Chapter 1

Defining the Structure of Arguments with an AI Model of Argumentation

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Abstract. The structure of arguments is an important issue in the field of informal logic and argumentation theory. In this paper we discuss how the "standard approach" of Thomas, Walton, Freeman and others can be analyzed from a formal perspective. We use the *ASPIC+* framework for structured argumentation for making the standard model of argument structure complete and for introducing a distinction between types of individual arguments and types of argument structures. We then show that Vorobej's extension of the standard model with a new type of hybrid arguments is not needed if our formal approach is adopted. We finally discuss the structure of so-called accrual of arguments.

Introduction

The structure of arguments is an important issue in the field of informal logic and argumentation theory. Many logicians have given their definitions according to different criteria. The main issue is to define the different ways in which premises and conclusions can be combined to generate different structural argument types. The first model can be traced back to the works of Beardsley (1950), Thomas (1986) and Copi and Cohen (1990). Many informal logicians contributed to this topic, for instance, Walton (1996) and Freeman (2011). Vorobej (1995) extended their models with an additional argument type called "hybrid arguments". The main aim of this paper is to show how formal AI models of argumentation can be used to further extend and clarify these informal models of the structure of arguments. In particular, we argue that although these models provide much insight in the structure of argumentation, they still have some limitations, since their classifications are incomplete and since they do not distinguish between types of individual arguments and structures consisting of several arguments. Moreover, we argue that Vorobej's proposal can be clarified by making a distinction between deductive and defeasible arguments.

We aim to achieve our aims by applying the ASPIC+ framework of Modgil & Prakken (2011a,b), and Prakken (2010), since it arguably currently is the most general AI framework for structured argumentation. The framework has been shown (Gijzel & Prakken, 2011, Modgil & Prakken, 2011a,b; Prakken, 2010) to capture a number of other approaches to structured argumentation, such as assumption-based argumentation (Dung, Mancarella, & Toni, 2007), forms of classical argumentation (Gorogiannis & Hunter, 2011) and Carneades (Gordon, Prakken, & Walton, 2007). In Prakken (2010) it is also shown that *ASPIC+* can capture reasoning with presumptive argument schemes. The *ASPIC+* framework is based on two ideas: the first is that conflicts between arguments are often resolved with explicit preferences, and the second is that arguments are built with two kinds of inference rules: strict, or deductive rules, whose premises guarantee their conclusion, and defeasible rules, whose premises only create a presumption in favor of their conclusion. The second idea implies that *ASPIC+* does not primarily see argumentation as inconsistency handling in a given base logic: conflicts between arguments may not only arise from the inconsistency of a knowledge base but also from the defeasibility of the reasoning steps in an argument. Accordingly, arguments can in *ASPIC+* be attacked in three ways: on their uncertain premises or on their defeasible inferences, and the latter by either attacking their conclusion or the inference itself. We will use the *ASPIC+* framework to make four specific contributions: (1) to make the standard classifications complete; to (2) indicate and explain why convergent and divergent arguments are not arguments but argument structures; (3) to indicate and explain why Vorobej's class of hybrid arguments is not needed if an explicit distinction is made between deductive and defeasible arguments; and (4) to analyze the structure of so-called accrual of arguments.

This paper is organized as follows. In section 2, we introduce the standard informal model of argument structure and Vorobej's (1995) extension with so-called hybrid arguments. In section 3, we present a simplified version of the *ASPIC+* framework. We then use this framework in section 4 to complete the standard model and to distinguish between types and structures of arguments. In section 5, we discuss Vorobej's notion of hybrid arguments and how it can be captured in *ASPIC+*. In section 6, we define the structure of a special kind of argument, namely accrual of arguments. Section 7 concludes the paper.

Approaches to Argument Structure

We first introduce the main approaches to argument structures, notably the approach by e.g. Walton (1996) and Freeman (2011), which we will call the standard approach and Vorobej's (1995) extension with so-called hybrid arguments.

1.1 Standard Approach

The standard approach to the structure of arguments was introduced by Stephen N. Thomas (1986). He divided the arguments into (1) *linked arguments*, which means that every premise is dependent on the others to support the conclusion, (2) *convergent arguments*, which means that premises support the conclusion individually, (3) *divergent arguments*, which means that one premise supports two or more conclusions, and (4) *serial arguments*, which means that one premise supports a conclusion which supports another conclusion.

Walton (1996) then further discussed the structure of arguments. We present the informal definitions of the concepts of structures of arguments according to his latest description (2006).

Definition1. The types of arguments are informally defined as follows:

(1) An argument is a *single argument* iff it has only one premise to give a reason to support the conclusion.
(2) An argument is a *convergent argument* iff there is more than one premise and where each premise functions separately as a reason to support the conclusion.
(3) An argument is a *linked argument* iff the premises function together to give a reason to support the conclusion.
(4) An argument is a *serial argument* iff there is a sequence $\{A_1,\cdots\cdots,A_n\}$ such that one proposition A_i acts as the conclusion drawn from other proposition A_{i-1} as premise and it also functions as a premise from which a new proposition A_{i+1} as conclusion is drawn.
(5) An argument is a *divergent argument* iff there are two or more propositions inferred as separate conclusions from the same premise.
(6) An argument is a *complex argument* iff it combines at least two arguments of types (2), (3), (4) or (5).

In order to show the diagrams of argument types and structures, we first need to define an inference graph. An inference graph is a labeled, finite, directed graph, consisting of statement nodes and supporting links indicating connecting relationships between nodes. In the diagrams of inference graphs, nodes are displayed as dots while supporting links are indicated using ordinary arrowheads. Then example diagrams of the above argument types are shown in Figure1 (for simplicity, the distinction between strict and defeasible supporting links will be left implicit).

