INSTITUTE OF LOGIC AND INTELLIGENCE
SOUTHWEST UNIVERSITY
CHONGQING
2010

Argumentation Logics

Answers to the exercises

Author:

HENRY PRAKKEN

UTRECHT UNIVERSITY & UNIVERSITY OF GRONINGEN

Chapter 1

Answers to exercises of chapter 1

EXERCISE 1.2.1

1. B and D are justified. B is reinstated by D.
2. A, C and E are justified. No argument is reinstated by D, since D is not justified. A and C are reinstated by E.

Chapter 2

Answers to exercises of chapter 2

EXERCISE 2.2.1

1. B and D are justified. B is reinstated by D.

2. A, C and E are justified. No argument is reinstated by D, since D is not justified. A and C are reinstated by E.

EXERCISE 2.8.1

(a): C is justified since it has no defeaters. B is not justified, since it is defeated by a justified argument, viz. by C. Therefefore, A is justified, since its only defeater, which is B, is not justified.
(b): The status of A and B cannot be determined: A is justified if and only if its only defeater, which is B, is not justifed. But B is not justified just in case A, which is its only defeater, is justified. Thus we enter a loop. And since the status of C depends on the status of its only defeater, which is B, the status of C cannot be determined either.

EXERCISE 2.8.2 Consider an arbitrary argument A. By assumption, there is an argument B such that B defeats A. So $A \in F(\varnothing)$ iff there is a $C \in \varnothing$ such that C defeats B. However, no such C exists, so $A \notin F(\varnothing)$. Since A was chosen arbitrarily, we can conclude that no argument is in $F(\varnothing)$. $\Box$.

EXERCISE 2.8.3

a:	b:	c:	d:
$F^0 = \varnothing$	$F^0 = \varnothing$	$F^0 = \varnothing$	$F^0 = \varnothing$
$F^1 = \{A\}$	$F^1 = F^0$	$F^1 = \{C\}$	$F^1 = \{A, E\}$
$F^2 = \{A, D\}$		$F^2 = \{C, B\}$	$F^2 = \{A, E, C\}$
$F^3 = F^2$		$F^3 = F^2$	$F^3 = F^2$

The grounded extensions are the fixed points of these sequences.

EXERCISE 2.8.4

1. To show that $F(X) = G^2(X)$, for every set of arguments X, it turns out that it is easier to show that the complements of the two sets are equal. This has to do with quantifying over arguments. Thus, suppose $x \notin G^2(X)$. By definition of G this means that there exists a $y \in G(X)$ defeating x, i.e., $x \leftarrow y$. Since

$y \in G(X)$, the argument y is not defeated by a member of X. Hence y shows that $x \notin F(X)$. Conversely, suppose that $x \notin F(X)$. Then x is defeated by a y that is not defeated by a $z \in X$. Thus x is defeated by a $y \in G(X)$, and hence $x \notin G^2(X)$.

2. The result that G is anti-monotonic follows from the fact that, if an argument is not defeated by a member of B, then it surely cannot be defeated by a member of any subset $A \subseteq B$.

3. Suppose $A \subseteq B$. Since G is anti-monotonic, it follows that $G(B) \subseteq G(A)$. Again by anti-monotonicity of G, we obtain $G^2(A) \subseteq G^2(B)$, which is equal to the expression $F(A) \subseteq F(B)$.

4. If $\{G_i\}_{i \geq 0}$ with $G_0 =_{Def} \varnothing$ and $G_i =_{Def} G(G_{i-1})$, then in particular

$$G_0 \subseteq G_1 \text{ and } G_0 \subseteq G_2. \tag{2.1}$$

Now apply the anti-monotonicity of G to (2.1) repeatedly, to obtain the chain of inclusions desired.

EXERCISE 2.8.5

- (a): justified: A, D; overruled: B, C; defensible: none.
- (b): justified: none; overruled: none; defensible: all.
- (c): justified: B, C; overruled: A, D; defensible: none.
- (d): justified: A, C, E; overruled: B, D; defensible: none.

EXERCISE 2.8.6

$\Rightarrow$:
Consider any stable extension E, and consider first any argument A not defeated by E. Then $A \in E$. Consider next any argument B defeated by E. Then, since E is conflict-free, $B \notin E$. So $E = \{A \mid A \text{ is not defeated by } E\}$.□
$\Leftarrow$:
Let $E = \{A \mid A \text{ is not defeated by } E\}$. Clearly, E is conflict-free. Furthermore, for all A, if $A \notin E$, then E defeats A. So E is a stable extension.□

EXERCISE 2.8.7

- Example 2.1.3: There is just one status assignment, which is maximal:
  - $S_1 = (\{A, C\}, \{B\})$
- Example 2.1.4: There are three status assignments:
  - $S_1 = (\varnothing, \varnothing)$
  - $S_2 = (\{A\}, \{B\})$
  - $S_3 = (\{B\}, \{A\})$

