sub-argument A_i of A, $(B, A_i) \in \mathcal{C}$. Suppose that for all such A_i, for all sub-arguments A'_i of A_i s.t. B rebuts or undermines A_i on A'_i, there exists $\phi \subseteq E$ s.t. $B \prec A'_i \in \mathcal{P}(\phi)$.

[1] $Cl_{\mathcal{R}_s}(P)$, where $P \subseteq \mathcal{L}$ is the smallest set containing P and the consequent of any strict rule in $\mathcal{R}_s$ whose antecedents are in $Cl_{\mathcal{R}_s}(P)$

Then there would be some $\psi \subseteq E$ s.t. $(\psi, (B, A))$ (since B cannot attack on a strict rule), contradicting $B \rightarrow^E A$. Hence, for some A_i, $B \rightarrow^E A_i$. Hence, there exists a reinstatement set for $C \rightarrow^E B$. Hence A is acceptable w.r.t. E, and since by Lemma 9 $E \cup \{A\}$ is conflict free, then $A \in E$ by assumption of E being complete.

Prior to the proof of Theorem 13, observe that [13] expresses the reasonable argument ordering in the context of arguments A, B and an argument $A+$ that is said to be a B'*-extension of* A defined as follows:

Definition 12 Let A and B be arguments with contradictory conclusions, where B's top rule r is strict, and A's top rule is defeasible. Then $\forall B' \in M(B)$, $A+$ is a B' extension of A where letting $B_1, \ldots, B_n$ = $M(B) - B'$:

$\texttt{TopRule}(A+)$ is a transposition $\texttt{Conc}(A), \texttt{Conc}(B_1), \ldots, \texttt{Conc}B_n \rightarrow -(\texttt{Conc}(B'))$

It follows of course that if $A+$ is a B'-extension of A, then $A+$ rebuts or undermines B on B'. Under the assumption of a reasonable argument ordering (expressed by i) and ii) in Section 3.2 of the main paper), and the admissibility of $\preceq$ stating that strict rules cannot weaken arguments, then it cannot be that B' is stronger than $A+$. For example, either:

- $A+ = [\Rightarrow p, \Rightarrow q, p, q \rightarrow \neg r$, $B' = [\Rightarrow r]$, $A+ \not\prec B'$, or
- $A+ = [\Rightarrow p, \Rightarrow r, p, r \rightarrow \neg q$, $B' = [\Rightarrow q]$, $A+ \not\prec B'$

Theorem 13 Let $(\mathcal{A}, \mathcal{C}, \mathcal{D})$ be a *bh-EAFC*$_{EAT}$ satisfying **Ass1-3**, **Ass5**, and **Ass6**. Let E be any of its extensions under a given semantics subsumed by complete semantics. Then $\{\texttt{Conc}(A) | A \in E\}$ is consistent.

Proof Suppose otherwise. Then $\exists A, B \in E$ s.t. $\texttt{Conc}(A) \in \overline{\texttt{Conc}(B)}$.

1) If A is firm and strict, and; i) if B is firm and strict this contradicts the closure of $\mathcal{K}_n$ under $\mathcal{R}_s$ being consistent (**Ass2**); ii) if B is plausible or defeasible, then $A \rightarrow^E B$, contradicting E is conflict free.

2) If A is an assumption, then by the well-formedness assumption (**Ass3**) B is an assumption, and so (A, B), $(B, A) \in \mathcal{C}$.

3) If A is plausible or defeasible, and; i) B is firm and strict, then this is case 1ii) under the assumption of well-formedness (**Ass3**); ii) if B's top rule is defeasible and $\texttt{Conc}(A)$ is a contrary of $\texttt{Conc}(B)$, then $A \rightarrow^E B$ (by b) in Def.16) contradicting E is conflict free; iii) A and B fall into neither of cases 3i) and 3ii).

Suppose then that A, B is an instance of case 2) or 3iii). We prove by induction that this leads to a contradiction. The proof proceeds by induction on the partition $E_1 \cup \ldots \cup E_n$ of E, where $X \in E_i$ iff $X \in \mathcal{A}_i$ in the partition of $(\mathcal{A}, \mathcal{C}, \mathcal{D})$, and so any attacks involving X are in $\mathcal{C}_i$.