Example 1. Walton gives the following examples of, respectively, a convergent, divergent and linked argument:

(1) (A) Tipping makes the party receiving the tip feel undignified; (B) Tipping leads to an underground, black-market economy; (C) Tipping is a bad practice.

(2) (A) Smoking has been proved to be very dangerous to health; (B) Commercial advertisements for cigarettes should be banned; (C) Warnings that smoking is dangerous should be printed on all cigarette packages.

(3) (A) Birds fly; (B) Tweety is a bird; (C) Tweety flies.

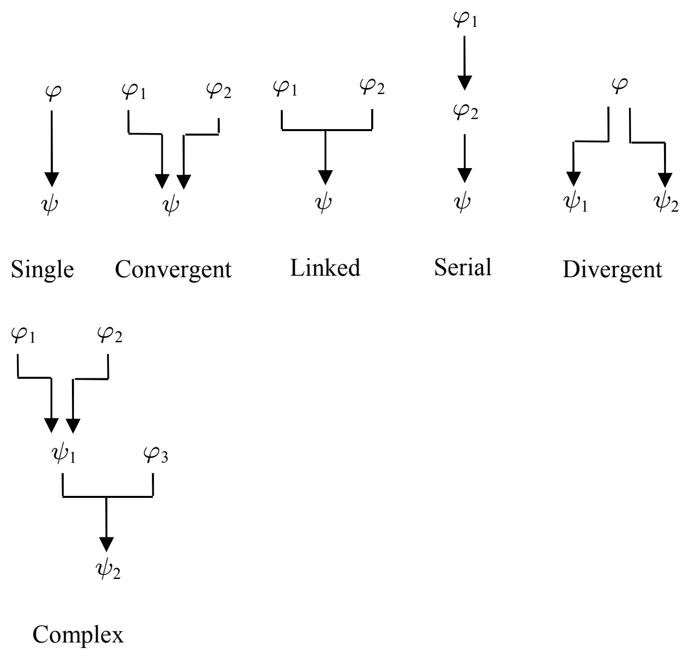


Figure 1. Diagrams of the Standard Approaches

In Example 1(1), the three statements form a convergent argument, since statements (A) and (B) function separately as a reason to support the conclusion (C). By contrast, in Example1(2) these three statements form a divergent argument, since statement (B) and (C) are inferred as separate conclusions from the same premise (A). Finally, Example1(3) is a linked argument, since neither premise alone gives any reason to accept the conclusion.

2.2. Hybrid Arguments

Mark Vorobej (1995) argued that the standard approach needs to be extended with a class of *hybrid* arguments. To discuss this class, we must first present Vorobej's basic definitions of types of arguments.

Definition2. An argument *A* is:

- *simple* iff *A* has exactly one conclusion. Otherwise, *A* is complex.
- *convergent* iff *A* is simple and each premise in *A* is relevant to *C*, where relevance is treated as a primitive dyadic relation obtaining in each instance between a set of propositions and a single proposition.

Definition3. A *linked* set and *linked* argument are defined as follows:

- A set of premises Δ forms a linked set *iff*

(1) Δ contains at least two members;

(2) Δ is relevant to *C*, and

(3) no proper subset of Δ is relevant to *C*.

- An argument A is *linked* iff A is simple and each premise in A is a member of some *linked* set.

Vorobej then motivates this new class of hybrid arguments with examples like the following one.

Example2. Consider example (F) as follows:

- (F): (1) All the ducks that I've seen on the pond are yellow. (2) I've seen all the ducks on the pond. (3) All the ducks on the pond are yellow.

Vorobej observes that (2) in isolation is not relevant to (3), so this is not a convergent argument. Secondly, (1) is relevant to (3), so (1) is not a member of any linked set, so this is also not a linked argument. Vorobej regards (F) as a hybrid argument, since (1) is relevant to the conclusion (3) and (2) is not relevant to the conclusion (3) but (1) and (2) together provide an additional reason for (3), besides the reason provided by (1) alone.

Vorobej provides the following definition of *hybrid* arguments in terms of a relation of supplementation between premises.

The relation of supplementation and *hybrid* argument are defined as follows:

- A set of premises Σ supplements a set of premises Δ iff

(1) Σ is not relevant to *C*;

(2) Σ is relevant to *C*;

(3) $\Sigma \cup \Delta$ offers an additional reason *R* in support of *C*, which Δ alone does not provide;

(4) Σ and Δ are the minimal sets yielding *R* which satisfy clauses (1),(2) and (3).

- An argument A is *hybrid* iff A is simple and contains at least one supplemented (or supplementing) set.

In Example2 premise (2) supplements premise (1). The argument is therefore a *hybrid* argument.

The *ASPIC+* Framework

The *ASPIC+* framework of Modgil & Prakken (2011a,b) and Prakken (2010) models arguments as inference trees constructed by two types of inference rules, namely, strict and defeasible inference rules. In this paper we use a simplified version of *ASPIC+* framework, with symmetric negation instead of an arbitrary contrariness function over the language and with just one instead of four types of premises. We also leave the various preference orderings on inference rules, the knowledge base and arguments implicit.

Definition5. [*Argumentation system*] An *argumentation system* is a tuple $AS = (\mathcal{L}, \mathcal{R})$, where

- is a logical language closed under negation ($\neg$). Below we write $\varphi = -\psi$ when either $\varphi = \neg\psi$ or $\psi = \neg\varphi$.
- $\mathcal{R}=\mathcal{R}_s \cup \mathcal{R}_d$ is a set of strict ($\mathcal{R}_s$) and defeasible ($\mathcal{R}_d$) inference rules such that $\mathcal{R}_s \cap \mathcal{R}_d = \emptyset$.