  Only S_2 and S_3 are maximal.
- Example 2.3.7: There is just one status assignment, which is maximal:
  - $S_1 = (\varnothing, \varnothing)$

EXERCISE 2.8.8

1. Consider any $A \in Out$. Then there is a $B \in In$ defeating A. But also $B \in In'$, so that $A \in Out'$. So $Out \subseteq Out'$.
2. Consider any C such that $C \notin In$ but $C \in In'$. Then there is a $B \notin In$ such that B defeats C (since otherwise C has to be in). Then $B \in Out'$. So (with 1) $Out \subset Out'$.

EXERCISE 2.8.9

- A is defensible iff is in *in* some but not all preferred status assignments, and A is overruled if A is *out* in all preferred status assignments. *This leaves open that there are arguments that neither justified, nor defensible, nor overruled. Cf. Example 2.3.7.*
- A is defensible iff is in *in* some but not all preferred status assignments, and A is overruled if there is no status assignment in which A is *in*. *With this definition all arguments are either justified, Xor defensible, Xor overruled.*

EXERCISE 2.8.10: The empty set, which is maximally admissible.

EXERCISE 2.8.11

1. (a) Preferred: $\{A, D\}$, also stable.
   (b) Preferred: $\{B, D, E\}$, also stable; $\{A, E\}$, also stable.
   (c) Preferred: $\varnothing$, no stable extensions.
   (d) Preferred: $\{A, C, E\}$, also stable.
   (e) (with slightly detailed explanation)
   (1) Preferred extensions:
   - $E_1 = \{A, B, D\}$
   - $E_2 = \{C\}$

   (2) Stable extensions. Both E_1 and E_2 are also stable extensions, since both sets defeat all arguments outside them. Furthermore, by Proposition 2.4.1 there are no other stable extensions.

2. (a) for preferred and stable semantics: A, D justified, B, C overruled.
   (b) for preferred and stable semantics: E justified, C overruled, A, B, D defensible.
   (c) for preferred semantics: neither is justified, defensible or overruled. For stable semantics: all are both justified and overruled.
   (d) For preferred and stable semantics: A, C, E justified, B, D overruled.
   (e) For preferred and stable semantics: all defensible

EXERCISE 2.8.12: The grounded extension is empty, while there are two preferred extensions, viz. $\{B, D\}$ and $\{A, C\}$. Note that one preferred extension concludes that Larry is rich, while the other concludes that Larry is not rich, so inbothsemantics no conclusion about Larry's richness is justified. Yet it may be argued that the conclusion that Larry is not rich is the intuitively justified conclusion, since all arguments for the opposite conclusion have a strict defeater. Anyone who adopts this analysis, will have to conclude that this example presents a problem for both grounded and preferred semantics. However, see Exercise 3.6.8 for a solution when the structure of arguments is made explicit.

Chapter 3

Answers to exercises of chapter 3

EXERCISE 3.4.1

1. The defeat graph is:

$$\begin{tikzcd}
 & & & F \arrow[dl] & & \\
G \arrow[r, bend left] & E \arrow[l, bend left] \arrow[r] & D \arrow[r] & A \arrow[r, bend left] & B \arrow[l, bend left] & C \arrow[l]
\end{tikzcd}$$

2. We are asked to list all strategies of P an O. There are two strategies for P ("?" indicates an unfortunate move, "‡" indicates the move that leads to a loss for the other party):

Strategy 1 for P
(responding to D with F
and winning)

$$\begin{tikzcd}
 & P_1\colon A \arrow[dl, shift left] & \\
O_1\colon B \arrow[ur, shift left] & & O_1'\colon D \arrow[ul] \\
P_2\colon C \arrow[u] & & P_2'\colon F \arrow[u]
\end{tikzcd}$$

Strategy 2 for P
(responding to D with E
and losing)

$$\begin{tikzcd}
 & P_1\colon A \arrow[dl, shift left] & \\
O_1\colon B \arrow[ur, shift left] & & O_1'\colon D \arrow[ul] \\
P_2\colon C \arrow[u] & & P_2'\colon E\ [\mathbf{?}] \arrow[u] \arrow[d, shift left] \\
 & & O_2\colon G\ [\ddagger] \arrow[u, shift left]
\end{tikzcd}$$

There are two strategies for O:

Strategy 1 for O (responding to A with B and losing)

P_1: $A \longleftrightarrow O_1$: $B \longleftarrow P_2$: C [‡]

