COSMOLOGY FORMULA SHEET

FRIEDMANN EQS: c

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3c^2}\rho - \frac{Kc^2}{a^2} + \frac{\Lambda}{3}$$

$$\dot{H} = -\frac{4\pi G_N}{c^2}(P+\rho) + \frac{Kc^2}{a^2}$$

COVARIANT CONSERVATION OF $T_{\mu\nu}$:

PERFECT FLUID:

$$T_{\mu\nu} = (P+\rho)\frac{u_\mu u_\nu}{c^2} - g_{\mu\nu}P$$

$$\dot{\rho} + 3H(\rho + P) = 0$$

FLUID REST FR: $u^\mu = (c, 0, 0, 0)$

EOS: $P = w\rho$

- MATTER: $w = 0$ (dust)
- RADIATION: $w = 1/3$ (relat. fluid)
- INFLATION: $w \simeq -1$ $\left(\epsilon = \frac{3}{2}(1+w) \ll 1\right)$
- CURVATURE: $w = -1/3$.

GENERAL METRIC:

$$ds^2 = c^2dt^2 - a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right]$$

K = spatial curvature

a(t) = scale factor

LUMINOSITY DISTANCE d_L: $\mathcal{F} = \frac{\mathcal{L}}{4\pi d_L^2}$

$\mathcal{F}$ = flux ; $\mathcal{L}$ = luminosity (source)

HOMOGENEOUS SCALAR FIELD

$$P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \; ; \; \rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

BOSE-EINSTEIN DISTR. / FERMI-DIRAC
(CHEMICAL EQUILIBRIUM)

$$n_{BE} = \frac{1}{e^{\beta(E-\mu)} - 1} \; ; \; n_{FD} = \frac{1}{e^{\beta(E-\mu)} + 1}$$

$$E = \sqrt{(\vec{p}c)^2 + (mc^2)^2} \; ; \; \mu = \text{chemical potential}$$

$(\beta = 1/(k_B T))$

ENERGY DENSITY / PRESSURE =

$$\rho = \sum_a g_a \int \frac{d^3p}{(2\pi\hbar)^3} E_a(\vec{p})\, n_a \; \Big| \; P = \sum_a g_a \int \frac{d^3p}{(2\pi\hbar)^3} \frac{c^2p^2}{3E_a(p)} n_a(p)$$

RELATIVISTIC FLUID:

$$\rho = g_* \frac{\pi^2 (k_B T)^4}{30 (\hbar c)^3} \; ; \; P = \frac{1}{3}\rho$$

$$g_* = \sum_{a=\text{bosons}} g_a \left(\frac{T_a}{T}\right)^4 + \frac{7}{8} \sum_{a=\text{fermions}} (T_a/T)^4$$

CFL CONDENSATE

$$\epsilon_{ab}\langle \psi^{a\alpha}_{iL}(\vec{p},p^0)\,\psi^{b\beta}_{jL}(-\vec{p},p^0)\rangle = -\langle\ \rangle_R = \frac{\Delta_{CFL}(\vec{p}^2,p^0)}{\hbar c}\,\epsilon^{\alpha\beta A}\epsilon_{ijA}$$

SYMMETRY BREAKING

$$\Delta_{CFL} \propto \exp\left(-\frac{3\pi^2}{\sqrt{2}\, g_s}\right)$$

$$SU(3)_C \times \underbrace{SU(3)_L \times SU(3)_R + U(1)_B}_{\text{GLOBAL}} \to SU(3)_{C+L+R} \times \mathbb{Z}_2$$

$\Rightarrow$ all gluons become massive

NEUTRINO TEMPERATURE TODAY

$$T_\nu \simeq \left(\frac{4}{11}\right)^{1/3} T_\gamma \simeq 1.95\,K$$

BOLTZMANN EQUATION

$$\frac{d}{dt} f(\vec{x},\vec{p},t) = \left(\partial_t + \vec{v}\cdot D_{\vec{x}} + \vec{F}\cdot D_{\vec{p}}\right) f = \mathrm{Coll}[f]$$

$\to$ IN CHEMICAL EQ, 2→2 scatt:

$$\Rightarrow \frac{1}{a^3}\left[a^3 n_1\right]^{\bullet} = -n_{10}\, n_{20} \langle \sigma v\rangle \left[\frac{n_1 n_2}{n_{10} n_{20}} - \frac{n_3 n_4}{n_{30} n_{40}}\right]$$

(2→2 scattering)

$z_{eq} \simeq 3300$ | $z_{dec} \simeq 1090 \pm 1$.

$\tau_{reion} \simeq 0.09$ | $z_{reion} \simeq 10$

COSMOLOGICAL PERTURBATIONS

GAUGE INVARIANT POTENTIALS:

$$w_\varphi = \varphi + \frac{\dot\varphi}{H}\psi \; ; \; w_\psi = \psi + \frac{H}{\dot\varphi}\varphi = \frac{H}{\dot\varphi} w_\varphi$$

(MUKHANOV - SASAKI FIELDS)

* ZERO CURVATURE GAUGE: $\psi = 0, \; w_\psi = \frac{H}{\dot\varphi}\varphi$
* COMOVING GAUGE: $\varphi = 0 \Rightarrow w_\psi = \psi$.

GAUGE PERTS: $g_{\mu\nu}(x) \to \tilde g_{\mu\nu}(x) = g_{\mu\nu}(x) - 2\nabla_{(\mu}\xi_{\nu)}$

$x^\mu \to \tilde x^\mu = x^\mu + \xi^\mu$; $\qquad \Phi(x) \to \tilde\Phi(x) = \Phi(x) - \xi^\mu \partial_\mu \Phi(x)$

CANONICAL QUANTIZATION

* SCALAR FIELD : $\pi_\varphi = a^2 \varphi'$

$$[\hat\varphi(\vec x, t), \hat\pi_\varphi(\vec x', t)] = i\hbar\, \delta^3(\vec x - \vec x')$$

* graviton : $\delta g_{\mu\nu} = a^2 h_{\mu\nu}$, $h_{ii} = 0 = \partial_i h_{ij}$.

$$\pi^{ij} = \frac{M_P^2}{4} a^2 h'_{ij} \; ;$$

$$[h_{ij}(\vec x, t), \pi^{kl}(\vec x', t)] = i\hbar \frac{1}{2}[P_{ik}P_{jl} + P_{il}P_{jk} - P_{ij}P_{kl}]\delta^3(\vec x - \vec x')$$

$$P_{ik} = \delta_{ij} - \frac{\partial_i \partial_j}{\nabla^2} \; ; \; \left(1/\nabla^2\right)\delta^3(\vec x - \vec x') = -\frac{1}{4\pi}\frac{1}{\|\vec x - \vec x'\|}$$

SPECTRA : SCALAR,

$$P_{w_\psi} = \left(\frac{H^4}{4\pi^2 \dot\phi^2}\right)_* \left(\frac{k}{k_*}\right)^{n_s - 1} ; \; n_s = 1 - 6\epsilon + 2\eta$$