Outline

About Me

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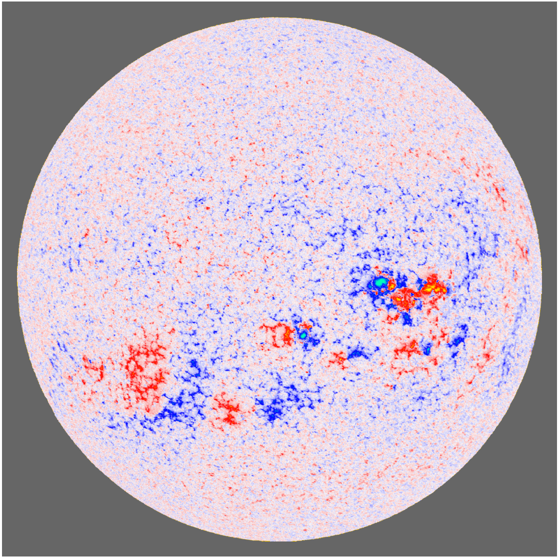
- www.astro.uu.nl/˜keller
- Solar Physics
  - observations of solar magnetic fields using polarization
  - realistic numerical, radiative MHD simulations (A. Vögler)
- Exoplanetary Systems
  - characterization of exoplanetary surfaces and atmospheres
  - polarimetric observations of circumstellar disks
- Instrumentation
  - Dutch Open Telescope (DOT) at La Palma, Canary Islands
  - polarimeters for solar magnetic field studies
  - imaging polarimeters for exoplanetary systems observations
  - high-resolution techniques such as Adaptive Optics

Literature

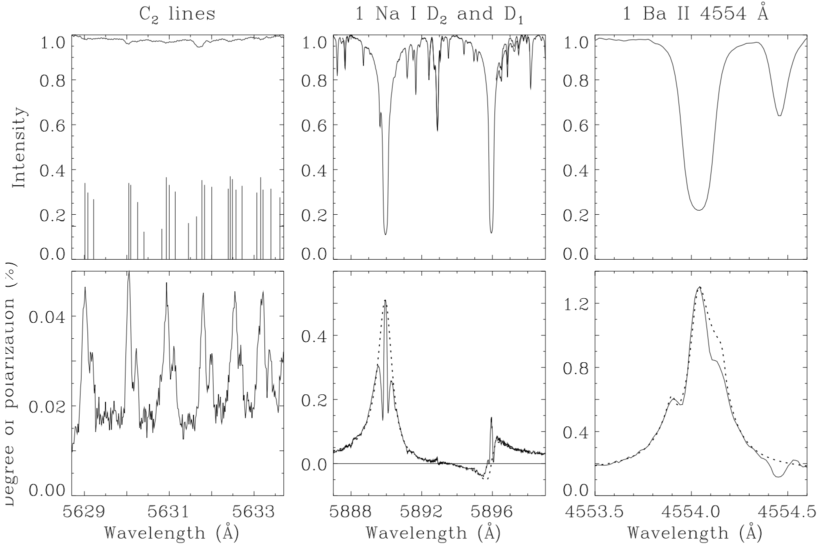
PDF file of this lecture with movies and links at
www.astro.uu.nl/˜keller/Teaching/USO2009/USO_2009_SpectroPolarimetry.pdf

The Sun in Polarized Light

Magnetic Field Maps from Longitudinal Zeeman Effect



Second Solar Spectrum from Scattering Polarization



Generation of Polarized Light

Polarization indicates *anisotropy* $\Rightarrow$ not all directions are equal

- geometry (not everything is spherically symmetric)
- temperature gradients
- magnetic fields
- electrical fields

Polarized Light in the Universe

- Cosmic Microwave Background (CMB) polarization
- unified model of Active Galactic Nuclei
- detection of circumstellar disks and exoplanets
- aerosols in solar-system planetary atmospheres
- interstellar magnetic field from polarized starlight
- supernova asymmetries
- stellar magnetic fields from Zeeman effect
- galactic magnetic field from Faraday rotation

Fundamentals of Polarized Light

Electromagnetic Waves in Matter

- *Maxwell's equations* $\Rightarrow$ electromagnetic waves
- optics: interaction of electromagnetic waves with matter as described by *material equations*
- polarization of electromagnetic waves are integral part of optics

Maxwell's Equations in Matter

$$\nabla \cdot \vec{D} = 4\pi\rho$$

$$\nabla \times \vec{H} - \frac{1}{c}\frac{\partial \vec{D}}{\partial t} = \frac{4\pi}{c}\vec{j}$$

$$\nabla \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$$

$$\nabla \cdot \vec{B} = 0$$

Symbols

- $\vec{D}$ *electric displacement*
- ρ *electric charge density*
- $\vec{H}$ *magnetic field*
- c *speed of light in vacuum*
- $\vec{j}$ *electric current density*
- $\vec{E}$ *electric field*
- $\vec{B}$ *magnetic induction*
- t time

Linear Material Equations

$$\vec{D} = \epsilon \vec{E}$$
$$\vec{B} = \mu \vec{H}$$
$$\vec{j} = \sigma \vec{E}$$

Symbols

- ϵ *dielectric constant*
- μ *magnetic permeability*
- σ *electrical conductivity*

Isotropic Media

- isotropic media: ϵ and μ are scalars
- for most materials: $\mu = 1$

Plane-Wave Solutions

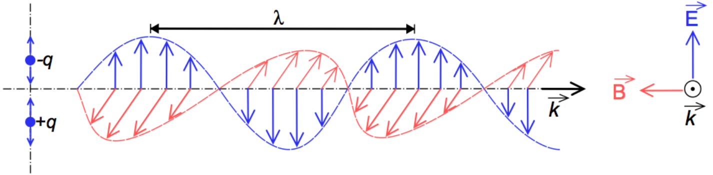
- Plane Vector Wave ansatz

$$\vec{E} = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$$

$\vec{k}$ spatially and temporally constant *wave vector*
$\vec{k}$ normal to surfaces of constant phase
$|\vec{k}|$ *wave number*
$\vec{x}$ spatial location
ω *angular frequency* ($2\pi\times$ frequency)
t time
$\vec{E}_0$ (generally complex) vector independent of time and space

- could also use $\vec{E} = \vec{E}_0 e^{-i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$
- damping if $\vec{k}$ is complex
- real electric field vector given by real part of $\vec{E}$

Transverse Waves



- plane-wave solution must fulfill Maxwell's equations

$$\vec{E}_0 \cdot \vec{k} = 0, \quad \vec{H}_0 \cdot \vec{k} = 0, \quad \vec{H}_0 = \frac{\tilde{n}}{\mu} \frac{\vec{k}}{|\vec{k}|} \times \vec{E}_0$$

- isotropic media: electric, magnetic field vectors normal to wave vector $\Rightarrow$ transverse waves
- $\vec{E}_0$, $\vec{H}_0$, and $\vec{k}$ orthogonal to each other, right-handed vector-triple
- $\vec{E}_0$ and $\vec{H}_0$ have constant relationship $\Rightarrow$ consider only $\vec{E}$

Polarization

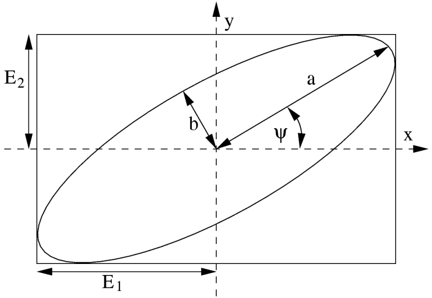
- Plane Vector Wave ansatz $\vec{E} = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$
- spatially, temporally constant vector $\vec{E}_0$ lays in plane perpendicular to propagation direction $\vec{k}$
- represent $\vec{E}_0$ in 2-D basis, unit vectors $\vec{e}_1$ and $\vec{e}_2$, both perpendicular to $\vec{k}$

  $$\vec{E}_0 = E_1 \vec{e}_1 + E_2 \vec{e}_2.$$

  E_1, E_2: arbitrary complex scalars
- damped plane-wave solution with given ω, $\vec{k}$ has 4 degrees of freedom (two complex scalars)
- additional property is called *polarization*
- many ways to represent these four quantities
- if E_1 and E_2 have identical phases, $\vec{E}$ oscillates in fixed plane

Description of Polarized Light

Polarization Ellipse



Polarization

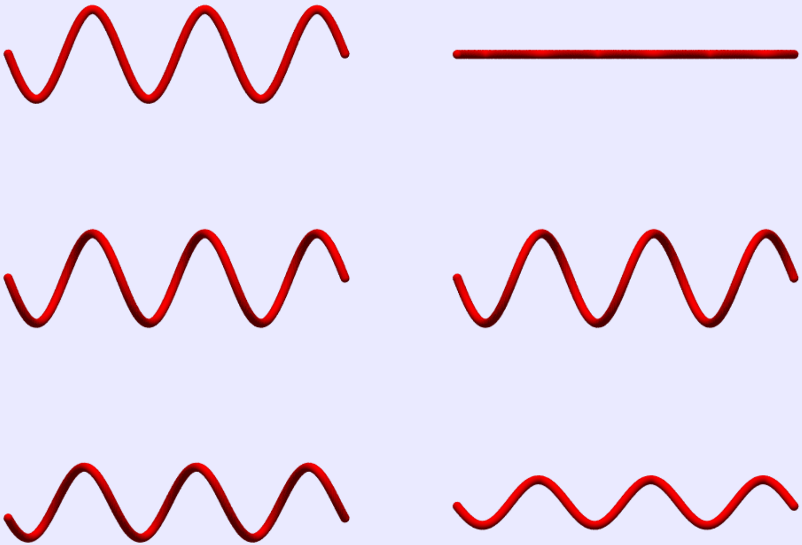
$$\vec{E}(t) = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$$

$$\vec{E}_0 = E_1 e^{i\delta_1}\vec{e}_x + E_2 e^{i\delta_2}\vec{e}_y$$

- wave vector in z-direction
- $\vec{e}_x$, $\vec{e}_y$: unit vectors in x, y
- E_1, E_2: (real) amplitudes
- $\delta_{1,2}$: (real) phases

Polarization Ellipse

- 2 complex scalars not the most useful description
- at given $\vec{x}$, time evolution of $\vec{E}$ described by *polarization ellipse*
- ellipse described by axes a, b, orientation ψ



Jones Vectors

$$\vec{E}_0 = E_x \vec{e}_x + E_y \vec{e}_y$$

- beam in z-direction
- $\vec{e}_x$, $\vec{e}_y$ unit vectors in x, y-direction
- complex scalars $E_{x,y}$
- Jones vector

$$\vec{e} = \left(\begin{array}{c} E_x \\ E_y \end{array} \right)$$

- phase difference between E_x, E_y multiple of π, electric field vector oscillates in a fixed plane $\Rightarrow$ *linear polarization*
- phase difference $\pm\frac{\pi}{2} \Rightarrow$ *circular polarization*
- Maxwell's equations are linear $\Rightarrow$ Jones vector of sum of two waves $=$ sum of Jones vectors
- elements of Jones vectors are not observed directly

Linear Polarization

- horizontal: $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
- vertical: $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
- 45°: $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Circular Polarization

- left: $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$
- right: $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$