Strategy 2 for O (responding to A with D and losing)

$$\begin{tikzcd}
 & & P_2\text{: } E\ [\mathbf{?}] \arrow[dl] \arrow[r, leftrightarrow] & O_2\text{: } G \\
P_1\text{: } A & O_1\text{: } D \arrow[l] & & \\
 & & P_2\text{: } F\ [‡] \arrow[ul] &
\end{tikzcd}$$

EXERCISE 3.4.2

1. $$\begin{tikzcd} A \arrow[r, bend left] & B \arrow[l, bend left] \end{tikzcd}$$

2. $$\begin{tikzcd} & C \arrow[dr, bend left] & \\ A \arrow[ur, bend left] & & B \arrow[ll, bend left] \end{tikzcd}$$

3. $A_1 \longleftarrow A_2 \longleftarrow A_3 \longleftarrow A_4 \longleftarrow A_5 \longleftarrow \cdots$

EXERCISE 3.4.3

1.

(a) P has winning strategies for A and D, but not for B:

- A winning strategy for A consists of putting forward A, after which O cannot respond because A has no defeaters.
- A winning strategy for B does not exist, because O can reply to B with A, after which P cannot move.
- A winning strategy for D is simple: put forward D; the only responses to D are B and C, which can both be countered with A, after which O cannot move.

(b) C is not provable. A simple winning strategy for O is:

P_1: C
O_1: B
P_2: A
O_2: B (P cannot move)

E is not provable either. A winning strategy for O is:

$$
\begin{tikzcd}
& P_1\colon E & \\
& O_1\colon C \arrow[u] & \\
P_2\colon A \arrow[ur] & & P_2'\colon B \arrow[ul] \\
O_2\colon D \arrow[u] & & O_2'\colon A \arrow[u]
\end{tikzcd}
$$

(c) A is not provable. A winning strategy for O is:

$$
\begin{array}{lll}
P_1: & A & \\
O_1: & B & \\
P_2: & D & \\
O_2: & C & (P \text{ cannot move})
\end{array}
$$

B is provable, as follows:

$$
\begin{array}{lll}
P_1: & B & \\
O_1: & D & \\
P_2: & C & (O \text{ cannot move})
\end{array}
$$

C is trivially provable, since it has no defeaters.

(d) C is provable. A winning strategy is:

$$
\begin{tikzcd}
& P_1\colon C & \\
O_1\colon B \arrow[ur] & & O_1'\colon D \arrow[ul] \\
P_2\colon A \arrow[u] & & P_2'\colon E \arrow[u]
\end{tikzcd}
$$

2. C is not provable: P has no moves when O replies with B.
   D is neither provable: P has no replies after O's reply with C.

3. We make the comparison for the proof of A in graph (a):

   $F^0 = \varnothing$
   $F^1 = \{A\}$
   $F^2 = \{A, D\}$

   Compared to a won dialogue on D, the order of stating A and D is reversed. With F, we start with the undefeated arguments and at each iteration add the arguments reinstated by the arguments added at the previous iteration. In a dialectical proof, P starts with an argument from F^i where i may be greater than 1, and at each next turn in a dialogue P moves an argument from F^{i-1} that can reinstate the argument of the previous move.

EXERCISE 3.4.4 P successively moves $A_1, A_3, \ldots, A_{2i-1}, A_{2i+1}, \ldots$ and O successively moves $A_2, A_4, \ldots, A_{2i}, A_{2i+2}, \ldots$ so they wil never repeat their own argument. And P always uses 'odd' arguments while O always uses 'even' arguments, so they will never repeat each other's argument. Finally, since the defeat chain is infinite, they will always have a new move.

EXERCISE 3.4.5 The simplest example is with two arguments A and B such that A defeats itself and there are no other defeat relations. B is provable since O has no reply if P starts with B, but this argumentation theory has no stable extensions.

EXERCISE 3.4.6

1.

   (a) All arguments except argument C are provable. Figure 3.1 contains a first attempt to display the winning strategies for P.

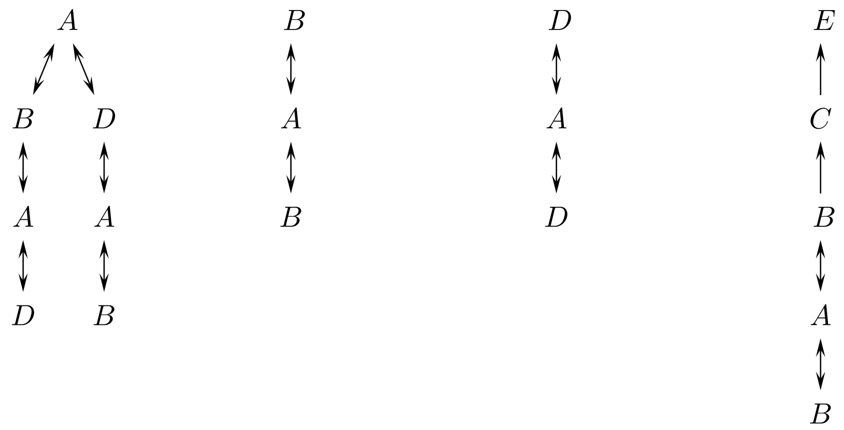

Figure 3.1: P's winning strategies (first attempt)

However, these trees are not yet strategies, since they do not contain all possible backtracking replies of O as children of a P move. (Note that a strategy is not a tree of *dispute lines* but a tree of *disputes*, so that a next move in a branch of a strategy may well reply not to the previous move but to an earlier move in the branch.) So the correct winning strategy for A is a lot more complex.