Definition6. [*Knowledge base*] A *knowledge base* in an argumentation system $(\mathcal{L}, \mathcal{R})$ is a set $\mathcal{K} \subseteq \mathcal{L}$.

Arguments can be constructed step-by-step by chaining inference rules into trees. In what follows, for a given argument the function *Prem* returns all its premises, *Conc* returns its conclusion *Sub* returns all its sub-arguments, while *TopRule* returns the last inference rule applied in the argument.

Definition7. [*Argument*] An argument A on the basis of a knowledge base $\mathcal{K}$ in an argumentation system $(\mathcal{L}, \mathcal{R})$ is:

1. φ *if* $\varphi \in \mathcal{K}$ with: $Prem(A) = \{\varphi\}$; $Conc(A) = \varphi$; $Sub(A) = \{\varphi\}$; $TopRule(A) = undefined$.
2. $A_1, \cdots\cdots, A_n \rightarrow / \Rightarrow \psi$ *if* $A_1, \cdots\cdots, A_n$ are arguments such that there exists a strict/defeasible rule $Conc(A_1), \cdots\cdots, Conc(A_n) \rightarrow / \Rightarrow \psi$ in $\mathcal{R}_s \cup \mathcal{R}_d$.
   $Prem(A) = Prem(A_1) \cup, \cdots\cdots, \cup Prem(A_n)$;
   $Conc(A) = \psi$;
   $Sub(A) = Sub(A_1) \cup, \cdots\cdots, \cup Sub(A_n) \cup \{A\}$;
   $TopRule(A) = Conc(A_1), \cdots\cdots, Conc(A_n) \rightarrow / \Rightarrow \psi$
   $DefRules(A) = DefRules(A_1) \cup, \cdots\cdots, \cup DefRules(A_n)$.

An argument is *strict* if all its inference rules are strict and *defeasible* otherwise.

Definition8. [*Maximal proper subargument*] Argument A is a *maximal proper subargument* of B iff A is a subargument of B and there does not exist any proper subargument C of B such that A is a proper subarugment of C.

The following example illustrates these definitions.

Example3. Consider a knowledge base in an argumentation system with $\mathcal{R}_s=\{p, q \rightarrow s; u, v \rightarrow w\}$; $\mathcal{R}_d= \{p \Rightarrow t; s, r, t \Rightarrow v\}$; $\mathcal{K}= \{p, q, r, u\}$.

The diagram of the argument for w is displayed in Figure 2. Strict inferences are displayed with solid lines and defeasible inferences with dotted lines. Formally the argument and its subarguments are written as follows:

$A_1=[p]$; $A_2=[q]$; $A_3=[r]$; $A_4=[t]$; $A_5=[m]$;
$A_6=[A_1, A_2 \rightarrow s]$;
$A_7=[A_3, A_4, A_6 \Rightarrow v]$;
$A_8=[A_5 \rightarrow n]$;
$A_9=[A_8 \Rightarrow u]$;
$A_{10}=[A_7, A_9 \rightarrow w]$.

We have that

$Prem(A_{10}) = \{p, q, r, t, m\}$;
$Conc(A_{10}) = w$;
$Sub(A_{10})=\{ A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}\}$;
$MaxSub(A_{10})= \{ A_7, A_9\}$;
DefRules $(A_{10}\}= \{ n \Rightarrow u;\ s, r, t \Rightarrow v\}$;
$Toprule(A_{10}\}= \{u, v \rightarrow w\}$.

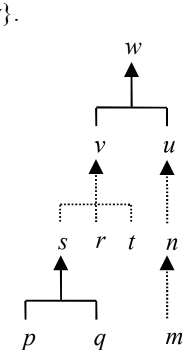


Figure 2. An Argument in *ASPIC+*

In the *ASPIC+* framework arguments can be attacked in three ways: attacking a premise, a defeasibly derived conclusion, or a defeasible inference. Attacks combined with an argument ordering yield a defeat relation, so that *ASPIC+* induces an abstract argumentation framework in the sense of Dung (1995). Since for present purposes the precise nature of the attack and defeat relations are irrelevant, we refer the reader for the formal definitions to Modgil & Prakken (2011) and Prakken (2010).

Types and Structures of Argument

We now give a new classification of arguments in terms of the *ASPIC+* framework and then define so-called argument structures, which are collections of arguments with certain features. We first define two kinds of *unit* arguments and then define several other argument notions consisting of these two *unit* types in different ways. We finally define various structures of argument in terms of the various definitions of argument types.

Definition9. [*Argument type*] The types of arguments can be defined as follows:

(1) An argument A is an *unit I* argument iff A has the form $B\rightarrow\psi$ or $B\Rightarrow\psi$ and subargument B is an atomic argument $B{:}\varphi$. We call the inference rule $\varphi\rightarrow\psi$ or $\varphi\Rightarrow\psi$ an *unit I* inference.

(2) An argument A is an *unit II* argument iff A has the form $B_1,\cdots\cdots,B_n\rightarrow\psi$ or $B_1,\cdots\cdots, B_n\Rightarrow\psi$ and subarguments $B_1,\cdots\cdots,B_n$ are atomic arguments $B_1{:}\varphi_1, \cdots\cdots,B_n{:}\varphi_n$. We call the inference rule $\varphi_1,\cdots\cdots,\varphi_n\rightarrow\psi$ or $\varphi_1,\cdots\cdots,\varphi_n\Rightarrow\psi$ an *unit II* inference.