Notes on Jones Formalism

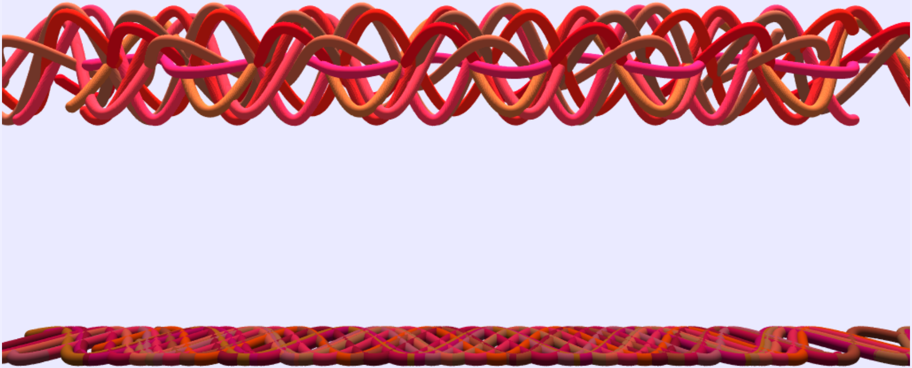
- Jones formalism operates on amplitudes, not intensities
- coherent superposition important for coherent light (lasers, interference effects)
- Jones formalism describes 100% polarized light

Quasi-Monochromatic Light

- monochromatic light: purely theoretical concept
- monochromatic light wave always fully polarized
- real life: light includes range of wavelengths $\Rightarrow$ *quasi-monochromatic light*
- quasi-monochromatic: superposition of mutually incoherent monochromatic light beams whose wavelengths vary in narrow range $\delta\lambda$ around central wavelength λ_0

$$\frac{\delta\lambda}{\lambda} \ll 1$$

- measurement of quasi-monochromatic light: integral over measurement time t_m
- amplitude, phase (slow) functions of time for given spatial location
- *slow*: variations occur on time scales much longer than the mean period of the wave



Polarization of Quasi-Monochromatic Light

- electric field vector for quasi-monochromatic plane wave is sum of electric field vectors of all monochromatic beams

$$\vec{E}\left(t\right) = \vec{E}_0\left(t\right) e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$$

- can write this way because $\delta\lambda \ll \lambda_0$
- measured intensity of quasi-monochromatic beam

$$\left\langle \vec{E}_x \vec{E}_x^* \right\rangle + \left\langle \vec{E}_y \vec{E}_y^* \right\rangle = \lim_{t_m -> \infty} \frac{1}{t_m} \int_{-t_m/2}^{t_m/2} \vec{E}_x(t)\vec{E}_x^*(t) + \vec{E}_y(t)\vec{E}_y^*(t) dt$$

$\langle\cdots\rangle$: averaging over measurement time t_m

- measured intensity independent of time
- quasi-monochromatic: frequency-dependent material properties (e.g. index of refraction) are constant within $\Delta\lambda$

Stokes Vector

- formalism to describe polarization of quasi-monochromatic light
- directly related to measurable intensities
- Stokes vector fulfills these requirements

$$\vec{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} E_x E_x^* + E_y E_y^* \\ E_x E_x^* - E_y E_y^* \\ E_x E_y^* + E_y E_x^* \\ i\left(E_x E_y^* - E_y E_x^*\right) \end{pmatrix} = \begin{pmatrix} E_1^2 + E_2^2 \\ E_1^2 - E_2^2 \\ 2E_1 E_2 \cos\delta \\ 2E_1 E_2 \sin\delta \end{pmatrix}$$

  Jones vector elements $E_{x,y}$, real amplitudes $E_{1,2}$, phase difference $\delta = \delta_2 - \delta_1$

- $$I^2 \geq Q^2 + U^2 + V^2$$

Stokes Vector Interpretation

$$\vec{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \text{intensity} \\ \text{linear } 0^\circ - \text{linear } 90^\circ \\ \text{linear } 45^\circ - \text{linear } 135^\circ \\ \text{circular left} - \text{right} \end{pmatrix}$$

- *degree of polarization*

$$P = \frac{\sqrt{Q^2 + U^2 + V^2}}{I}$$

  1 for fully polarized light, 0 for unpolarized light
- summing of Stokes vectors = *incoherent* addition of quasi-monochromatic light waves

Linear Polarization

- horizontal: $\begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$
- vertical: $\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$
- 45°: $\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

Circular Polarization

- left: $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$
- right: $\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$

Mueller Matrices

- 4×4 real Mueller matrices describe (linear) transformation between Stokes vectors when passing through or reflecting from media

$$\vec{I}' = \mathrm{M}\vec{I} \, ,$$

$$\mathrm{M} = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix}$$

- N optical elements, combined Mueller matrix is

$$\mathrm{M}' = \mathrm{M}_N \mathrm{M}_{N-1} \cdots \mathrm{M}_2 \mathrm{M}_1$$

Rotating Mueller Matrices

- optical element with Mueller matrix M
- Mueller matrix of the same element rotated by θ around the beam given by

$$\mathrm{M}(\theta) = \mathrm{R}(-\theta)\mathrm{M}\mathrm{R}(\theta)$$

with

$$\mathrm{R}(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta & 0 \\ 0 & -\sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

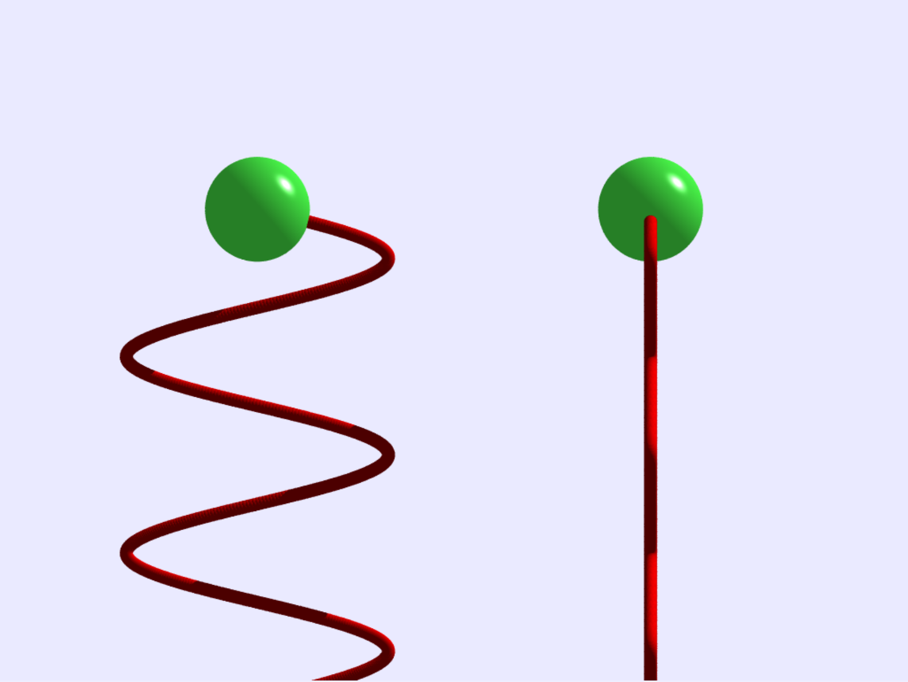
Polarization Summary

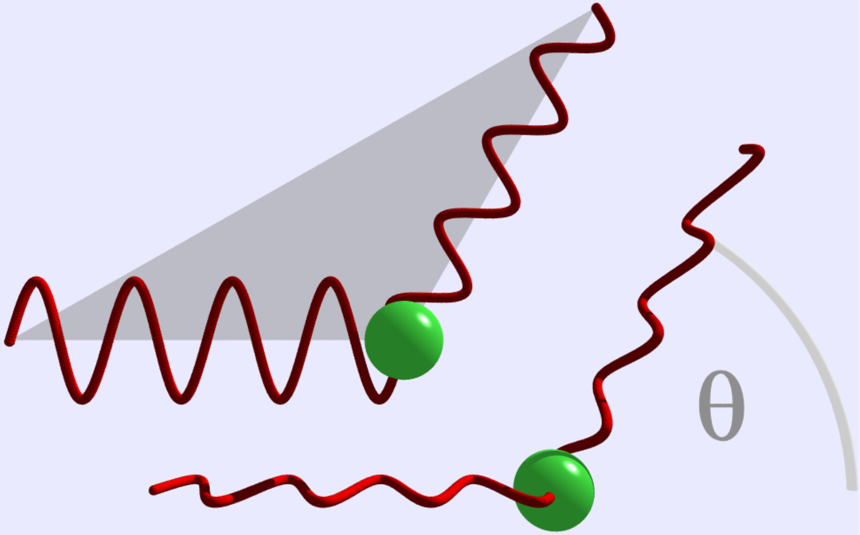
- polarization is an intrinsic property of light
- polarization properties and intensity of light can be described by 4 parameters
- degree of polarization is the fraction of the intensity that is fully polarized
- typical values for degree of polarization:
  - 45 degree reflection off aluminum mirror: 5%
  - clear blue sky: up to 75%
  - 45 degree reflection off glass: 90%
  - LCD screen: 100%
  - solar scattered polarization: 1% to 0.001%
  - exoplanet signal: 0.001%

Scattering Polarization

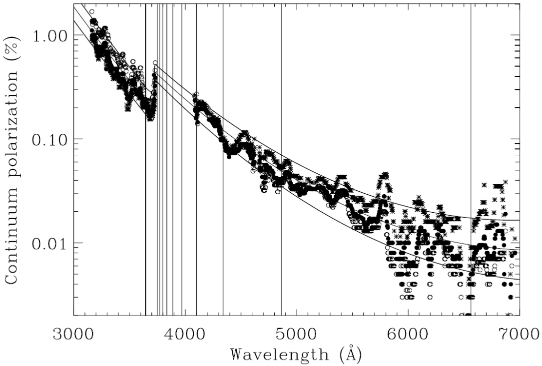
Single Particle Scattering

- light is absorbed and re-emitted
- if light has low enough energy, no energy transfered to electron, but photon changes direction ⇒ elastic scattering
- for high enough energy, photon transfers energy onto electron ⇒ inelastic (Compton) scattering
- Thomson scattering on free electrons
- Rayleigh scattering on bound electrons
- based on very basic physics, scattered light is linearly polarized




θ

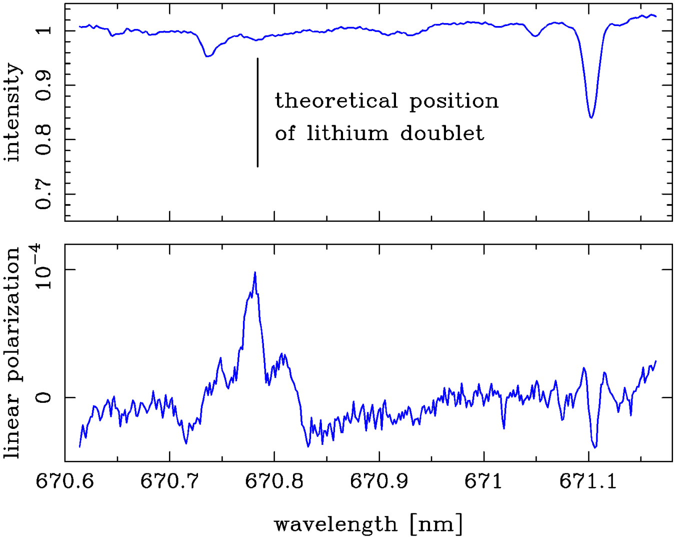
Solar Continuum Scattering Polarization



(from Stenflo 2005)