Base case: $A, B \in E_n$. Since E is conflict free and $\mathcal{D}_n$ = $\emptyset$, no two arguments in E_n attack each other. Contradiction.

Inductive hypothesis: For $j > i$, no two arguments in E_j attack each other.

General case: $A, B \in E_i$.

- We deal first with case 2. Since (A, B), $(B, A) \in \mathcal{C}_i$, $\exists \phi, \phi' \subseteq E_{i+1}$ s.t. $(\phi, (A, B)), (\phi', (B, A)) \in \mathcal{D}_i$.

  Since A and B are assumptions that attack each other on their conclusions, then it must be that $A \prec B \in \mathcal{P}(\phi)$, $B \prec A \in \mathcal{P}(\phi')$. By **Ass5**, $\exists X \in \phi$, $Y \in \phi'$, such that either:

  - X and Y have contradictory conclusions, and so cases 1,2 and 3 can be applied to $X, Y \in E_{i+1}$ to demonstrate the existence of an attack between X and Y, contradicting the inductive hypothesis;

  - X is extended by some strict rules to $X+$ (where $X+ \in E$ as shown in the proof of Theorem 11), and $X+$ and Y have contradictory conclusions, so contradicting the inductive hypothesis as above.

- We deal with case 3iii), where both A and B are plausible or defeasible:

  - If both the top rules of A and B are defeasible, then by the well-formedness assumption (**Ass3**), $\texttt{Conc}(A)$ and $\texttt{Conc}(B)$ contradict each other, and so (A, B), $(B, A) \in \mathcal{C}$, and we can apply the same reasoning as in case 2.

  - If both the top rules of A and B are strict, then make it a case of 3rd dash ...

  - If B's top rule is strict and A's top rule is defeasible, then by the well-formedness assumption (**Ass3**), $\texttt{Conc}(A)$ and $\texttt{Conc}(B)$ contradict each other, and we have an asymmetric attack $(B, A) \in \mathcal{C}$ (where by assumption of E being conflict free $\exists \xi \subseteq E, (\xi, (B, A)) \in \mathcal{D}$).

  We can guarantee that there is a $B' \in M(B)$ that is a $\preceq$ minimal argument in E, since suppose otherwise. Then $\exists \phi, \psi \subseteq E$ s.t. $B' \prec B'' \in \mathcal{P}(\phi)$, $B'' \prec B' \in \mathcal{P}(\psi)$, and so by **Ass5**, $\exists X \in \phi$, $Y \in \psi$, such that either X and Y have contradictory conclusions, or X is extended by some strict rules to $X+$ s.t. $X+$ and Y have contradictory conclusions. Applying the same reasoning as for case 2, either case contradicts the inductive hypothesis.

  Hence, by **Ass6**, $\forall A'$ s.t. A' is a sub-argument of A and B rebuts or undermines A on A', $\exists B' \in M(B)$ that is $\preceq$ minimal argument in E, $\exists \phi' \subseteq \xi$ s.t. $B' \prec A' \in \mathcal{P}(\phi')$. Since $\texttt{Conc}(A)$ and $\texttt{Conc}(B)$ contradict each other, such an A' must include A itself, and so $\exists B' \in M(B)$ that is $\preceq$ minimal argument in E, $\exists \phi' \subseteq \xi$ s.t. $B' \prec A \in \mathcal{P}(\phi')$.

  Let $A+$ be the B'-extension of A as defined in Definition 12. By Theorem 10 every such sub-argument of $A+$ is in E.

  > We show that $A+ \in E$. Suppose $X \rightarrow^E A+$, and so $\neg \exists \phi \subseteq E$ s.t. $(\phi, (X, A+)) \in \mathcal{D}$. Let $V_1 \ldots V_n$ be the sub-arguments of $A+$ on which X rebuts or undermines $A+$, and suppose that for $i = 1 \ldots n$: $X \not\rightarrow^E V_i$ and so $\exists \psi_i \subseteq E$ s.t. $(\psi_i, (X, V_i)) \in \mathcal{D}$. But then this contradicts $\neg \exists \phi \subseteq E$ s.t. $(\phi, (X, A+)) \in \mathcal{D}$. Hence, for some sub-argument V of $X+$, $X \rightarrow^E V$, and so there is a reinstatement set for some $Z \rightarrow^E X$. Hence $A+$ is acceptable w.r.t. E, and since by Lemma 9 $E \cup \{A+\}$ is conflict free, then $A+ \in E$ by assumption of

E being complete.