(3) An argument A is a *multiple unit I* argument iff all inferences $r_1,\cdots\cdots,r_n$ in the argument A are *unit I* inferences.

(4) An argument A is a *multiple unit II* argument iff all inferences $r_1,\cdots\cdots,r_n$ in the argument A are *unit II* inferences.

(5) An argument A is a *mixed* argument iff A has at least one *unit I* subargument and *unit II* subargument.

We display the diagrams of argument types in Figure3. For simplicity, we assume n=2 in these diagrams and show only one case of a *mixed* argument.

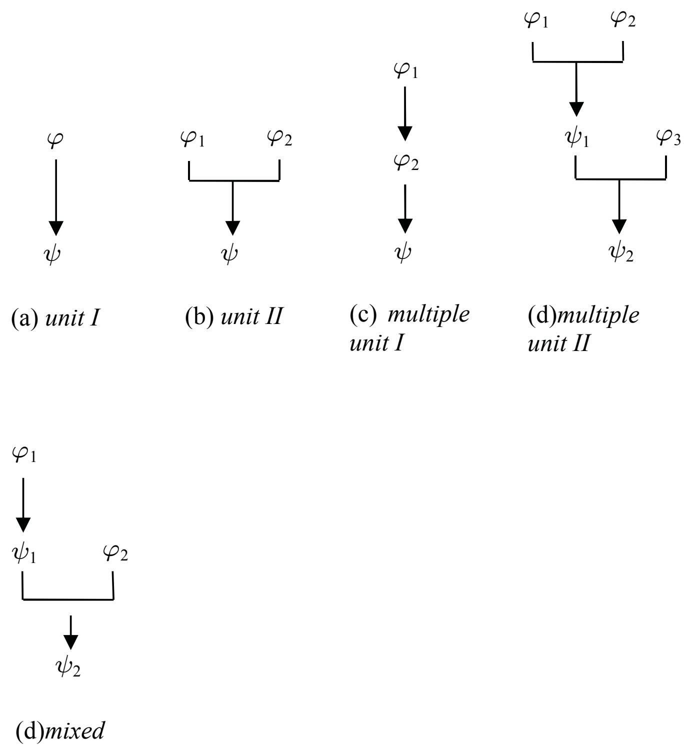


Figure 3. Diagrams of Argument Types in the New Classification

Proposition1. Every argument is of exactly one argument type.

Proof. Firstly, we prove the existence of an argument type by induction on the number of unit inferences. For n=1, argument A corresponds to an *unit I* argument. For n=k>1, argument A corresponds to a *multiple unit I* argument, a *multiple unit II* argument, or a *mixed* argument. For n=k+1, we represent argument A as $B_1,\cdots\cdots,B_n \Rightarrow \psi$, where $m \leqslant n$. Consider the following cases:

(1) If B_i is a *multiple unit I* argument and r_{k+1} is an *unit I* inference, then according to definition7 and definition9(3), A is a *multiple unit I* argument.

(2) If B_i is a *multiple unit I* argument and r_{k+1} is an *multiple unit II* inference, then according to definition7 and definition9(5), A is a *mixed* argument.

(3) If B_i is a *multiple unit II* argument and r_{k+1} is an *unit I* inference, then according to definition7 and definition9(5), A is a *mixed* argument.
(4) If B_i is a *multiple unit II* argument and r_{k+1} is an *unit II* inference, then according to definition7 and definition9(4), A is a *multiple unit II* argument.
(5) If B_i is a *mixed* argument and r_{k+1} is an *unit I* or *unit II* inference, then according to definition7 and definition9(5), A is a *mixed* argument.

Secondly, we prove the property of uniqueness of argument type. Assume there exists an argument A corresponding to two or more argument types: then there must exist two or more top rules in the argument, and then there are two or more conclusions in A, which contradict the definition of argument.

Consider again Example 3. We have that A_1, A_2, A_3, A_4, A_5 are atomic arguments, A_8 is an *unit I* argument, A_6 is an *unit II* argument, A_9 is a *multiple unit I*, A_7 is a *multiple unit II*, and A_{10} is a *mixed* argument.

We next define several argument structures, which are sets of arguments with certain properties. We should first define connected arguments and an interconnected argument set as follows:

Definition10. Argument A and B are connected iff there exist $A' \in Sub(A)$ and $B' \in Sub(B)$, such that $Conc(A') \in Prem(B')$ or $Conc(A')=Conc(B')$ or $Prem(A') \subseteq Prem(B')$.

Proposition2. An argument A is connected with any of its subarguments.

Proof. For any $A_i \in Sub(A)$ it holds that $Prem(A_i) \subseteq Prem(A)$. From definition10, it follows that A_i is connected with A.

In Example 3, argument A_{10} is connected with any subargument A_i, where $i \in \{1,\cdots\cdots,9\}$. But minimal subarguments of A_{10} are not connected.

Definition11. A set of arguments $S=\{A_1,\cdots\cdots,A_n\}$ ($n \geqslant 2$) is *interconnected* iff for any argument A_i and $A_j \in S$, there exists a sequence of arguments $B_1,\cdots\cdots,B_k$, $B_{k+1},\cdots\cdots,B_m$ in S, where B_k is connected with B_{k+1} ($1 \leqslant m \leqslant n$), such that A_i is connected with B_1 and A_j is connected with B_m.