Let us illustrate this with a simpler example, viz. the graph of Exercise 3.8.11(1a). At first sight, a winning strategy for D would look as in Figure 3.2. However, the correct winning strategy is as displayed in Figure 3.3 (where the replied-to move is indicated between brackets).

Let us now return to the arguments in graph (b). Firstly, C is not provable. A simple winning strategy for O is:

P_1:	C	
O_1:	B	
P_2:	A	
$O_2[P_1]$:	A	(eo ipso)

$$
\begin{tikzcd}[column sep=small]
& P_1\colon D & \\
O_1\colon B \arrow[ur] & & O'_1\colon C \arrow[ul] \\
P_2\colon A \arrow[u] & & P'_2\colon A \arrow[u]
\end{tikzcd}
$$

Figure 3.2: P's seeming winning strategy for D in 3.8.11(1a)

$$
\begin{tikzcd}[column sep=small]
& P_1\colon D & \\
O_1(P_1)\colon B \arrow[ur] & & O'_1(P_1)\colon C \arrow[ul] \\
P_2(O_1)\colon A \arrow[u] & & P'_2(O'_1)\colon A \arrow[u] \\
O_2(P_1)\colon C \arrow[u] & & O'_2(P_1)\colon C \arrow[u] \\
P_3(O_2)\colon A \arrow[u] & & P'_3(O'_2)\colon A \arrow[u]
\end{tikzcd}
$$

Figure 3.3: P's correct winning strategy for D in 3.8.11(1a)

But E is provable. A winning strategy for P is:

P_1:	E
O_1:	C
P_2:	B
O_2:	A
P_3: B	(O cannot move)

B is also simply provable:

P_1:	B	
O_1:	A	
P_2:	B	(O cannot move)

The proof for D is completely analogous. B is also provable, but the proof is much more complex, since O can backtrack several times.

(b) In graph (c), the argument C is not provable since O has the following winning strategy:

P_1:	C	
O_1:	A	
P_2:	B	
O_2:	C	(eo ipso)

For A and B O has analoguous winning strategies. For D O also has a simple winning strategy:

P_1: D
O_1: C
P_2: A
O_2: B (P has no moves)

(c) In graph (d), the argument C is provable. A winning strategy for P is displayed in Figure 3.4.

$$\begin{tikzcd}
& P_1\colon C & \\
O_1(P_1)\colon B \arrow[ur] & & O'_1(P_1)\colon D \arrow[ul] \\
P_2(O_1)\colon A \arrow[u] & & P'_2(O'_1)\colon E \arrow[u] \\
O_2(P_1)\colon D \arrow[u] & & O'_2(P_1)\colon B \arrow[u] \\
P_3(O_2)\colon E \arrow[u] & & P'_3(O'_2)\colon A \arrow[u]
\end{tikzcd}$$

Figure 3.4: P's winning strategy for C in 3.8.11(1.d)

2. In Figure 3.2, C is not provable. A winning strategy for O is:

P_1: c
O_1: m
P_2: l
O_2: k (P has no moves)

Chapter 4

Answers to exercises of chapter 4

EXERCISE 4.6.1.

1. The following argument for Ra can be created.

$$\begin{array}{ll} A_1: & \forall x(Px \supset Qx) \\ A_2: & Pa \\ A_3: & A_1, A_2 \rightarrow Qa \\ A_4: & \forall x(Qx \supset Rx) \\ A_5: & A_3, A_4 \rightarrow Ra \end{array}$$

2. $\texttt{Prem}(A) = \{Pa, \forall x(Px \supset Qx), \forall x(Qx \supset Rx)\}$
   $\texttt{Conc}(A) = Ra$
   $\texttt{Sub}(A) = \{A_1, A_2, A_3, A_4\}$
   $\texttt{DefRules}(A) = \varnothing$
   $\texttt{TopRule}(A) = Qa, \forall x(Qx \supset Rx) \rightarrow Ra$

3. The argument is strict and plausible.

EXERCISE 4.6.2. The following argument for t can be created.