- due to anisotropy of the radiation field
- anisotropy due to limb darkening
- limb darkening due to decreasing temperature with height
- last scattering approximation without radiative transfer

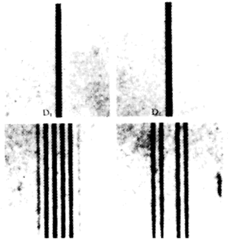
Solar Spectral Line Scattering Polarization



resonance lines exhibit “large” scattering polarization signals

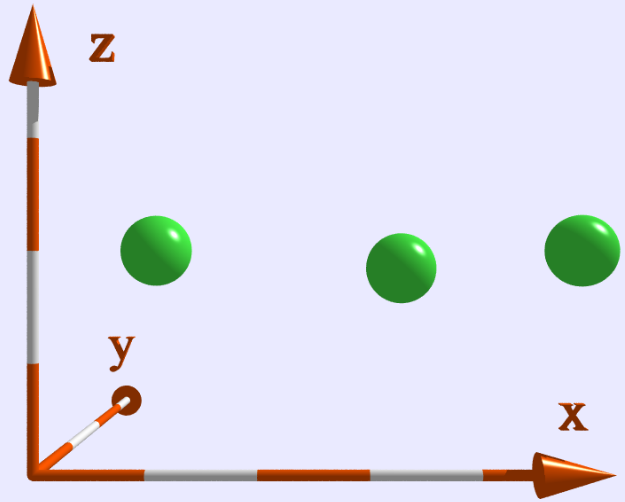
Zeeman Effect

photos.aip.org/



Splitting/Polarization of Spectral Lines

- discovered in 1896 by Dutch physicist Pieter Zeeman
- different spectral lines show different splitting patterns
- splitting proportional to magnetic field
- split components are polarized
- *normal Zeeman effect* with 3 components explained by H.A.Lorentz using classical physics
- splitting of sodium D doublet could not be explained by classical physics (*anomalous Zeeman effect*)
- quantum theory and electron's intrinsic spin led to satisfactory explanation


z
y
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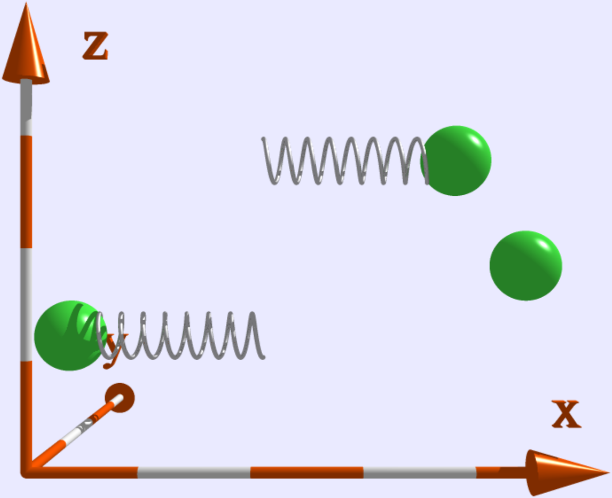
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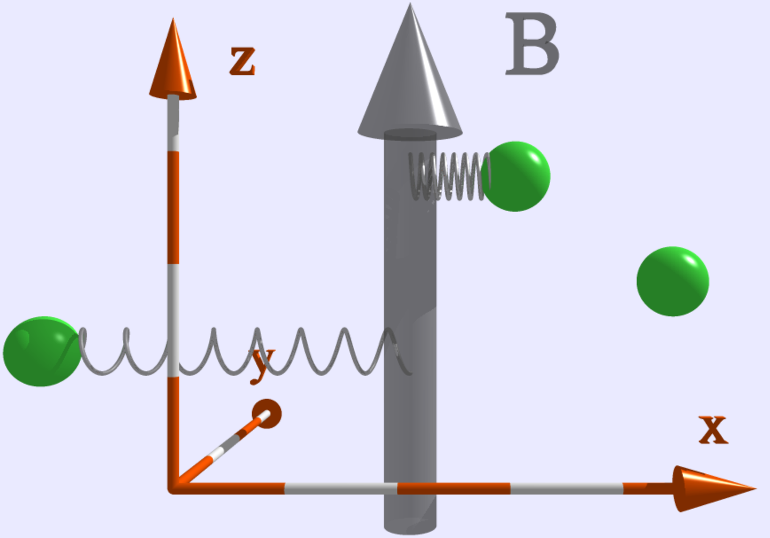

B
z
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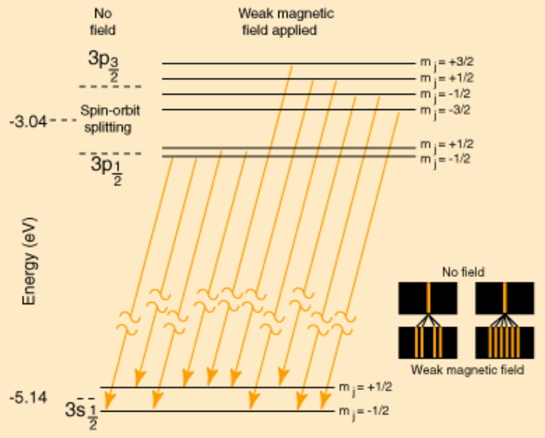


hyperphysics.phy-astr.gsu.edu/hbase/quantum/sodzee.html

Spectral Lines - Transitions between Energy States

- spectral lines are due to transitions between energy states:

  lower level with $2J_l + 1$ sublevels M_l

  upper level with $2J_u + 1$ sublevels M_u

- not all transitions occur (selection rules)

Effective Landé Factor and Polarized Components

- each component can be assigned an effective Landé g-factor, corresponding to how much the component shifts in wavelength for a given field strength
- components are also grouped according to the linear polarization direction for a magnetic field perpendicular to the line of sight
  - π components are polarized parallel to the magnetic field (**p**i for *parallel*)
  - σ components are polarized perpendicular to the magnetic field (**s**igma for German *senkrecht*)
- for a field parallel to the line of sight, the π-components are not visible, and the σ components are circularly polarized

The Landé g Factor

- based on pure mathematics (group theory, Wiegner Eckart theorem), one obtains

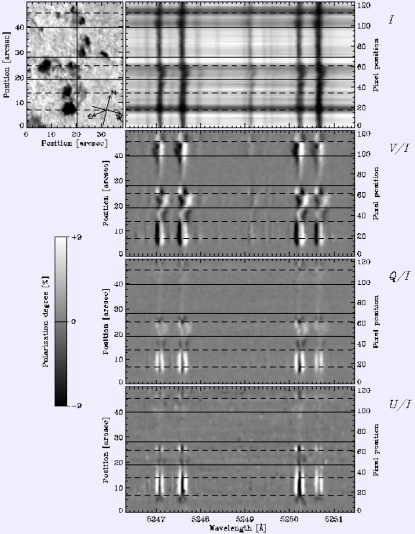
$$\Delta E_{NLSJ}(M) = \mu_0 g_L B M$$

$\mu_0 = \frac{e\hbar}{2m}$	Bohr magneton
g_L	Landé g-factor
B	magnetic field strength
M	eigenvalue of J_z in state $\langle NLSJM|$

- LS coupling, B small compared to spin-orbit splitting field

$$g_L = 1 + \frac{J(J+1) + L(L+1) - S(S+1)}{2J(J+1)}$$

  - $L(L+1)$, eigenvalue of $\vec{L}^2$ (orbital angular momentum)
  - $S(S+1)$, eigenvalue of $\vec{S}^2$ (spin angular momentum)
  - $J(J+1)$, eigenvalue of $\vec{J}^2$ (total angular momentum)

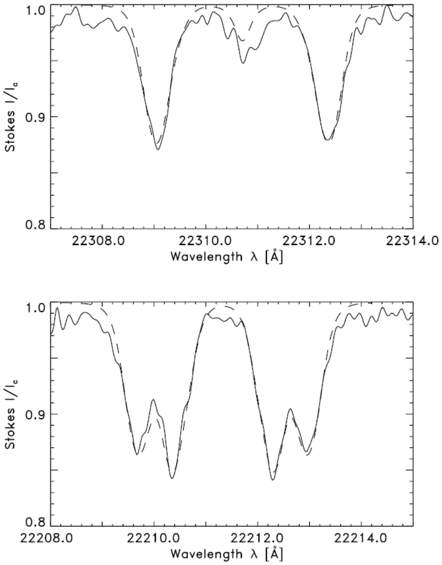


Bernasconi et al. 1998

Zeeman Effect in Solar Physics

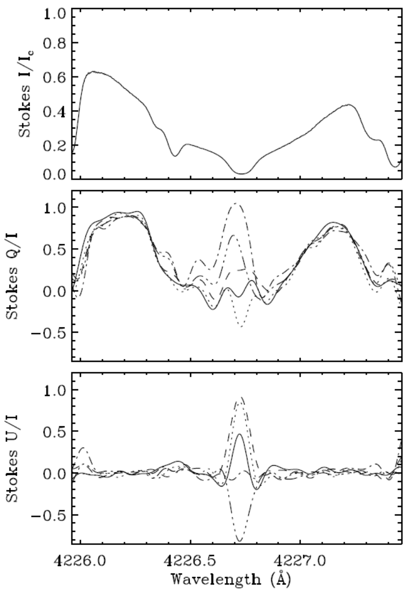
- discovered in sunspots by G.E.Hale in 1908
- splitting small except for in sunspots
- much of intensity profile due to non-magnetic area ⇒ filling factor
- a lot of strong fields outside of sunspots
- full Stokes polarization measurements are key to determine solar magnetic fields
- 180 degree ambiguity