In Example3, $\{A_1,\cdots\cdots,A_{10}\}$ is interconnected. Moreover, let $S=\{A, B, A', B'\}$, where $A=[p]$, $B=[q]$, $A'=[A \Rightarrow p]$ with inference rule $p \Rightarrow q$ and $B'=[B \Rightarrow q]$ with inference rule $r \Rightarrow s$. From definition11, it follows that S is not interconnected, since (1) there is no argument to connect A with B or B' and (2) there is no argument to connect B with $A'$and (3) there is no argument to connect A' and B'.

Corollary1. A set of arguments S consisting of an argument A and all of its subarguments is interconnected.
Proof. Follows from proposition2 and definition11.

We call the argument set which consists of an argument and its subarguments as classic interconnected set and non-classic interconnected set otherwise. For instance, in Example3, $\{A_1,\cdots\cdots,A_{10}\}$ is classic interconnected.

Definition12. The set of argument structures[1] is defined as follows:

(1) A set of arguments $\{A_1,\cdots\cdots,A_n\}$ is a *serial convergent structure SCS* iff there are only *unit I* arguments in the set of arguments $\{A_1,\cdots\cdots,A_n\}$ and for any A_i and A_j we have $Conc(A_i)=Conc(A_j)$, where $i\neq j$.
(2) A set of arguments $\{A_1,\cdots\cdots,A_n\}$ is a *serial divergent structure SDS* iff there are only *unit I* arguments in the set of arguments $\{A_1,\cdots\cdots,A_n\}$ and for any A_i and A_j we have $Prem(A_i)=Prem(A_j)$, where $i\neq j$ and $A_i \notin Sub(A_j)$.
(3) A set of arguments $\{A_1,\cdots\cdots,A_n\}$ is a *linked convergent structure LCS* iff it contains only *unit II* arguments and for any A_i and A_j we have $Conc(A_i)=Conc(A_j)$, where $i\neq j$.
(4) A set of arguments $\{A_1,\cdots\cdots,A_n\}$ is a *linked divergent structure LDS* iff it contains only *unit II* arguments and for any A_i and A_j we have $Prem(A_i)=Prem(A_j)$, where $i\neq j$.
(5) A set of arguments $\{A_1,\cdots\cdots,A_n\}$ is a *mixed structure MS* iff it is non-classic interconnected and it is not of the form of either *SCS*, *SDS*, *LCS* or *LDS*.

We display the diagrams of argument structures in Figure4. For simplicity, we assume n=2 in the diagrams and show only one case of *mixed structure*.

[1] The structure here is different from the structure in informal approaches, where it refers to the structure of an *individual* argument.

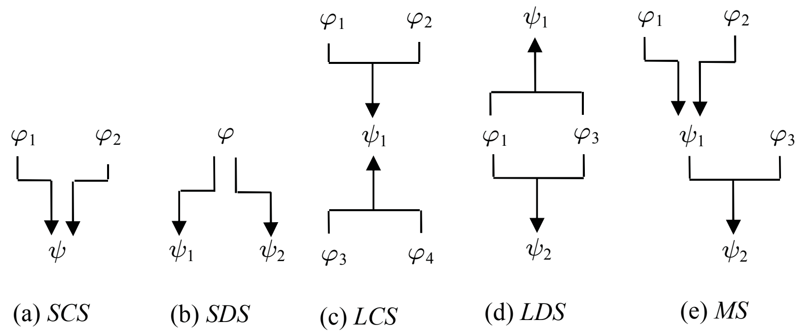


Figure 4. Diagrams of Argument structures in the New Classification

Proposition3. All of argument structures (*SCS*, *SDS*, *LCS*, *LDS* or *MS*) are non-classic interconnected.

Proof. Follows from definition 11 and definition 12.

Proposition4. If a set of arguments *S* is a *mixed* structure, then there exist at least one *unit I* argument and one *unit II* argument in it.

Proof. Suppose for contradiction that there does not exist *unit I* argument and one *unit II* argument in a set of arguments *S*. From definition 9, we have all cases as follows:

S does not contain *unit I* argument and one *unit II*. It contradicts definition 9, since all of argument types consists of *unit I* argument or one *unit II*.

S only contains *unit I* argument. If S is not interconnected, then *S* is not in any argument structure. If *S* is interconnected, then *S* is a *SCS* or *SDS* according to definition 12.

S only contains *unit II* argument. If *S* is not interconnected, then *S* is not in any argument structure. If *S* is interconnected, then S is a *LCS* or *LDS* according to definition 12.

Corollary2. If any two proper subsets of interconnected argument set *S* are of two different types of *SCS*, *SDS*, *LCS* and *LDS*, then *S* is a *mixed structure*.

Proof. Follows from proposition 4.

4.1. Reconsidering the Standard Approach

First, we consider the correspondence between the standard approach and our new approach. It is easy to see that single, linked and serial arguments, respectively, correspond to *unit I*, *unit II* and *multiple unit I* arguments.

However, *convergent* and *divergent* arguments are not arguments any more, since a *convergent* "argument" now is an argument structure consisting of a number of distinct *unit I* arguments for the same conclusion, while a divergent "argument" now is an argument structure consisting of a number of distinct *unit I* argument with the same premise. For instance, in Example1(1) there are two arguments $A \Rightarrow (C)$ and $B \Rightarrow (C)$ for the same conclusion (C), and in Example1(2), there are two arguments $A \Rightarrow (B)$ and $A \Rightarrow (C)$ with the same premise (A) where but different conclusions.