$$\begin{array}{ll} A_1: & p \\ A_2: & q \\ A_3: & A_1, A_2 \Rightarrow r \\ A_4: & A_3 \rightarrow r \vee s \\ A_5: & A_4 \Rightarrow t \end{array}$$

It is undermined by the following argument for $\neg q$:

$$\begin{array}{ll} B_1: & u \\ B_2: & B_1 \Rightarrow v \\ B_3: & \neg(q \wedge v) \\ B_4: & B_2, B_3 \rightarrow \neg q \end{array}$$

Since q is an assumption, we have that B_4 successfully undermines A_5 on A_2 and thus strictly defeats A_5. However, B_4 is undermined by the following argument for $\neg u$:

$$\begin{array}{ll} C_1: & w \\ C_2: & C_1 \Rightarrow \neg u \end{array}$$

Since $\leq' = \varnothing$ we have that C_2 successfully undermines B_4 on B_1. However, we also have that B_1 rebuts C_2 and since $\leq' = \varnothing$ we have that B_1 successfully rebuts C_2. Therefore B_1 and C_2 defeat each other. It is then easy to see that the grounded extension does not contain arguments A_5, B_1 or C_2. Moreover, no defeater of A_5 is thus in the grounded extension, so A_5 is defensible, which makes t defensible also.

EXERCISE 4.6.3.

1. r is overruled. The argument A for r is undercut by an argument B for $\neg d_1$. In turn, B is rebutted by an argument C for $\neg t$. B uses one defeasible rule, namely, d_2, while C uses two defeasible rules, namely, d_3 and d_4. Since $d_3 < d_2$ we have that $C \prec B$ so B is undefeated, so A is overruled.

2. Now r is justified, since arguments B and C are now compared on d_2 and d_4 and since $d_2 < d_4$. So C reinstates A.

EXERCISE 4.6.4. The following argument for t can be created.

A_1: s
A_2: $A_1 \Rightarrow t$

A_2 is rebutted by the following argument for $\neg t$:

B_1: p
B_2: $B_1 \Rightarrow q$
B_3: $B_1, B_2 \Rightarrow r$
B_4: $B_2, B_3 \rightarrow q \wedge r$
B_5: $(q \wedge r) \supset \neg t$
B_6: $B_4, B_5 \rightarrow \neg t$

(Note that since B_6 is strict, A_2 does not in turn rebut B_6.) We have that $\mathtt{LastDefRules}(A_2) = \{d_3\}$ while $\mathtt{LastDefRules}(B_6) = \{d_1, d_2\}$. Since $d_2 < d_3$ we have that $\mathtt{LastDefRules}(B_6) \prec_s \mathtt{LastDefRules}(A_2)$, so $B_6 \prec A_2$. Hence B_6 does not defeat A_2. Since A_2 has no other defeaters, we can concude at this point that A_2 will be *in* in all maximal status assignments, which makes it justified. Then t is a justified conclusion.

It is interesting to verify the status of argument B_6 for $\neg t$. Since the present argumentation theory satisfies the conditions of Theorem 3.3.10, it is to be expected that this conclusion is not justified. This turns out to be indeed the case. First of all, A_2 can be extended to a rebuttal of B_3:

A_3: $(q \wedge r) \supset \neg t$
A_4: $A_2 \rightarrow \neg(q \wedge r)$
A_5: p
A_6: $A_5 \Rightarrow q$
A_7: $A_4, A_6 \rightarrow \neg r$

We have that $\mathtt{LastDefRules}(A_7) = \{d_1, d_3\}$ while $\mathtt{LastDefRules}(B_3) = \{d_1, d_2\}$. Since $<$ is transitive we have $d_2 < d_1$ so $\{d_1, d_2\} \prec_s \{d_1, d_3\}$ and $B_3 \prec A_7$. Hence A_7 successfully rebuts and thus strictly defeats B_3. But then A_7 also defeats B_4, B_5 and B_6.

Yet another relevant argument can be constructed, which starts in the same way as A_7:

A_3: $(q \wedge r) \supset \neg t$
A_4: $A_2 \rightarrow \neg(q \wedge r)$
A_5: p
A_6: $A_5 \Rightarrow q$
A_8: $A_5, A_6 \Rightarrow r$
A_9: $A_4, A_8 \rightarrow \neg q$

A_9 rebuts B_2 (and not vice versa). We have $\texttt{LastDefRules}(A_9) = \{d_2\}$ while $\texttt{LastDefRules}(B_2) = \{d_1\}$. Since $d_2 < d_1$ so $A_9 < B_2$ we have that A_9 does not defeat B_2. Since $A_6 = B_2$ we also have that A_9 does not defeat A_6. Finally, A_7 rebuts A_8. Recall that $\texttt{LastDefRules}(A_7) = \{d_1, d_3\}$; moreover, $\texttt{LastDefRules}(A_8) = \{d_2\}$ and we have seen that $\{d_2\} \prec_s \{d_1, d_3\}$ so $A_8 \prec A_7$, for which reason A_7 strictly defeats A_8.