We thus have that $A+ \in E$, $(A+, B) \in \mathcal{C}$, and since E is conflict free, $\exists \omega \subseteq E$ s.t. $(\omega, (A+, B)) \in \mathcal{D}$. Since $A+$ rebuts or undermines B on B', $A+ \prec B' \in \mathcal{P}(\omega')$, for some $\omega' \subseteq \omega$. By construction of $A+$, the fact that for all $B'' \in (M(B) - B')$, $\neg\exists \psi \subseteq E$ s.t. $B'' \prec B' \in \mathcal{P}(\psi)$, and the top strict rule of $A+$ cannot weaken any of $A+$'s maximal defeasible sub-arguments A and $(M(B) - B')$, it must be the case that $A \prec B' \in \mathcal{P}(\omega')$.

- We thus have $\exists \omega' \subseteq E$ s.t. $A \prec B' \in \mathcal{P}(\omega')$, $\exists \phi' \subseteq E$ s.t. $B' \prec A \in \mathcal{P}(\phi')$, and so by **Ass5**, $\exists X \in \omega'$, $Y \in \phi'$, such that either X and Y have contradictory conclusions, or X is extended by some strict rules to $X+$ s.t. $X+$ and Y have contradictory conclusions. We thus apply the same reasoning as for case 2 to show that either case contradicts the inductive hypothesis.

Theorem 14 Let $(\mathcal{A}, \mathcal{C}, \mathcal{D})$ be a *bh-EAFC*$_{EAT}$ satisfying **Ass1-3**, **Ass5**, and **Ass6**. Let E be any of its extensions under a given semantics subsumed by complete semantics. Then $Cl_{\mathcal{R}_s}(\{\texttt{Conc}(A) | A \in E\})$ is consistent.

Proof Follows from Theorems 11 and 13.

3 Proofs for Section 3.3

We define what it means for a set of arguments to conclude an ordering $\prec_s$ over sets of named wff of $\mathcal{L}$:

Definition 15 [Conclusion of $\prec_s$ by a set of arguments]
Let $\Gamma = r_1 : l_1, \ldots, r_n : l_n$ be a set of named wff of $\mathcal{L}$, and $\geq$ a partial ordering on Γ (with its strict counterpart $>$ defined in the usual way). Let $\Gamma' \subseteq \Gamma, \Gamma'' \subseteq \Gamma$. Then for some set ϕ of arguments:

> ϕ is said to conclude that $\Gamma' \prec_s \Gamma''$, iff $\exists r_i : l_i \in \Gamma'$ s.t. $\forall r : l \in \Gamma''$, $r > r_i$ is the conclusion of an argument in ϕ.

Finally, recall in the example formalism in Section 3.3 of the paper we assume that the strict rules contain the axioms of a strict partial order (x,y,z are meta-variables ranging over rule names):

- $o1 : (y > x) \wedge (z > y) \rightarrow (z > x)$
- $o2 : (y > x) \wedge \neg(z > x) \rightarrow \neg(z > y)$
- $o3 : (z > y) \wedge \neg(z > x) \rightarrow \neg(y > x)$
- $o4 : (y > x) \rightarrow \neg(x > y)$

3.1 Last Link Principle

Definition 16 [$\mathcal{P}$ defined under the last link principle]
Let $(\mathcal{A}, \mathcal{C}, \mathcal{D})$ be a *bh-EAFC*$_{EAT}$, $A, B \in \mathcal{A}$, $\phi \subseteq \mathcal{A}$. Then $B \prec A \in \mathcal{P}(\phi)$ under the last link principle iff

1. ϕ concludes $\texttt{LastDefRules}(B) \prec_s \texttt{LastDefRules}(A)$; or
2. $\texttt{LastDefRules}(B)$ and $\texttt{LastDefRules}(A)$ are empty and ϕ concludes $\texttt{Prem}(B) \prec_s \texttt{Prem}(A)$

Proposition 17 Let $(\mathcal{A}, \mathcal{C}, \mathcal{D})$ be a *bh-EAFC*$_{EAT}$, where the strict rules in EAT include $o1 \ldots 04$. Let $\mathcal{P}$ be defined under the last link principle. Then $(\mathcal{A}, \mathcal{C}, \mathcal{D})$ satisfies **Ass5**.