	Argument types				
Standard approach	*single*	*linked*	*serial*		*complex*
New approach	*unit I*	*unit II*	*multiple unit I*	*multiple unit II*	*mixed*

Table 1. Comparison of Argument Types

	Argument structures				
Standard approach	*convergent*	*divergent*			*complex*
New approach	*serial convergent structure*	*serial divergent structure*	*linked convergent structure*	*linked divergent structure*	*mixed structure*

Table 2. Comparison of Argument Structures

Therefore, the classes of convergent, divergent "arguments" are not arguments but argument *structures*. Actually, they correspond to the serial convergent structure *SCS* and the serial divergent structure *SDS*. Moreover, the class of complex arguments in the standard approach is not an argument if it contains *SCS* or *SDS*, but instead corresponds to the mixed argument structure *MS*. Otherwise, it corresponds to a mixed argument.

From the above analysis we see that the standard approach is incomplete and, moreover, does not distinguish types of individual argument from types of argument structures. We can show the comparisons between the standard approach and our new approach in *Table1* and *Table2*. We can also conclude that the new classification in terms of the *ASPIC+* framework is helpful in clarifying and complementing the standard approach.

5. The Problem of Hybrid Arguments

In this section we analyze why Vorobej's class of hybrid arguments is not needed if our approach is adopted. In our new approach, Vorobej's hybrid "argument" are not arguments but argument structures consisting a number of arguments. More specifically, they are of type mixed structure MS or linked convergent structure *LCS*.

We first make a notion explicit and redefine a definition. In Vorobej (1995) the notion of relevance is implicit and treated as a primitive dyadic relation. We note that there are two kinds of relevance: *defeasible relevance* indicates the support from a set of arguments to the conclusion via a defeasible inference, while *strict relevance* indicates the support form a set of arguments to the conclusion via a strict inference.

In the *ASPIC+* framework, we write $S \vdash \varphi$ if there exists a strict argument for φ with all premises taken from *S*, and $S \mid\!\sim \varphi$ if there exists a defeasible argument for φ with all premises taken from *S*. Then Definition 4 can be rewritten as follows:

Definition13. A set of premises Σ supplements a set of premises Δ iff (1) $\Sigma \not\mid\!\sim$ C and $\Sigma \neq \emptyset$; (2) $\Delta \mid\!\sim C$; (3) $\Sigma \cup \Delta \vdash C$ or $\Sigma \cup \Delta \mid\!\sim C$, and (4) $\Sigma \cup \Delta$ is the minimal set satisfying clauses (1),(2) and (3) when $\cup \Delta \vdash C$.

If a set of premises $\Sigma=\{P_1,\cdots\cdots,P_m\}$ supplements a set of premises $\Delta=\{Q_1,\cdots\cdots,Q_n\}$, then we have two arguments *A* and *B*, where argument *A* is of the form $Q_1,\cdots\cdots,Q_n \Rightarrow C$ and argument *B* is of the form $P_1,\cdots\cdots,P_m,Q_1,\cdots\cdots,Q_n \Rightarrow C$ or $P_1,\cdots\cdots,P_m,Q_1,\cdots\cdots,Q_n \rightarrow C$.

Thus, the hybrid argument here is a (1) mixed structure *MS* consisting of a *unit I* argument and a *unit II* argument, if m=1, or (2) a linked convergent structure *LCS* consisting of two linked arguments, if m>1.

We now first reconsider Example 2.

- (F): (1) All the ducks that I' ve seen on the pond are yellow. (2) I've seen all the ducks on the pond. (3) All the ducks on the pond are yellow.

Arguably, (1) supports (3) because of the defeasible inference rule of enumerative induction:

- All observed F's are G's $\Rightarrow$ all F's are G's.

Moreover, (1) and (2) together arguably support (3) because of a deductive version of enumerative induction:

- All observed F's are G's, all observed F's are all F's $\Rightarrow$ all F's are G's.

We then see that the apparently hybrid argument is in fact a convergent structure consisting of two separate arguments for the same conclusion, sharing one premise:

- $A = [1 \Rightarrow (3)]$ with a defeasible inference rule: All observed F's are G's $\Rightarrow$ all F's are G's;
- $B = [1,2 \Rightarrow (3)]$ with a strict inference rule: All observed F's are G's, all observed F's are all F's $\rightarrow$ all F's are G's.

Actually, all examples in Prakken (2010) can be reconstructed in terms of these two kinds of structures:

Example4. Consider examples (*G*) and (*J*) as follows:

(G): (1) My duck is yellow. (2) Almost without exception, yellow ducks are migratory. (3) My duck is no exception to any rule. (4) My duck migrates.

(H): (1) My duck is yellow. (2) Most yellow ducks, especially those born in Ontario, are migratory. (3) My duck was born in Enterprise. (4) Enterprise is in Ontario. (5) My duck is migratory.

In example (G), we have that $\{(1),(2)\} \mid\!\sim (4)$ and $\{(1),(2),(3)\} \vdash (4)$, so we have two arguments A and B for the same conclusion:

- $A = [1,2 \Rightarrow (4)]$ with a defeasible inference rule: almost without exception X's are Y's, a is a $X \Rightarrow a$ is a Y;
- $B = [1,2\ ,3\ \rightarrow (4)]$ with a strict inference rule: almost without exception X's are Y's, a is a X, a is no exception to any rule $X \rightarrow a$ is a Y.

In example (H), there are two arguments A and B based on $\{(1),(2)\} \mid\!\sim (5)$ and $\{(1),(2),(3),(4)\} \mid\!\sim (5)$:

- $A = [1,2 \Rightarrow (5)]$ with a defeasible inference rule: Most X's are Y's, especially X's born in Z, a is a $X \Rightarrow a$ is a Y;
- $B = [1,2,3,4 \Rightarrow (5)]$ with a defeasible inference rule: Most X's are Y's, especially X's born in Z, a is a X, a born in y, y is in $Z \Rightarrow a$ is a Y.