Fully Split Titanium Lines at 2.2μm



Rüedi et al. 1998

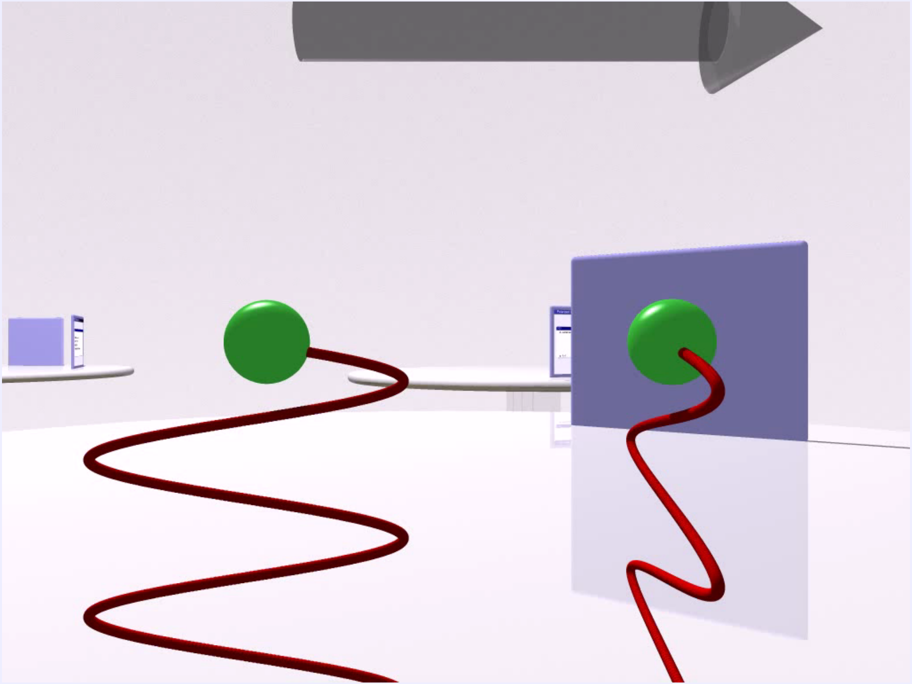
Hanle Effect

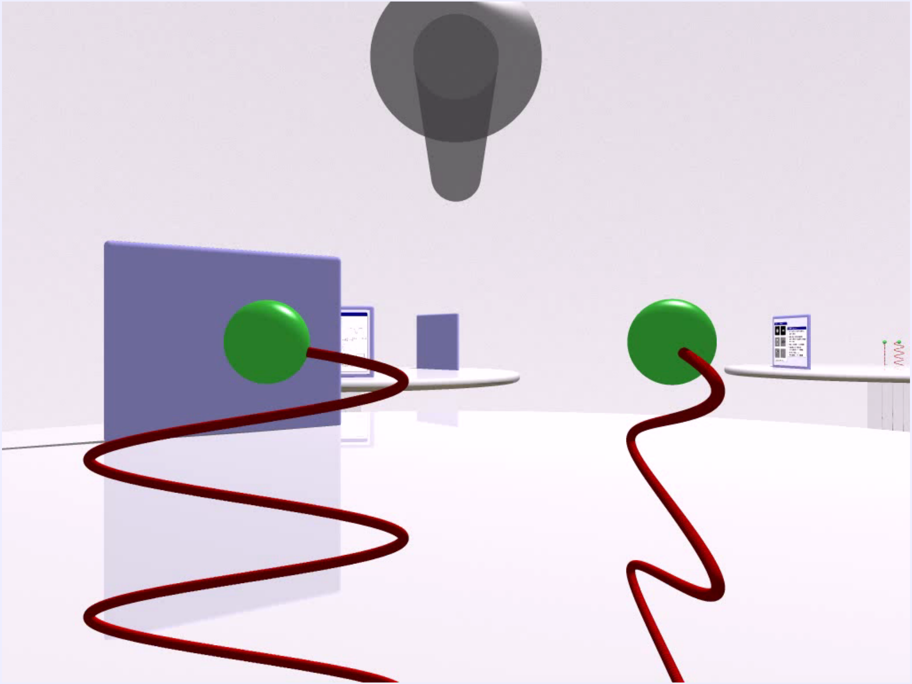


Bianda et al. 1998

Depolarization and Rotation

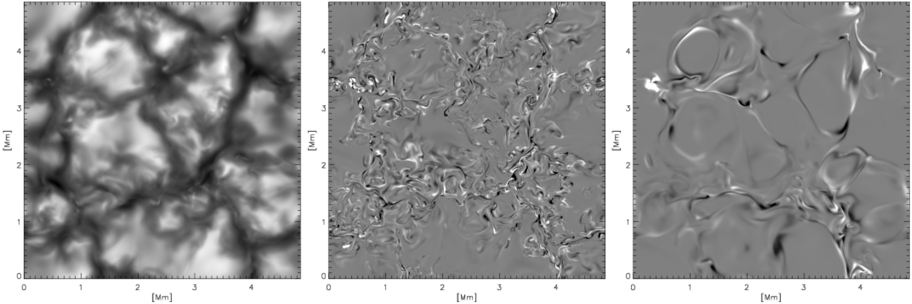
- scattering polarization modified by magnetic field
- precession around magnetic field depolarizes and rotates polarization
- sensitive $\sim 10^3$ times smaller field strengths that Zeeman effect
- measureable effects even for isotropic field vector orientations





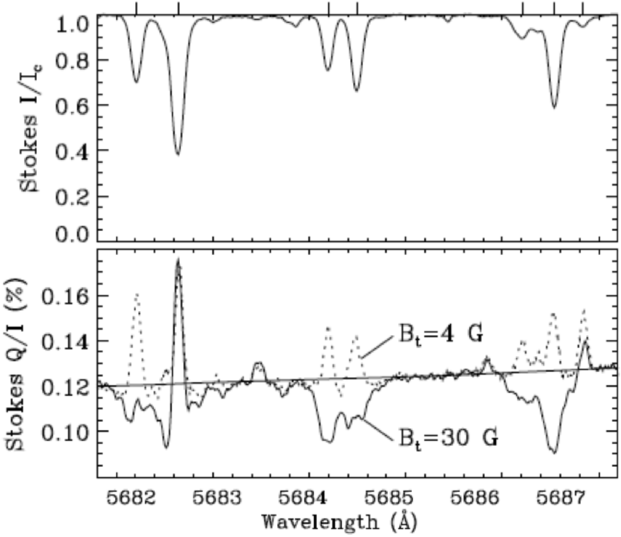
Ubiquitous, Turbulent Magnetic Field

- scattering polarization observations in spectral lines indicate ubiquitous magnetic field
- turbulent field is *hidden* from Zeeman effect observations
- relation to 11-year sunspot cycle is unknown
- realistic radiative MHD simulations show field generation in surface layers
- local surface dynamo or sunspot leftovers?



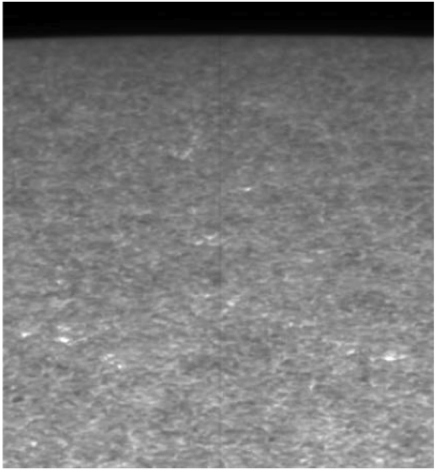
Courtesy A. Voegler, Utrecht University

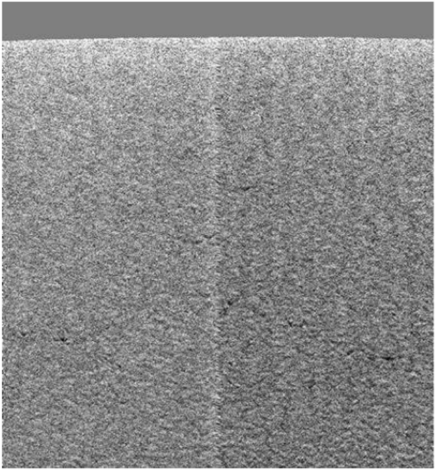
Spatial Variation of Turbulent Field



Stenflo, Keller & Gandorfer 1998

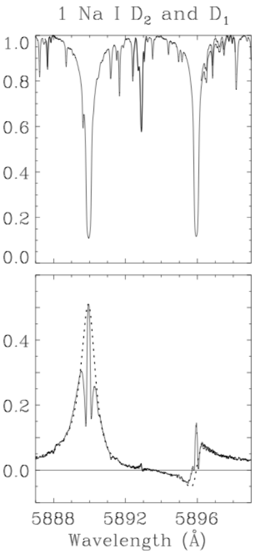
CN Scattering Polarization with Hinode





Courtesy Frans Snik, Utrecht University

NaI D_1 589.594 nm Scattering Polarization



Stenflo and Keller (1996)

- overall polarization pattern due to quantum interference between NaI D_1 and NaI D_2
- NaI D_1: J=1/2 to J=1/2 to J=1/2 has $w_2 = 0$
- stable isotope of sodium has a nuclear spin I=3/2
- hyperfine levels F=1 and F=2 can be polarized
- $w_2 = 0$ even with HFS (if lower level is not polarized)

Polarizers and Retarders

Polarizers

- polarizer: optical element that produces polarized light from unpolarizer input light
- linear, circular, or in general elliptical polarizer, depending on type of transmitted polarization
- linear polarizers by far the most common
- large variety of polarizers

Vertical Linear Polarizer

$$M_{\mathrm{pol}}(\theta) = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Horizontal Linear Polarizer

$$M_{\mathrm{pol}}(\theta) = \frac{1}{2}\begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Mueller Matrix for Ideal Linear Polarizer at Angle θ

$$M_{\mathrm{pol}}(\theta) = \frac{1}{2}\begin{pmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\theta & \cos^2 2\theta & \sin 2\theta \cos 2\theta & 0 \\ \sin 2\theta & \sin 2\theta \cos 2\theta & \sin^2 2\theta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Fixed Retarders

- retarder: splits beam into 2 components, retards phase of one component, combines components at exit into single beam
- ideal retarder does not change intensity of light or degree of polarization
- any retarder is characterized by two (not identical, not trivial) Stokes vectors of incoming light that are not changed by retarder $\Rightarrow$ *eigenvectors* of retarder
- depending on polarization described by eigenvectors, retarder is
  - *linear retarder*
  - *circular retarder*
  - *elliptical retarder*
- linear, circular retarders are special cases of elliptical retarders
- circular retarders sometimes called *rotators*
- linear retarders by far the most common type of retarder

Mueller Matrices for Linear Retarders

- corresponding Mueller matrix is given by

$$M_r = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\delta & -\sin\delta \\ 0 & 0 & \sin\delta & \cos\delta \end{pmatrix}$$

- linear retarder, fast axis angle θ, retardance δ

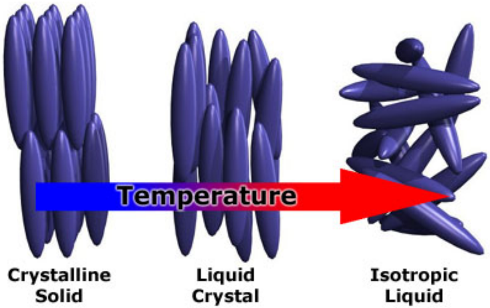
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\theta + \cos\delta \sin^2 2\theta & \cos 2\theta \sin 2\theta - \cos 2\theta \cos\delta \sin 2\theta & \sin 2\theta \sin\delta \\ 0 & \cos 2\theta \sin 2\theta - \cos 2\theta \cos\delta \sin 2\theta & \cos\delta \cos^2 2\theta + \sin^2 2\theta & -\cos 2\theta \sin\delta \\ 0 & -\sin 2\theta \sin\delta & \cos 2\theta \sin\delta & \cos\delta \end{pmatrix}$$

- 2 or more linear retarders in series $\Rightarrow$ (in general) equivalent to 1 elliptical retarder

Variable Retarders

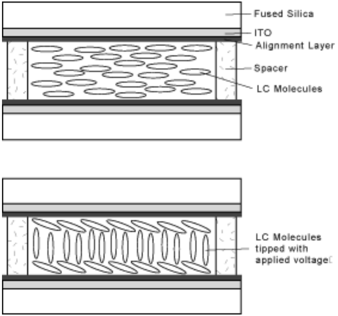
- sensitive polarimeters requires retarders whose properties (retardance, fast axis orientation) can be varied quickly (*modulated*)
- retardance changes (change of birefringence):
  - liquid crystals
  - Faraday, Kerr, Pockels cells
  - piezo-elastic modulators (PEM)
- fast axis orientation changes (change of *c*-axis direction):
  - rotating fixed retarder
  - ferro-electric liquid crystals (FLC)