Proof:
Suppose $\phi, \psi \subseteq \mathcal{A}$ s.t. $B \prec A \in \mathcal{P}(\phi)$, $A \prec B \in \mathcal{P}(\psi)$ and either $LastDefRules(B)$ or $LastDefRules(A) \neq \emptyset$. Then ϕ concludes $\texttt{LastDefRules}(B) \prec_s \texttt{LastDefRules}(A)$, ψ concludes $\texttt{LastDefRules}(A) \prec_s \texttt{LastDefRules}(B)$. Hence for some $r_A \in \texttt{LastDefRules}(A)$, $r_B \in \texttt{LastDefRules}(B)$, $\exists X \in \phi$ that concludes $r_A > r_B$, $\exists Y \in \psi$ that concludes $r_B > r_A$. Hence, X can be extended with $o4$ to an argument $X+$ concluding $\neg(r_B > r_A)$ that contradicts Y's conclusion.
Suppose $\phi, \psi \subseteq \mathcal{A}$ s.t. $B \prec A \in \mathcal{P}(\phi)$, $A \prec B \in \mathcal{P}(\psi)$ and $LastDefRules(B) = \emptyset$, $LastDefRules(A) = \emptyset$. Then the above proof straightforwardly applies to the case where ϕ concludes $\texttt{Prem}(B) \prec_s \texttt{Prem}(A)$, ψ concludes $\texttt{Prem}(A) \prec_s \texttt{Prem}(B)$.

Proposition 18 Let $(\mathcal{A}, \mathcal{C}, \mathcal{D})$ be a *bh-EAFC*$_{EAT}$, where the strict rules in EAT include $o1 \ldots 04$. Let $\mathcal{P}$ be defined under the last link principle. Then $(\mathcal{A}, \mathcal{C}, \mathcal{D})$ satisfies **Ass6**.

Proof:
1) Let $(\phi, (B, A)) \in \mathcal{D}$ and suppose either $\texttt{LastDefRules}(B) \neq \emptyset$ or for some non-empty set $\{A'_1, \ldots, A'_n\}$ of sub-arguments of A on which B rebuts or undermines A, for $i = 1 \ldots n$, $\texttt{LastDefRules}(A'_i) \neq \emptyset$ (the case where $\texttt{LastDefRules}(B) = \emptyset$ and there are rebutted or undermined sub-arguments of A that do not have defeasible last rules is dealt with in case 2) below).
Let A' be any argument in $\{A'_1, \ldots, A'_n\}$. We show:
$\exists B' \in M(B)$, $\exists \phi' \subseteq \phi$ s.t. $\texttt{LastDefRules}(B') \prec_s \texttt{LastDefRules}(A')$ is concluded by ϕ' **(i)**
By definition, $\texttt{LastDefRules}(A')$ is the single top rule $r_{A'}$ of A'.
Since $(\phi, (B, A)) \in \mathcal{D}$ then $\exists \phi' \subseteq \phi$ s.t. $B \prec A' \in \mathcal{P}(\phi')$. Hence, $\texttt{LastDefRules}(B) \prec_s \{r_{A'}\}$ is concluded by ϕ'.
Hence, $\exists B' \in M(B)$ s.t. $r_{B'}$ is the top (defeasible) rule of B', $r_{B'} \in \texttt{LastDefRules}(B)$, and, $r_{A'} > r_{B'}$ is the conclusion of an argument in ϕ'_i.
Hence, $\texttt{LastDefRules}(B') \prec_s \texttt{LastDefRules}(A')$ is concluded by ϕ', $\phi' \subseteq \phi$.
It remains to show that the $B' \in M(B)$ identified above can indeed be an argument that is $\preceq$ minimal in E. Suppose otherwise. Then $\exists B'' \in M(B)$ s.t. $\texttt{LastDefRules}(B'') \prec_s \texttt{LastDefRules}(B')$ is concluded by some $\xi \subseteq E$. Since both sets are singleton top rules $r_{B''}$ and $r_{B'}$ of arguments in $M(B)$, then ξ is the singleton set $\{X\}$, where X concludes $r_{B'} > r_{B''}$. But then since $r_{A'} > r_{B'}$ is the conclusion of an argument X in E, then X can be extended with $o1$ to $X+$ with conclusion $r_{A'} > r_{B''}$, so that $\exists B''$, $\exists \phi' \subseteq \phi$ s.t. $\texttt{LastDefRules}(B'') \prec_s \texttt{LastDefRules}(A')$ is concluded by ϕ'.
So we have shown that if the $B' \in M(B)$ identified above is not $\preceq$ minimal in E, we