Now to evaluate the status of the arguments, A_7 and all its subarguments can be made *in* since they have no defeaters. Since A_7 strictly defeats A_8 and thus also A_9, the latter two arguments can be made *out*. Moreover since A_7 strictly defeats B_3 and thus also B_4, B_5 and B_6, the latter four arguments can also be made *out*. No alternative status assignments are possible, while moreover the present assignment is complete. So B_6 is out in all maximal status assigments, which makes $\neg t$ an overruled conclusion.

EXERCISE 4.6.5.

1. No.

2. Δ can be translated into an argumentation theory as follows:

   $\mathcal{R}_s = \varnothing$;
   $\mathcal{R}_d = \{p,\ q \Rightarrow r;\ r, s \Rightarrow t;\ p, u \Rightarrow \neg s\}$
   $\mathcal{K}_p = \{p\}$
   $\mathcal{K}_a = \{q, s, u\}$

3. t is overruled. An argument for t is

   A_1: p
   A_2: q
   A_3: $A_1, A_2 \Rightarrow r$
   A_4: s
   A_5: $A_3, A_4 \Rightarrow t$

   But it is undermined and thus strictly defeated by

   $B_1 =$ A_1
   B_2: u
   B_3: $B_1, B_2 \Rightarrow \neg s$

   Since B_3 is not defeated by any argument, no stable extension contains A_5.

EXERCISE 4.6.6.

1. It can be verified that there is no status assignment that assigns a status to A_2 or A_3.

Firstly, to make A_2 *in*, its defeater A_3 must be *out*. To make A_3 *out*, one of its defeaters must be *in*. However, the only defeater of A_3 is A_3 itself (by undercutting its subargument A_2) and A_3 cannot be both *in* and *out*. So A_2 cannot be made *in*.

Next, to make A_2 *out*, it must have a defeater that is *in*. Its only defeater is A_3. To make A_3 *in*, all its defeaters must be *out*. However, A_3 defeats itself and A_3 cannot be both *in* and *out*. So A_2 cannot be made *out*.

So there is only one maximal status assignment, in which A_1 is *in*, since A_1 has no defeaters. Moreover, this set is also the grounded extension.

2. Add $\texttt{Says}(John, \text{“}\texttt{StabbedWithKnife}(Suspect, Victim)\text{”})$ to $\mathcal{K}_p$. Then the following argument can be constructed:

B_1: $\texttt{Says}(John, \text{“}\texttt{StabbedWithKnife}(Suspect, Victim)\text{”})$
B_2: $\texttt{StabbedWithKnife}(Suspect, Victim)$

This argument is undercut by A_3. Since, as we have seen, no status assignment assigns a status to A_3, argument B_2 cannot have a status either. $E = \{A_1\}$ is then still the only preferred and grounded extension of the extended argumentation theory. Then according to preferred semantics B_2 is neither justified, nor defensible, nor overruled while according to grounded semantics it is defensible.

EXERCISE 4.6.7

1. $\mathcal{K}_p$ consists of:

$\forall x(\texttt{BornInNL}(x) \leadsto \texttt{Dutch}(x))$
$\forall x(\texttt{NorwegianName}(x) \leadsto \texttt{Norwegian}(x))$
$\forall x((\texttt{Dutch}(x) \vee \texttt{Norwegian}(x)) \leadsto \texttt{LikesIceSkating}(x))$
$\texttt{BorninNL}(b)$
$\texttt{NorwegianName}(b)$
$\forall x \neg(\texttt{Dutch}(x) \wedge \texttt{Norwegian}(x))$

The following relevant arguments can be constructed:

A_1: $\texttt{BorninNL}(b)$
A_2: $\forall x\, (\texttt{BornInNL}(x) \leadsto \texttt{Dutch}(x))$
A_3: $A_2 \rightarrow \texttt{BornInNL}(b) \leadsto \texttt{Dutch}(b)$
A_4: $A_1, A_3 \Rightarrow \texttt{Dutch}(b)$
A_5: $A_4 \rightarrow \texttt{Dutch}(b) \vee \texttt{Norwegian}(b)$
A_6: $\forall x((\texttt{Dutch}(x) \vee \texttt{Norwegian}(x)) \leadsto \texttt{LikesIceSkating}(x))$
A_7: $A_6 \rightarrow (\texttt{Dutch}(b) \vee \texttt{Norwegian}(b)) \leadsto \texttt{LikesIceSkating}(b)$
A_8: $A_5, A_7 \Rightarrow \texttt{LikesIceSkating}(b)$

