1- $\boxed{\text{GR 2013 · MIDTERM: LIST FORMULAE}}$

VECTORS/FORMS: $V = V^\mu \partial_\mu$; $\omega = \omega_\mu dx^\mu$

* BASES ON T_p & T^*_p : $\{\partial_\mu\}$ & $\{dx^\mu\}$.

* STRESS ENERGY TENSOR: PERFECT FLUID

$$T^{\mu\nu} = (\rho + p) u^\mu u^\nu + p g^{\mu\nu} ; \quad u^\mu = \frac{dx^\mu}{d\tau}$$

$$\cancel{u^\mu} \quad u^\mu u_\mu = -c^2 ; \quad p^\mu p_\mu = -m^2 c^2 ; \cancel{p^\mu = mc}$$

* PARAMETRIC DERIVATIVE. $\boxed{\frac{d}{d\lambda} = \frac{dx^\mu}{d\lambda}\partial_\mu \;\Big|\; \frac{D}{d\lambda} = \frac{dx^\mu}{d\lambda} D_\mu}$

* TRANSFORMATIONS OF TENSORS;

Ex: $(0,2)$ tensor $g_{\mu\nu}(x)$ transforms as

$$g_{\mu\nu}(x) \to \tilde{g}_{\mu\nu}(\tilde{x}) = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu}\frac{\partial x^\beta}{\partial \tilde{x}^\nu} g_{\alpha\beta}(x)$$

* $g^{\mu\nu} g_{\nu\rho} = \delta^\nu{}_\rho$

LEVI CIVITA TENSOR: $\epsilon = \frac{\sqrt{|g|}\,\tilde{\epsilon}_{\mu_1 \dots \mu_n}}{n!} dx^{\mu_1} \wedge \dots \wedge dx^{\mu_n}$

INTEGRATION; ϕ = scalar field on M.

$$\mathrm{Vol}(\Sigma) = \int_\Sigma \epsilon \doteq \int_\Sigma \sqrt{|g|}\, d^n x$$

$$I_\Sigma[\phi] = \int_\Sigma \epsilon \phi \doteq \int_\Sigma d^n x \sqrt{|g|}\, \phi$$

GEODESIC EQ: $\frac{D}{d\lambda}\frac{dx^\mu}{d\lambda}=\frac{d^2x^\mu}{d\lambda^2}+\Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda}=0$

PARALLEL TRANSPORT OF A VECTOR:

$$\frac{D}{d\lambda}V^\mu=\frac{dV^\mu}{d\lambda}+\Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\lambda}V^\beta=0.$$

LEVI-CIVITA or CHRISTOFFEL CONNECTION

$$\Gamma^\rho_{\mu\nu}(x)=\frac{1}{2}g^{\rho\sigma}\left[\partial_\mu g_{\nu\sigma}+\partial_\nu g_{\sigma\mu}-\partial_\sigma g_{\mu\nu}\right];\ \nabla_\alpha g_{\mu\nu}=0$$

$\nabla_\alpha V^\mu=\partial_\alpha V^\mu+\Gamma^\mu_{\alpha\beta}V^\beta$

$\nabla_\alpha\omega_\beta=\partial_\alpha\omega_\beta-\Gamma^\rho_{\alpha\beta}\omega_\rho.$

} GENERALISE TO (k,l) TENSORS.

STOKES: $\int_V(\nabla_\mu A^\mu)\sqrt{|g|}\,d^nx=\int_{\partial V}n_\mu A^\mu\sqrt{|\gamma|}\,d^{n-1}x$

γ = induced metric on ∂V

ACTION FOR A PARTICLE IN EM FIELD:

$$S[x,A]=-mc\int_1^2\sqrt{-g_{\mu\nu}dx^\mu dx^\nu}-\frac{e}{c}\int_1^2A_\mu dx^\mu$$

RIEMANN TENSOR / TORSION TENSOR

$$[\nabla_\mu,\nabla_\nu]V^\rho=R^\rho{}_{\sigma\mu\nu}V^\sigma-T_{\mu\nu}{}^\alpha\nabla_\alpha V^\rho$$

$R^\rho{}_{\sigma\mu\nu}=\partial_\mu\Gamma^\rho_{\nu\sigma}+\Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma}-(\mu\rightleftarrows\nu)$ | $T^\alpha_{\mu\nu}=2\Gamma^\alpha_{[\mu\nu]}$

Bianchi: $\nabla_{[\lambda}R_{\rho\sigma]\mu\nu}=0$; $\nabla^\mu G_{\mu\nu}=0$,

Einstein TENSOR: $G_{\mu\nu}=R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}$; $R_{\mu\nu}=R^\alpha{}_{\mu\alpha\nu}$; $R=g^{\mu\nu}R_{\mu\nu}$

Killing equation

$$\boxed{D_{(\mu}K_{\nu)}=0} \Rightarrow K^{\nu}D_{\nu}R=0.$$

Ricci scalar / tensor

$$R = g^{\mu\nu}R_{\mu\nu} \Leftarrow \boxed{R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu}}$$