Liquid Crystals



- liquid crystals are fluids whose molecules are elongated
- at high temperatures, liquid crystal is isotropic
- at lower temperature, molecules become ordered in orientation and sometimes also space in one or more dimensions
- liquid crystals can line up parallel or perpendicular to external electrical field

Liquid Crystal Retarders



- dielectric constant anisotropy often large $\Rightarrow$ very responsive to changes in applied electric field
- birefringence δn can be very large (larger than typical crystal birefringence)
- liquid crystal layer only a few μm thick $\Rightarrow$ true zero-order retarder
- birefringence strongly temperature dependent

Ferro-Electric Liquid Crystal Retarders

- respond much more quickly to externally applied fields than nematic liquid crystals
- can be used to make fast, bistable electro-optic devices
- FLCs act like retarders with fixed retardation where fast axis direction can be switched by about 45° (switching angle) by alternating sign of applied electrical field
- achromatic modulators with FLCs in Pancharatnam configuration
- switching times on the order of 150 μs
- switching angle is temperature sensitive
- retardance rather insensitive to temperature variations

Liquid Crystal Advantages and Disadvantages

Advantages:

- arbitrary (optimized) modulation schemes
- large, uniform apertures available
- retardation or fast axis changes possible
- FLC allow fast modulation (<10 kHz)
- require only low voltages at moderate driving powers ($\sim$ 1 W)

Disadvantages:

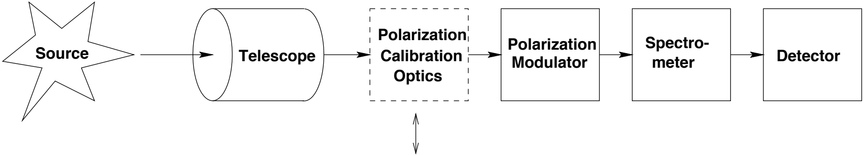
- degrades quickly under UV irradiation
- requires temperature control
- nematic have slow modulation frequency (<50 Hz)
- FLCs cannot change retardation

Comparison of Variable Retarders

Modulator	Advantages	Disadvantages
rotating retarder	high stability	relatively slow modulation
	large wavelength range	beam motion
		needs 8 measurements for all Stokes parameters
liquid crystal	relatively fast modulation	narrow simultaneous wavelength range
	only 4 measurements for all Stokes parameters	limited temporal stability
	no moving parts	damaged by strong UV light

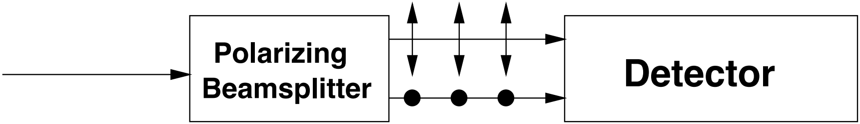
Polarimeters

General Polarimeters



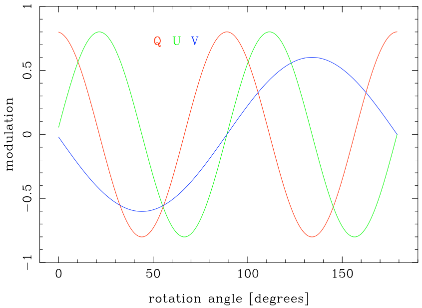
- polarimeters: optical elements (e.g. retarders, polarizers) that change polarization state of incoming light in controlled way
- detectors always measure only intensities
- intensity measurements combined to retrieve polarization state of incoming light
- polarimeters vary by polarization modulation scheme
- polarimeter should also include polarization calibration optics

Polarizing Beam-Splitter Polarimeter



- simple linear polarimeter: polarizing beam-splitter producing 2 beams corresponding to 2 orthogonal linear polarization states
- full linear polarization information from rotating assembly
- *spatial modulation*: simultaneous measurements of two (or more) Stokes parameters

Rotating Waveplate Polarimeters



$$I' = \frac{1}{2}\left(I + \frac{Q}{2}\left((1+\cos\delta) + (1-\cos\delta)\cos 4\theta\right) + \frac{U}{2}(1-\cos\delta)\sin 4\theta - V\sin\delta\sin 2\theta\right)$$

- Q, U modulated at twice the frequency of V
- phase shift in modulation between Q and U is 90°⇒ measurements at 8 angles to determine all 4 Stokes parameters
- most frequently used *temporal modulation scheme*

Comparison of Temporal and Spatial Modulation Schemes

Modulation	Advantages	Disadvantages
temporal	negligible effects of flat field and optical aberrations	influence of seeing if modulation is slow
	potentially high polarimetric sensitivity	limited read-out rate of array detectors
spatial	off-the-shelf array detectors	requires up to four times larger sensor
	high photon collection efficiency	influence of flat field
	allows post-facto reconstruction	influence of differential aberrations

schemes rather complementary ⇒ modern, sensitive polarimeters use both to combine advantages and minimize disadvantages

Double-Ratio Technique

- combination of spatial and temporal modulation
- data reduction minimizes effects of many artifacts
- rotatable quarter-wave plate, polarizing beam-splitter
- consider case of circularly polarized light
- quarter-wave plate switches between +45° or -45° to polarizing beam-splitter
- both beams recorded simultaneously
- four measurements are combined to obtain estimate of Stokes V/I ratio largely free of effects from seeing and gain variations between different detector areas
- excellent if polarization signal is small
- frequently used in stellar polarimetry
- can be applied to any polarized Stokes parameter
- works very well for solar applications where the spectrum in the first and the second exposures are different

Double-Ratio Technique (continued)

- measured intensities in two beams in first exposure

$$S_1^l = g_l \alpha_1 (I_1 + V_1), \quad S_1^r = g_r \alpha_1 (I_1 - V_1)$$

  - subscript 1 indicates first exposure
  - subscripts *l*, *r* indicate left and right beams of polarizing beam-splitter
  - *S*: measured signal
  - *g*: gain in particular beam
  - α: average transmission of atmosphere and instrument for a given exposure
- second exposure

$$S_2^l = g_l \alpha_2 (I_2 - V_2), \quad S_2^r = g_r \alpha_2 (I_2 + V_2)$$

- second exposure may be different from first exposure
- also includes beam-wobble induced by rotation of wave plate

Double-Ratio Technique (continued)

- combination of 4 measured intensities removes effect of transmission changes and differential gain variations of different detector areas

$$\frac{1}{4}\left(\frac{S_1^l}{S_2^l}\frac{S_2^r}{S_1^r}-1\right)=\frac{1}{2}\frac{I_2 V_1 + I_1 V_2}{I_1 I_2 - I_2 V_1 - I_1 V_2 + V_1 V_2}$$

- if $V \ll I$

$$\frac{1}{2}\left(\frac{V_1}{I_1}+\frac{V_2}{I_2}\right)$$

- obtain average V/I signal of two exposures
- no spurious polarization signals are introduced

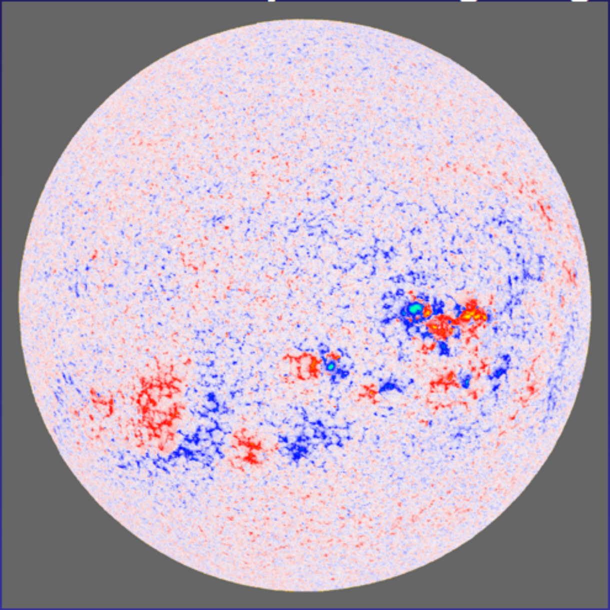
Liquid Crystal Polarimeters

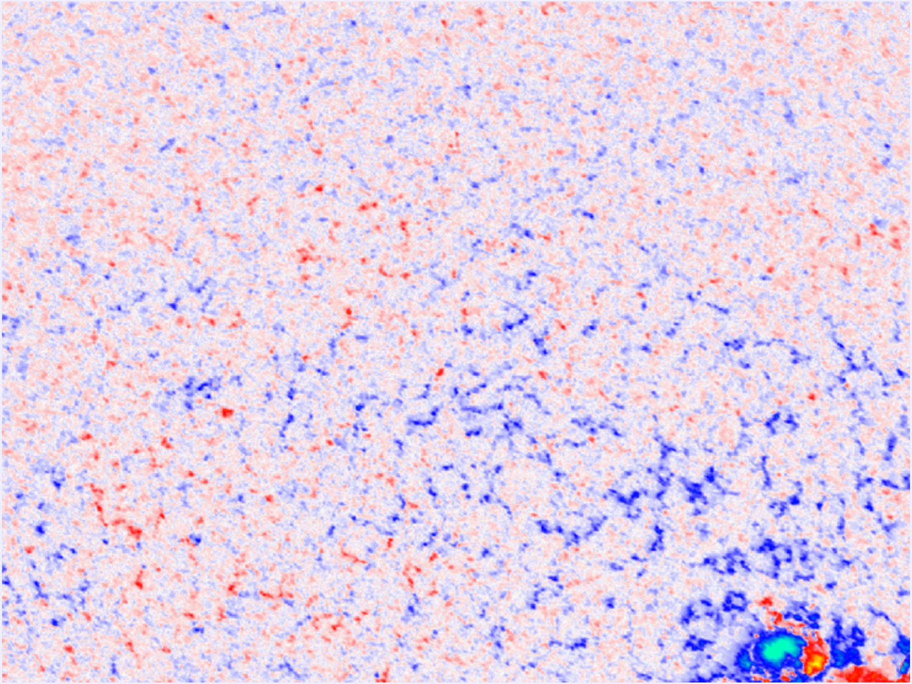
- many systems in operation since 1983
- variety of liquid crystyal types and arrangements
- often combinations of variable liquid crystal retarders and fixed retarders
- requires only 4 measurements for complete Stokes vector
- modulation scheme can be optimized for maximum efficiency
- example 1: SOLIS Vector-SpectroMagnetograph (VSM)
- example 2: Small Synoptic Second Solar Spectrum Telescope (S^5T)

SOLIS Vector-Spectromagnetograph (VSM)