B_1: $\texttt{BorninNL}(b)$
B_2: $\forall x\, (\texttt{BornInNL}(x) \leadsto \texttt{Dutch}(x))$
B_3: $B_2 \rightarrow \texttt{BornInNL}(b) \leadsto \texttt{Dutch}(b)$
B_4: $B_1, B_3 \Rightarrow \texttt{Dutch}(b)$
B_5: $\forall x \neg(\texttt{Dutch}(x) \wedge \texttt{Norwegian}(x))$
B_6: $B_4, B_5 \rightarrow \neg\texttt{Norwegian}(b)$

C_1: $\texttt{NorwegianName}(b)$
C_2: $\forall x\ (\texttt{NorwegianName}(x) \rightsquigarrow \texttt{Norwegian}(x))$
C_3: $C_2 \rightarrow \texttt{NorwegianName}(b) \rightsquigarrow \texttt{Norwegian}(b)$
C_4: $C_1, C_3 \Rightarrow \texttt{Norwegian}(b)$
C_5: $C_4 \rightarrow \texttt{Dutch}(b) \vee \texttt{Norwegian}(b)$
C_6: $\forall x((\texttt{Dutch}(x) \vee \texttt{Norwegian}(x)) \rightsquigarrow \texttt{LikesIceSkating}(x))$
C_7: $C_6 \rightarrow (\texttt{Dutch}(b) \vee \texttt{Norwegian}(b)) \rightsquigarrow \texttt{LikesIceSkating}(b)$
C_8: $C_5, C_7 \Rightarrow \texttt{LikesIceSkating}(b)$

D_1: $\texttt{NorwegianName}(b)$
D_2: $\forall x\ (\texttt{NorwegianName}(x) \rightsquigarrow \texttt{Norwegian}(x))$
D_3: $D_2 \rightarrow \texttt{NorwegianName}(b) \rightsquigarrow \texttt{Norwegian}(b)$
D_4: $D_1, D_3 \Rightarrow \texttt{Norwegian}(b)$
D_5: $\forall x \neg(\texttt{Dutch}(x) \wedge \texttt{Norwegian}(x))$
D_6: $D_4, D_5 \rightarrow \neg\texttt{Dutch}(b)$

(If the example is formalised in a propositional language, then the steps A_7 and C_7 must be omitted.)

2. Note first that if no preference relation is specified, it does not hold. Then the relevant defeat relations are as follows:

   - B_6 defeats C_4 and thus also $C_5 - C_8$
   - D_6 defeats B_4 and thus also B_5 and B_6
   - D_6 defeats A_4 and thus also $A_5 - A_8$
   - B_6 defeats D_4 and thus also D_5 and D_6

   Let us first concentrate on B_6 and D_6. Since they defeat each other and have no other defeaters, it is possible to assign no status to them. Then in the grounded status assignments they have no status. But then the same holds for the arguments defeated by one of them. This includes A_8 and C_8. Hence the conclusion $\texttt{LikesIceSkating}(b)$ only has defensible arguments and is therefore itself defensible.

   (The same answer in terms of the fixpoint definition: Since B_6 and D_6 defeat each other and have no other defeaters, they are in no F^i. But then the arguments defeated by one of them also are in no F^i.)

3. Let us again first concentrate on B_6 and D_6. Argument B_6 can be made *in* by making D_6 *out* and vice versa. Then there is a maximal status assignment in which B_6 is *in* and D_6 is *out*. In this status assignment also $C_4 - C_8$ are *out* and $A_1 - A_8$ are *in*. So an argument for the conclusion $\texttt{LikesIceSkating}(b)$ is *in*, namely, A_8. Conversely, there is also a maximal status assignment in which D_6 is *in* and B_6 is *out*. In this status assignment also $A_4 - A_8$ are *out* and $C_1 - C_8$ are *in*. So again an argument for the conclusion $\texttt{LikesIceSkating}(b)$ is *in* but this time it is not A_8 but C_8. So both A_8 and C_8 are defensible, so the conclusion $\texttt{LikesIceSkating}(b)$ is also defensible.

4. Since both preferred extensions contain an argument for the conclusion $\texttt{LikesIceSkating}(b)$, this conclusion is f-justified, even though there is no justified argument for it.

EXERCISE 4.6.8 The following formalisation is based on the intuition that the conclusion that Larry is not rich is justified. The undercutters in the example are based on the principle that statistical defaults about subclasses have priority over statistical defaults about superclasses.

$\mathcal{R}_s$ consists of all valid propositional and first-order inferences.