can always find a $B'' \in M(B)$ s.t. $\texttt{LastDefRules}(B'') \prec_s \texttt{LastDefRules}(B')$ is concluded by some $\xi \subseteq E$, and B'' satisfies **(i)**. This implies, given that the definition of **Ass6** assumes at least one argument in $M(B)$ that is $\preceq$ minimal in E, that we can identify a $\preceq$ minimal argument satisfying **(i)**.

2) Let $(\phi, (B, A)) \in \mathcal{D}$ and suppose $\texttt{LastDefRules}(B) = \emptyset$ and for some subset $\{A'_1, \ldots, A'_n\}$ of sub-arguments of A on which B rebuts or undermines A, $\texttt{LastDefRules}(A'_i) = \emptyset$.

Let A' be any argument in $\{A'_1, \ldots, A'_n\}$. We show:
$\exists B' \in M(B)$, We show $\exists B' \in M(B)$, $\exists \phi' \subseteq \phi$ s.t. $\texttt{Prem}(B') \prec_s \texttt{Prem}(A')$ is concluded by ϕ' **(ii)**

Since $(\phi, (B, A)) \in \mathcal{D}$ then $\exists \phi' \subseteq \phi$ s.t. $B \prec A' \in \mathcal{P}(\phi')$. Hence, $\texttt{Prem}(B) \prec_s \texttt{Prem}(A')$ is concluded by ϕ'.

Hence, for some premise named $r_{B'}$ in $\texttt{Prem}(B)$, where $r_{B'}$ must be some premise in some $B' \in M(B)$, for all premises $r_{A'}$ in A', $r_{A'} > r_{B'}$ is the conclusion of an argument in ϕ'.

Hence, $\texttt{Prem}(B') \prec_s \texttt{Prem}(A')$ is concluded by ϕ', $\phi' \subseteq \phi$.

It remains to show that the $B' \in M(B)$ identified above can indeed be an argument that is $\preceq$ minimal in E. Suppose otherwise. Then $\exists B'' \in M(B)$ s.t. $\texttt{Prem}(B'') \prec_s \texttt{Prem}(B')$ is concluded by some $\xi \subseteq E$. Hence, there is a premise named $r_{B''}$ in B'' s.t. for all premises $r_{B'}$ in B', $X \in \xi$ concludes $r_{B'} > r_{B''}$. But then since for all premises $r_{A'}$ in A', $r_{A'} > r_{B'}$ is the conclusion of an argument X in E for some premise $r_{B'}$ in B', then each such X can be extended with $o1$ to $X+$ with conclusion $r_{A'} > r_{B''}$, so that $\exists \phi' \subseteq \phi$ s.t. $\texttt{Prem}(B'') \prec_s \texttt{Prem}(A')$ is concluded by ϕ'.

So we have shown that if the $B' \in M(B)$ identified above is not $\preceq$ minimal in E, we can always find a $B'' \in M(B)$ s.t. $\texttt{Prem}(B'') \prec_s \texttt{Prem}(B')$ is concluded by some $\xi \subseteq E$, and B'' satisfies **(ii)**. This implies, given that the definition of **Ass6** assumes at least one argument in $M(B)$ that is $\preceq$ minimal in E, that we can find a $\preceq$ minimal argument satisfying **(ii)**.

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