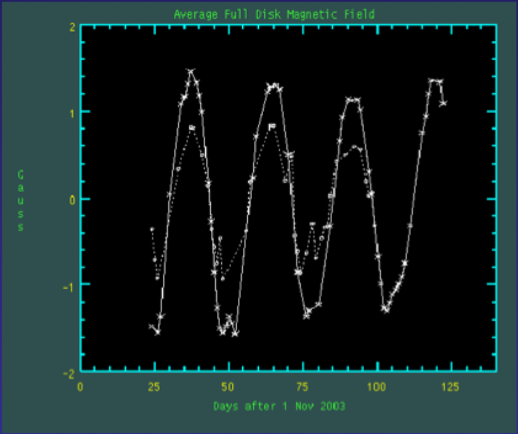
- SOLIS = Synoptic Optical Long-term Investigations of the Sun
- 3 instruments: Vector SpectroMagnetograph, Full-Disk Patrol, and Integrated Sunlight Spectrometersun-as-a-star spectrometer) attached to single equatorial mount
- located on top of old Kitt Peak Vacuum Telescope
- http://solis.nso.edu
- VSM operates in four different observing modes at three different wavelengths:
  1. photospheric full-disk longitudinal magnetograms in FeI 630.15 and 630.25 nm
  2. photospheric full-disk vector-magnetograms in FeI 630.15 and FeI 630.25 nm
  3. chromospheric full-disk magnetograms in CaII 854.2 nm
  4. full-disk HeI 1083.0 nm line characteristics

Full-Disk Photospheric Magnetogram

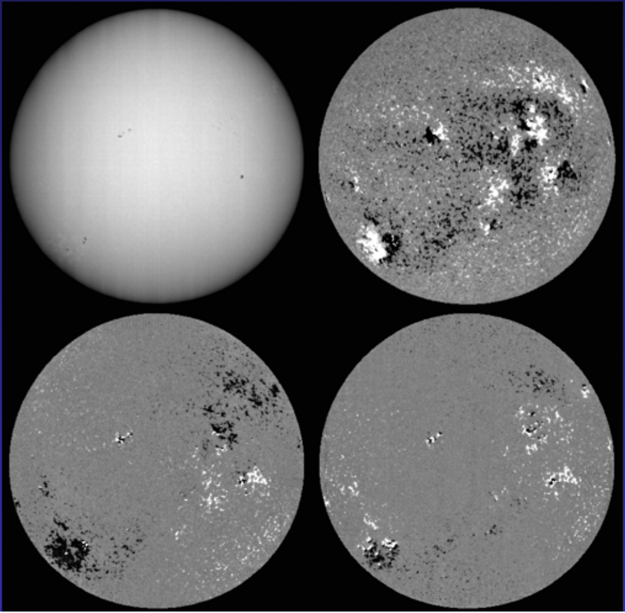




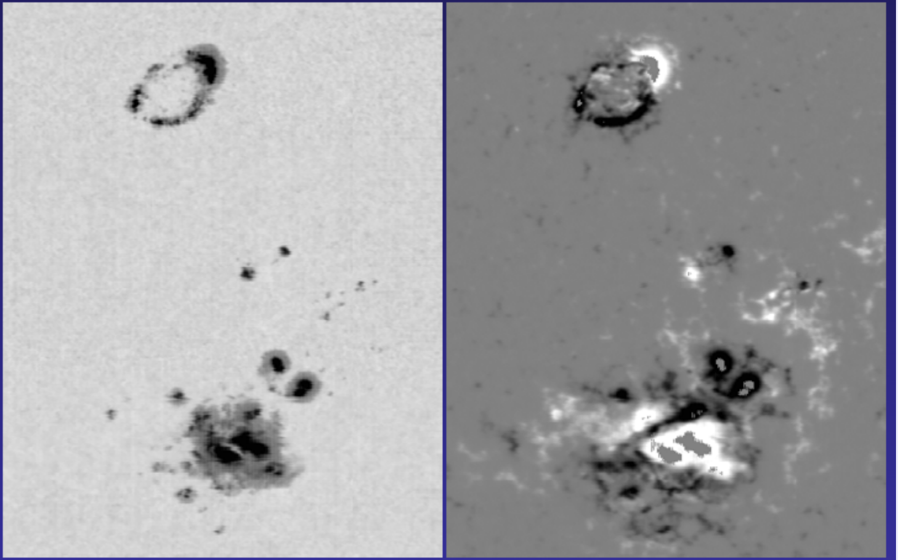
Sun-as-a-Star Magnetic Field



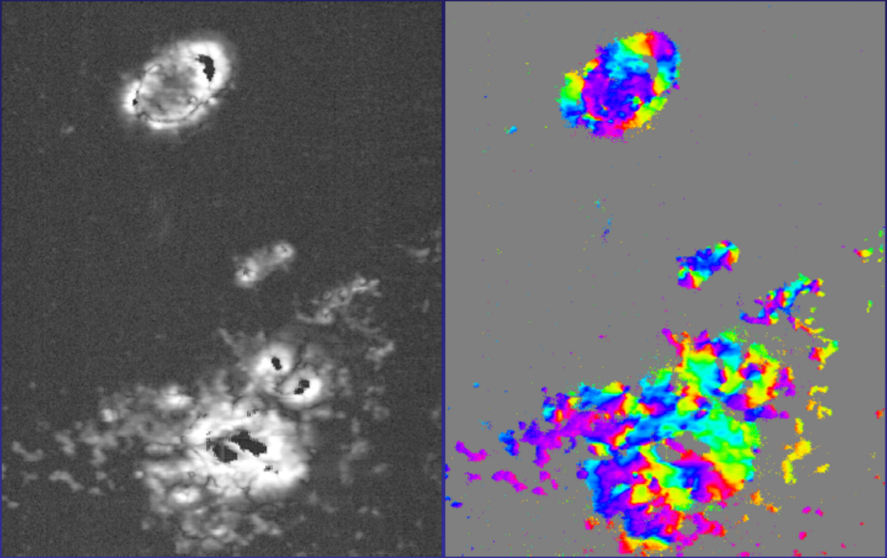
Full-Disk Vector-Polarimetry



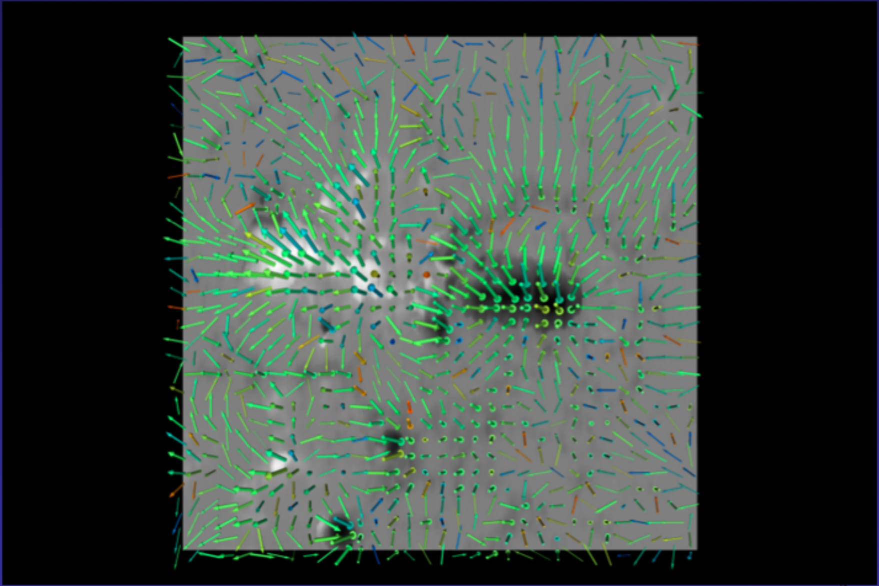
More Vector-Polarimetry



More Vector-Polarimetry



Field Vector, Filling Factor, and Helicity

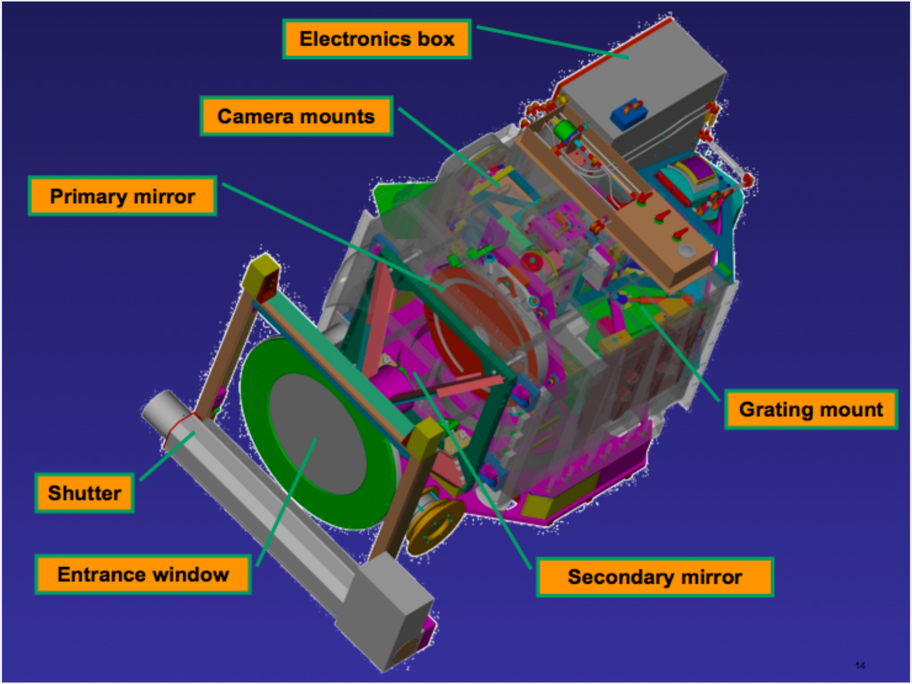


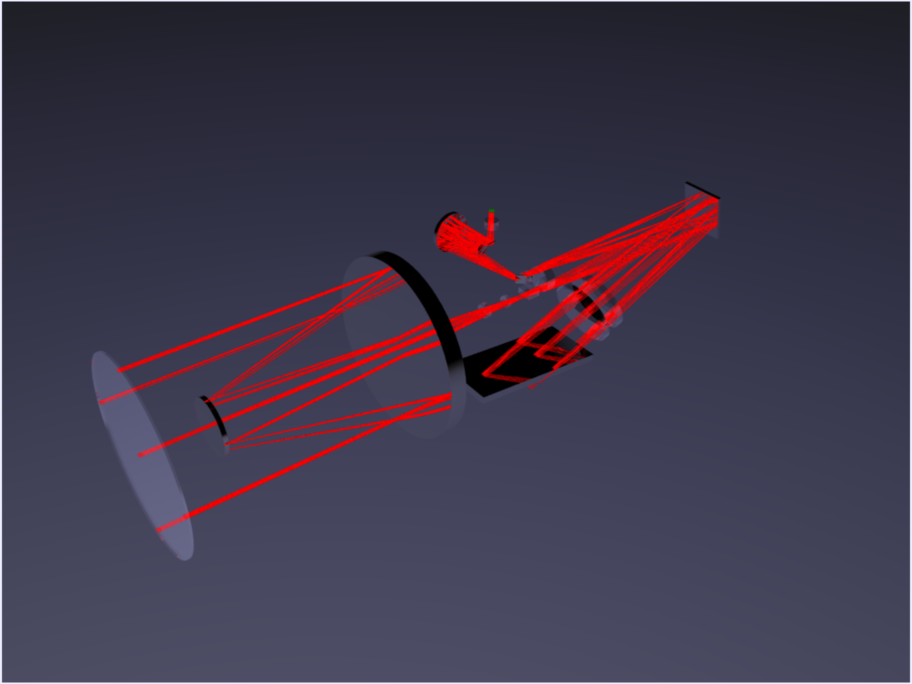
Specifications

Parameter	Specification
Effective pixel size	1 arcsec by 1 arcsec (1.125 by 1.125 arcsec initially)
Angular coverage	2048 arcsec by 2048 arcsec
Geometric accuracy	0.5 arcsec rms after data reduction
Scan rate	0.2 to 5.0 seconds/arcsec
Timing accuracy	Better than 1 second
Time stamping	Better than 1 ms
Spectral resolution	238,000 (at 630 nm)
Wavelengths	630 nm, 854 nm, 1083 nm
Polarimetry	• FeI 630.15 and FeI 630.25 nm: I,V,Q,U • CaII 854 nm: I,V • HeI 1083.0 nm: I
Polarimetric sensitivity	0.0002 at 0.5 seconds/arcsec scanning rate
Polarimetric accuracy	Better than 0.001