$\mathcal{R}_d$ consists of:

d_1. $\texttt{Lawyer}(x) \Rightarrow \texttt{Rich}(x)$
d_2. $\texttt{LivesInHollywood}(x) \Rightarrow \texttt{Rich}(x)$
d_3. $\texttt{PublicDefender}(x) \Rightarrow \neg\, \texttt{Rich}(x)$
d_4. $\texttt{RentsinHollywood}(x) \Rightarrow \neg\, \texttt{Rich}(x)$
d_5. $\texttt{PublicDefender}(x) \Rightarrow \neg d_1(x)$
d_6. $\texttt{RentsinHollywood}(x) \Rightarrow \neg d_2(x)$

$\mathcal{K}_p$ consists of

p_1. $\texttt{PublicDefender}(L)$
p_2. $\texttt{RentsInHollywood}(L)$

$\mathcal{K}_n$ consists of

n_1. $\forall x(\texttt{PublicDefender}(x) \supset \texttt{Lawyer}(x))$
n_2. $\forall x(\texttt{RentsInHollywood}(x) \supset \texttt{LivesInHollywood}(x))$

The following relevant arguments can be constructed:

A_1: $\texttt{PublicDefender}(L)$
A_2: $\forall x(\texttt{PublicDefender}(x) \supset \texttt{Lawyer}(x))$
A_3: $A_1, A_2 \rightarrow \texttt{Lawyer}(L)$
A_4: $A_3 \Rightarrow \texttt{Rich}(L)$

B_1: $\texttt{PublicDefender}(L)$
B_2: $B_1 \Rightarrow \neg\texttt{Rich}(L)$

C_1: $\texttt{RentsInHollywood}(L)$
C_2: $\forall x(\texttt{RentsInHollywood}(x) \supset \texttt{LivesInHollywood}(x))$
C_3: $C_1, C_2 \rightarrow \texttt{LivesInHollywood}(L)$
C_4: $C_3 \Rightarrow \texttt{Rich}(L)$

D_1: $\texttt{RentsInHollywood}(L)$
D_2: $B_1 \Rightarrow \neg\texttt{Rich}(L)$

E_1: $\texttt{PublicDefender}(L)$
E_2: $E_1 \Rightarrow \neg d_1(L)$

F_1: $\texttt{RentsInHollywood}(L)$
F_2: $F_1 \Rightarrow \neg d_2(L)$

Let us apply preferred semantics (but in grounded semantics the outcome is the same). Note first that E_2 undercuts A_4 and F_2 undercuts C_4. Moreover, neither E_2 nor F_2 has a defeater, so both of them are in all preferred extensions. But then A_4 and C_4 are not in any preferred extension, so that B_2 and D_2 are in all these extensions. So the conclusion $\neg\texttt{Rich}(L)$ is justified.

EXERCISE 4.6.9. The point of this exercise is that closure under transposition does not imply closure under contraposition.

1. $Cl_{tp}(R_s) = R_s \cup \{\overline{q} \rightarrow \overline{p};\ \overline{r} \rightarrow \overline{p};\ p, \overline{s} \rightarrow \overline{r};\ r, \overline{s} \rightarrow \overline{p}\}$.

2. Yes.

3. No.

EXERCISE 4.6.10. The point of this exercise is that closure under contraposition does not imply closure under transposition.

1. No: $\mathcal{R}_s$ contains $p \rightarrow q$ but not $\neg q \rightarrow \neg p$.

2. Yes. We have:

   $\{p\} \vdash q$ and $\{\neg q\} \vdash \neg p$
   $\{p\} \vdash \neg r$ and $\{r\} \vdash \neg p$
   $\{\neg r\} \vdash q$ and $\{\neg q\} \vdash r$
   $\{\neg q\} \vdash r$ and $\{\neg r\} \vdash q$

   So $\mathcal{R}_s$ satisfies contraposition.

EXERCISE 4.6.11.

1. C_2 rebuts D_2 and not vice versa. Since both arguments use defeasible rules and no preference relations hold between them, C_2 successfully rebuts and therefore defeats D_2. Argument C_2 in turn has two defeaters: its subarguments A_2 and B_2 defeat each other and thus also defeat C_2. Since there are no undefeated arguments that defeat A_2 or B_2, none of A_2, B_2, C_2 and D_2 are in the grounded extension. (In terms of status assignments: it is possible to give none of them a status so in the grounded extension, which maximises undecidedness, none of them have a status.) However, none of these arguments are defeated by an argument that is in the grounded extension, so they are all defensible.

2. Note that A_2 can be made *in* if B_2 is made *out* and vice versa. Then at least one preferred status assignment makes A_2 *in* and B_2 *out*, since such assignments minimise undecidedness. But since A_2 defeats C_2, this assignment also makes C_2 *out*. But then it makes D_2 *in*, since its only defeater is C_2. Conversely, a second preferred status assignment makes B_2 *in* and A_2 *out* so it also makes C_2 *out* and D_2 *in*. Since there are no other preferred status assignments, in all such assigments C_2 is *out* and D_2 is *in*. But then C_2 is overruled and D_2 is justified.