On our account arguments in example (G) and (H) are both linked convergent structures.

Example5. Consider examples (*H*) and (*I*) as follows:

- (I): (1) All the ducks that Data has seen on the pond are yellow. (2) All the ducks that Dax has seen on the pond are yellow. (3) Data has seen 96% of the ducks on the pond. (4) All the ducks on the pond are yellow.

- (J): (1) Data quacks. (2) Data has webbed feet. (3) 95% of those creatures who both quack and have webbed feet are ducks. (4) Data is a duck.

In example (I), there are two arguments *A*, *B* based on {(1)}|~(4), {(1),(3)}|~(4) (Note that argument based on {(1),(2),(3)}|~(4) is not an argument, since {(1),(2)} is not the minimal set yielding (4) and then (3) does not supplement {(1),(2)}:

- *A* = [1 ⇒(4)] with a defeasible inference rule: All observed *F*'s are *G*'s ⇒ all *F*'s are *G*'s;
- *C* = [1, 3 ⇒(4)] with a defeasible inference rule: All observed *F*'s are *G*'s, 95% observed *F* ⇒ all *F*'s are *G*'s.

In example (J), there are four arguments *A*, *B*, *C* and *D* based on {(1)}|~(4), {(2)}|~(4), {(1),(2)}|~(4) and {(1),(2),(3)}|~(4):

- *A* = [1⇒(4)] with a defeasible inference rule: *x* quacks ⇒ *x* is a duck;
- *B* = [2 ⇒(4)] with a defeasible inference rule: *x* has webbed feet ⇒ *x* is a duck;
- *C* = [1, 2 ⇒(4)] with a defeasible inference rule that aggregates the two previous inference rules;
- *D* = [1, 2, 3 ⇒(4)] with a defeasible inference rule: *a* is a *Y*, *a* is a *Z*, 95% of *x*'s who are both *Y* and *Z* are *T* ⇒*a* is a *T*.

On our account arguments in example (I) is a linked convergent structure and (J) is a mixed structure.

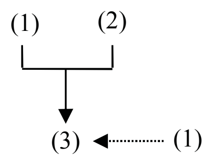
Example6. Consider example (*K*) as follows:

- (K): (1) Data and Dax have the same diet. (2) Data and Dax receive the same amount of exercise. (3) Data is a healthy duck. (4) Dax is a healthy duck.

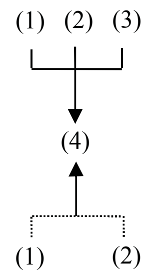
In example (K), there are three arguments *A*, *B* and C based on {(1),(3)}|~(4), {(2),(3)}|~(4) and {(1),(2),(3)}|~(4) (Note that no matter {(2)} supplements {(1),(3)} or {(1)} supplements {(2),(3)}, it would follow {(1),(2),(3)}|~(4)):

- *A* = [1,3 ⇒(4)] with a defeasible inference rule: *x* and *y* have the same diet, *x* is a healthy duck ⇒ *y* is a healthy duck;
- *B* = [2,3 ⇒ (4)] with a defeasible inference rule: *x* and *y* receive the same amount of exercise, *x* is a healthy duck ⇒ *y* is a healthy duck;
- *C* = [1,2,3 ⇒ (4)] with a defeasible inference rule that aggregates the two previous inference rules.

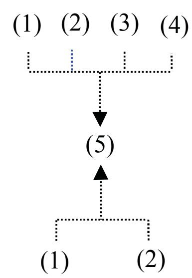
On our account arguments in example (K) is a linked convergent structure. The diagrams of the arguments in above examples are displayed in Figure5.



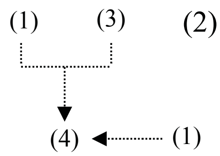
(a) *Example F*



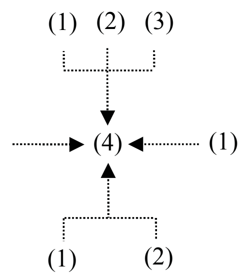
(b) *Example G*



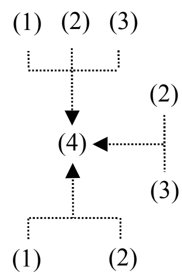
(c) *Example H*



(d) *Example I*



(e) *Example J*



(f) *Example K*

Figure 5. Diagrams of the Vorobej's examples

6. The Problem of Accrual of Arguments

In this section, we discuss the structure of so-called accrual of arguments. It often happens in natural arguments that several reasons support one proposition in such a way that each reason is offered as additional support for the proposition. This is called accrual of reasons or accrual of arguments.

In the AI literature many contributions (Gomez et al., 2009; Prakken, 2005; Verheij, 1995) focus on the formalisation of accrual of arguments. For example, Prakken (2010) presents the accrual of arguments as a form of inference in a standard logical framework for defeasible argumentation.

We first show an example of accural of arguments. Suppose that P is "witness a testifies φ" and Q is "witness b testifies φ", then we have two arguments:

A=[$A_1 \Rightarrow \varphi$], where A_1=[P] and B=[$B_1 \Rightarrow \varphi$], where B_1=[Q] . Thus these two arguments have the same conclusion \varphi with defeasible rule r: *testifies*(x,φ)$\Rightarrow \varphi$. According to the treatment of Prakken (2005) accrual is a new form of inference, and the basic idea towards formalizing this special inference can be divided into two points.