Technical Challenges

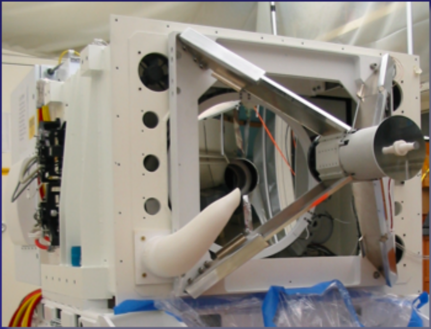
Challenge	Solution
Compact instrument no longer than 2.5 m	**Folded f/6.6 beam**
Good and stable spatial resolution	**Helium-filled, active M2**
High guiding accuracy of better than 0.5 arcsec rms	**Guider in slit plane, active secondary mirror**
Low instrumental polarization of less than $1 \cdot 10^{-3}$	**Axially symmetric design**
Fixed image size, low distortion from 630 to 1090 nm	**Quasi RC with correctors**
Stable high spectral resolution of 200,000	**Large, active grating**
Highest possible throughput	**Silver, multilayer coatings, CMOS hybrid cameras**
Energy densities of up to 0.2 MW/m^2	**Copper-silicon carbide plate**
High data rate of up to 320 Mbyte/s	**DSP array, Storage Area Network**

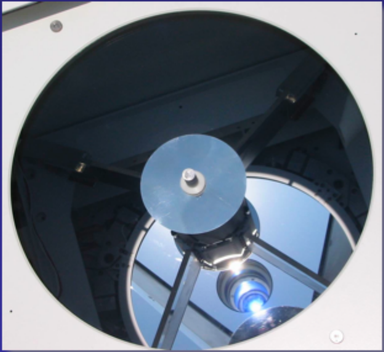

Electronics box
Camera mounts
Primary mirror
Grating mount
Shutter
Entrance window
Secondary mirror
14



Telescope

- Helium-filled f/6.6 Ritchey-Chrétien with field corrector lenses
- Entrance window provides environmental protection
  - 6-mm thick oversized, fused silica to minimize edge effects
  - 'Floats' in RTV to minimize stress birefringence





- 575-mm f/1.4 ULE primary mirror
- Single crystal silicon secondary
  - 40 Hz tip/tilt closed-loop bandwidth piezo platform
  - Slow closed-loop focus control
  - Cooled by helium flow

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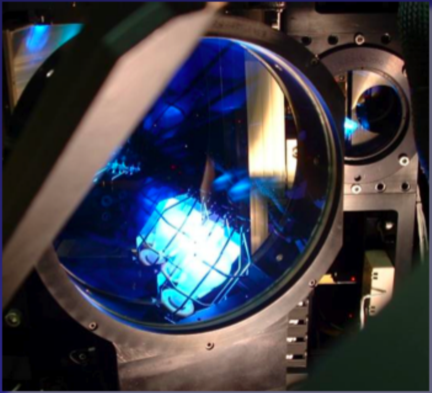
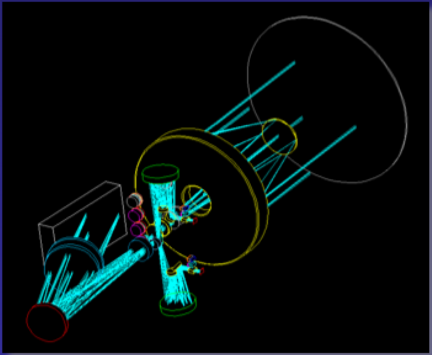
Folded Littrow Spectrograph



Littrow lens
- Air-spaced doublet
- Athermal design
- Moves to adjust for different wavelengths
- Dual Offner reimaging optics

Grating
- 79 lines/mm on 204 mm by 408 mm fused silica blank
- Almost no instrumental polarization
- Rotates for different wavelengths
- Active adjustment in 2 axes to compensate for flexure



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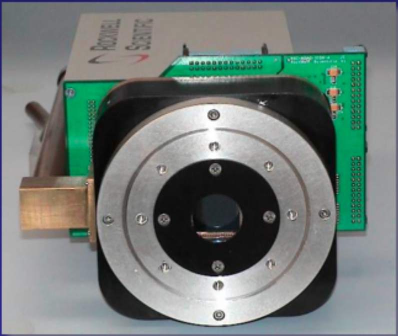
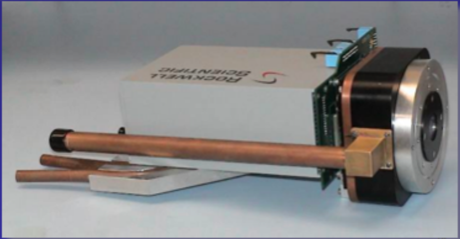
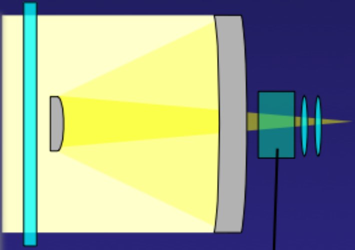
CMOS Hybrid Cameras



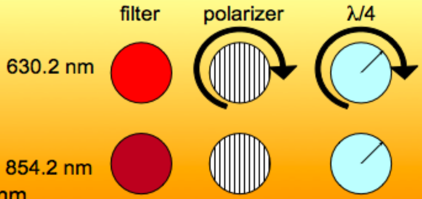
- Interim replacement for cancelled PixelVision & SITe CCD cameras
- Made by Rockwell Scientific
- 1024 by 1024 18 μm pixels
- 92 frames/s at 1024 by 256
- > 2,000,000 e- full well depth
- Silicon on CMOS multiplexer
- Quantum efficiency 85% at 630 and 854 nm, 5% at 1083 nm



Polarization Calibration



- ‘Polarization-free’ optics before polarization calibration
- Polarization calibration occurs as early as possible
- interference filters to limit solar flux
- rotating polarizers and retarders at 630 nm, fixed at 854 nm



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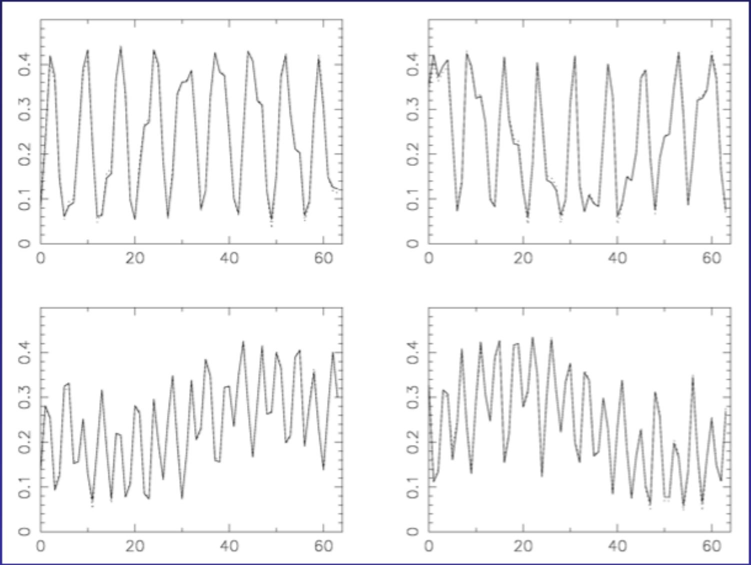
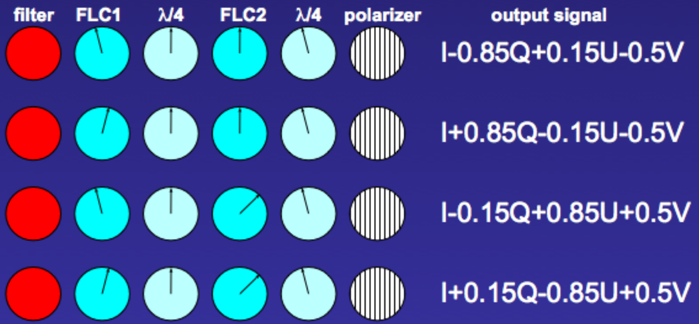
Vector-Calibration

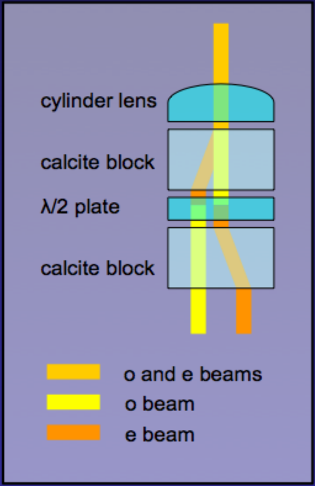


Polarization Modulation

- Ferroelectric liquid crystal (FLC) variable retarders (all λ/2 at 630 nm)
- Fixed λ/4 (at 630 nm) and λ/6 (at 854 nm) polymer retarders
- All true zero-order retarders to cope with fast f/6.6 beam
- Full vector modulation similar to Gandorfer and Rabin schemes
- Exact position angles optimized based on measured FLC properties
- After modulation, both polarization states pass the same low-polarization optics
- Solar-B spectropolarimeter and Diffraction-Limited Spectro-Polarimeter (DLSP) at Dunn Solar Telescope are based on VSM concept



Polarization Analysis



- Modified Savart plate
- Crystal astigmatism is a major issue for an f/6.6 beam, corrected by cylinder lens
- Provides high quality polarizing beam-splitting for fast beam and large field of view
- Different beamsplitters for 630.2 nm and 854.2 nm
  - Calcite splitting is wavelength dependent
  - Can use simple mica retarder

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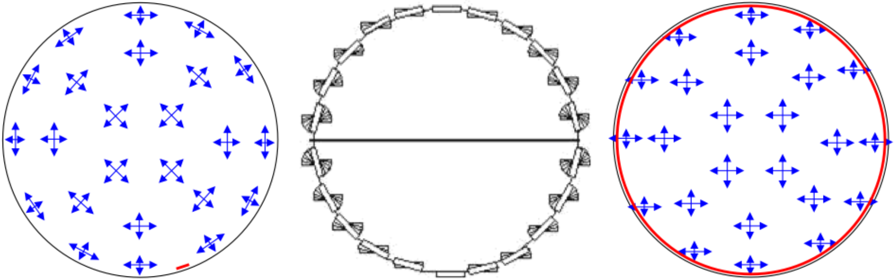
Small Synoptic Second Solar Spectrum Telescope (S^5T)