First, the conclusion of each individual defeasible inference step is labelled with the premises of the applied defeasible inference rule. Given a set of defeasible inference rules, they are slightly reformulated to the effect that their conclusions are labelled with the set of their premises. So, defeasible modus ponens for $\Rightarrow$ can be defined as follows:

$$\varphi, \varphi \Rightarrow \psi | \sim \psi^{\{\varphi, \cdots\cdots, \varphi \Rightarrow \psi\}}.$$

Second, a new defeasible inference rule is introduced that takes any set of labelled versions of a certain formula and produces the unlabelled version:

$$\varphi^{l_1}, \cdots\cdots, \varphi^{l_n} | \sim \varphi$$

Note that the above labels will for readability often be abbreviated to $l_1, \cdots\cdots, l_n$ and the rule is a scheme for any natural number i such that $1 \leqslant i \leqslant n$. Therefore, in the above example we have a new inference rule: $\varphi^{P,P \Rightarrow \varphi}, \varphi^{Q,Q \Rightarrow \varphi} | \sim \varphi$ and the accrual of arguments of this example can be presented as C=[A,$B \Rightarrow \varphi$], where A=[$A_1 \Rightarrow \varphi$] and B=[$B_1 \Rightarrow \varphi$].

The accrual of arguments as formalized in Modgil and Prakken (2011) is different from the notion of convergent arguments in informal approaches, since as we have discussed, the so called convergent argument is an argument structure rather than an argument. Actually, accrual of arguments is a mixed argument or *multiple unit II* argument with defeasible inference rule $\varphi^{l_1}, \cdots\cdots, \varphi^{l_n} | \sim \varphi$ as toprule. It should be noted that all subarguments of the accrual of arguments only have defeasible top rules, since accrual would not make sense for strictly derived conclusions. Then we can conclude the definition of the accrual of arguments as follows:

Definition13. An argument A is an accrual of arguments with conclusion φ iff A is a *multiple unit II* argument or *mixed* argument with *Toprule*(A)= $\varphi^{l_1}, \cdots\cdots, \varphi^{l_n} \Rightarrow \varphi$.

We now reconsider example (K) and show the accrual of arguments inside. According to the analysis in Example 6, there are two linked arguments for a same conclusion with two inferences rules:

- r_1: x and y have the same diet, x is a healthy duck $\Rightarrow$ y is a healthy duck;

- r_2: x and y receive the same amount of exercise, x is a healthy duck $\Rightarrow$ y is a healthy duck.

Arguably, there is an accrual argument for statement (4), accruing two arguments A and B for (4) based on two labelled versions of (4), viz. $(4)^{\{(1),(3),r_1\}}$ and $(4)^{\{(2),(3),r_2\}}$. Applying the accrual rule: $(4)^{\{(1),(3),r_1\}}$, $(4)^{\{(2),(3),r_2\}}$|~(4) to the above two reasons results in a defeasible argument for (4), namely accrual argument C, which can be represented as C=[A,B⇒(4)], where A=[1,3⇒$(4)^{\{(1),(3)\}}$] and B=[2,3⇒$(4)^{\{(2),(3)\}}$]. Actually, C is a *multiple unit II* argument and its structure is shown in Figure 6:

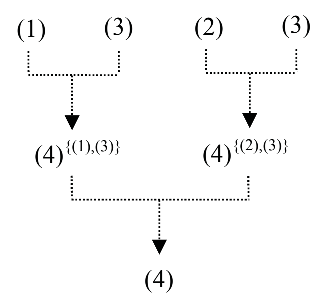


Figure 6. An accrual of arguments

Proposition5. If a non-classic interconnected argument set S contains an argument A which is an accrual of arguments, then

- it is a *mixed* structure, if S contains both *unit I* inferences and *unit II* inferences; or
- it is a *linked convergent* structure, if S only contains *unit II* inferences.

Proof. Firstly, since argument A is an accrual of arguments, then by definition 14, A contains *unit II* inferences.

Secondly, suppose for contradiction that S is not a mixed structure. By assumption and definition 12, S should be *SCS*, *LCS*, *SDS* or *LDS*, since S is non-classic interconnected. But S contains either *unit I* or *unit II*, contradicting S contains both *unit I* inferences and *unit II* inferences.

Thirdly, if S only contains *unit II* inferences and it is non-classic interconnected, then by definition 12 it is easy to conclude that S is a *linked convergent* structure.

Moreover, the interconnected argument set S consists of arguments in example (K) which contains three arguments in Example 6 and one accrual of arguments we have discussed. From the above proposition, since S only contains *unit II* inferences, we have that S is a *linked convergent* structure.

Corollary3. If an argument is an accrual of arguments, then it must belong to a set of arguments which is a *LCS* or *MS*.

Proof. Follows from proposition 5.

7. Conclusion

In this paper we showed how AI models of argumentation can be used to clarify and extend informal-logic approaches to the structure of arguments. We indicated that the standard approach is incomplete and then defined a complete classification of types of arguments in terms of the *ASPIC+* framework. We highlighted that *convergent* and *divergent* "arguments" in the standard approach are not arguments but sets of arguments, which we have classified as argument structures. We also showed that Vorobeij's *hybrid* arguments can be defined in terms of our classification if the distinction between deductive and defeasible inferences is made explicit, thus obviating the need to introduce a new type of argument to handle the examples discussed by Vorobeij. Finally, we applied the new approach to analyze the structure of accrual of arguments and we defined it as two possible argument types. We believe that our contributions are particularly relevant for argumentation theory, since we have clarified and extended terminology concerning classifications of arguments which is often used by argumentation theorists. Thus we have shown how formal methods can be of use not just in AI but also in argumentation theory and informal logic.

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