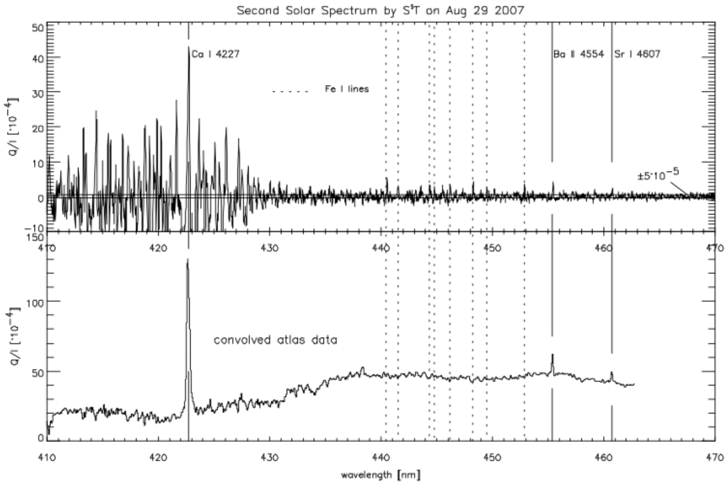
Theta Cell

- liquid-crystal device transforms radially oriented linear polarization into uniform orientation
- scattering polarization measurements required large aperture, measured small areas
- theta cell ⇒ small telescope, large area



Courtesy Frans Snik, Utrecht University

S5T Prototype First Light Results (Snik et al. 2008)



Stokes Inversion (based on lecture by C.E.Fischer)

Intensity Radiative Transfer in LTE

radiative transfer equation for intensity

$$\cos\theta \frac{dI_\nu}{d\tau_c} = (1+\eta_\nu)(I_\nu - B_\nu)$$

with τ_c the continuum optical depth, η the ratio of spectral line absorption coefficient to continuum absorption coefficient.

Polarized Radiative Transfer in LTE

radiative transfer equation for Stokes vector

$$\cos\theta \frac{d\vec{I}}{d\tau_c} = (1+\eta)\left(\vec{I} - \vec{B}_\nu\right)$$

with $\vec{B}_\nu^T = (B_\nu, 0, 0, 0)$ and absorption matrix η.

Absorption Matrix for Zeeman Effect

$$\eta = \begin{pmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_v & \eta_I & \rho_Q \\ \eta V & \rho_U & -\rho_Q & \eta_I \end{pmatrix}$$

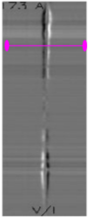
with

$$\begin{aligned} \eta_I &= \frac{1}{2}\eta \sin^2\gamma + \frac{1}{4}\left(\eta^+ + \eta^-\right)\left(1 + \cos^2\gamma\right) \\ \eta_Q &= \left(\frac{1}{2}\eta - \frac{1}{4}\left(\eta^+ + \eta^-\right)\right)\sin^2\gamma \cos 2\phi \\ \eta_U &= \left(\frac{1}{2}\eta - \frac{1}{4}\left(\eta^+ + \eta^-\right)\right)\sin^2\gamma \sin 2\phi \\ \eta_V &= \frac{1}{2}\left(\eta^+ - \eta^-\right)\cos\gamma \end{aligned}$$

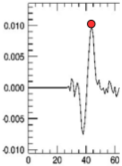
with magnetic field inclination γ and azimuth ϕ

Simple B field reconstruction

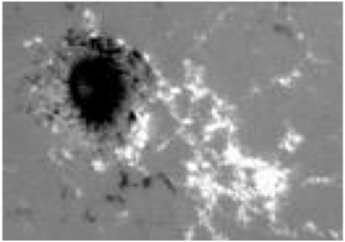
Weak field V ~ longitudinal B
Magnetogram in V
Only approximation



Themis



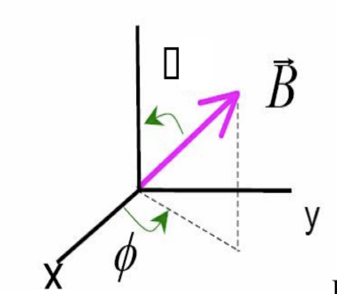
For every pixel



Taking maxima or mask and integration

Hinode webpage

Full magnetic field vector

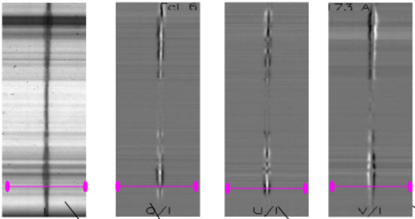


sanchez

Need to do full analysis in order to get
inclination angle
azimuth angle
and field strength

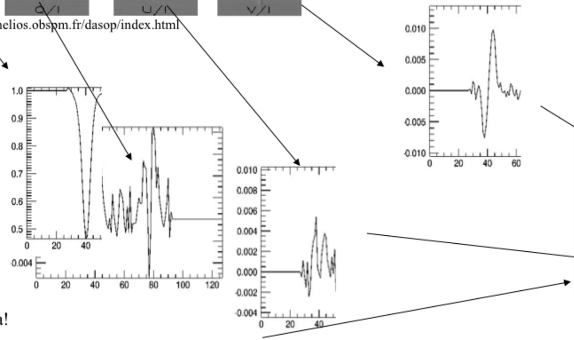
Problems:
For azimuth 180 degree ambiguity
Magnetic elements not resolved

Themis!

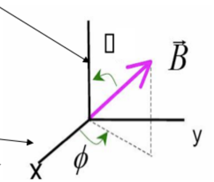


$$\vec{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \text{intensity} \\ \text{linear } 0^\circ - \text{linear } 90^\circ \\ \text{linear } 45^\circ - \text{linear } 135^\circ \\ \text{circular left} - \text{right} \end{pmatrix}$$

http://helios.obspm.fr/dasop/index.html



SOLIS data!



but 180 degree ambiguity!

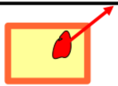
Stokes Inversion

Any method used to infer the physical conditions of the atmosphere from the interpretation of Stokes profiles

Tools:

- model of the atmosphere
- Radiative transfer
- Directional dependance of Stokes profiles

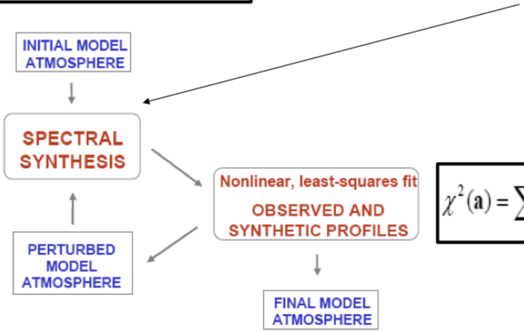
Least square inversion



Filling factor,

I=(1-f)I(mag) + f I(no mag)

$$\mathbf{a} \equiv (B, \gamma, \square, v_{LOS}, T, P_e, v_{mic})$$



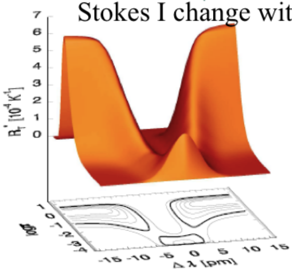
$$\chi^2(\mathbf{a}) = \sum \left[I_{obs}(\lambda_i) - I_{syn}(\lambda_i, \mathbf{a}) \right]^2$$

The inversion process minimizing Chi squared

$$\chi^2(\mathbf{a}) = \sum \left[I_{obs}(\lambda_i) - I_{syn}(\lambda_i, \mathbf{a}) \right]^2$$

Need to minimize Chi squared!
Use Levenberg-Marquardt algorithm.

Stokes I change with T



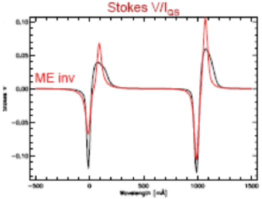
Derivative can be expressed in terms of response functions.
Response functions tell us about how the observed spectrum responds to modifications in the physical parameters of the model.

Milne eddington inversion

Assumptions : Source function linear with optical depth
Absorption matrix does not vary with optical depth

Flat atmosphere
Solve RTE analyticaly

Fast and SIMPLE treatment of the RTE
BUT
Can not account for asymmetric Stokes profiles



Inversion height dependence

Gradients of the physical parameters along LOS can cause asymmetric profiles

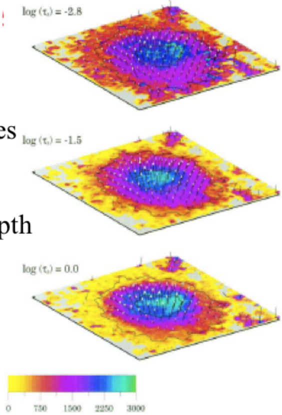
Solve RTE numericaly!

Inversion codes capable of dealing with asymmetries

-are based on numerical solution of RTE

-Provide reliable thermal information

-Infer stratifications of physical parameters with depth



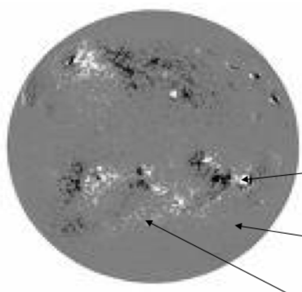
Examples:

SIR Cobo, del Toro Iniesta

LILIA H.Navarro

Westendorp Plaza et al. 2001, Astrophysical Journal 547, 1130

Problems



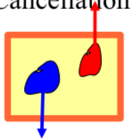
Sensitivity

$\frac{V}{I}$ Sunspot about 30%

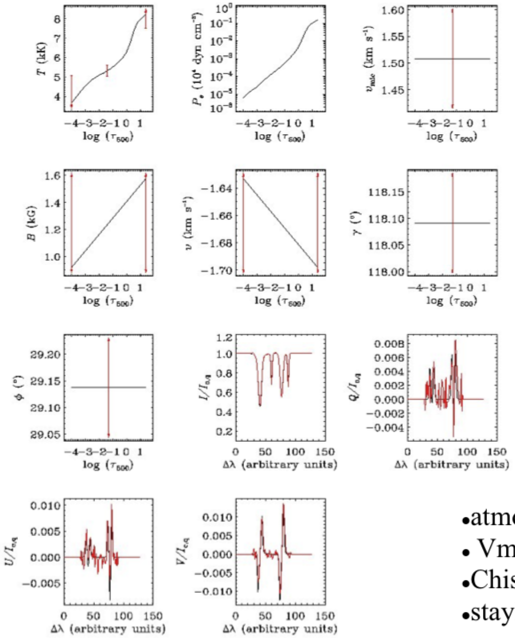
Network 1%

Plage 10 %

Cancellation



What happens if magnetic elements with opposite polarity in one resolution element?



- atmospheric model from best fit
- Vmac
- Chisq.
- staylight/filling factor

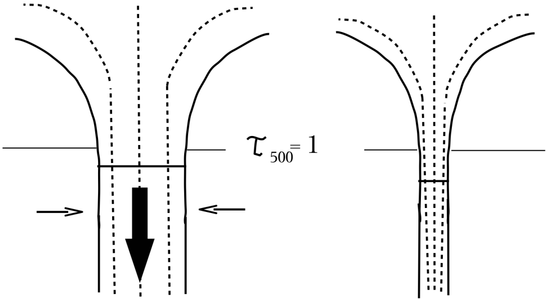
Weak Field Approximation

- magnetic splitting $\ll$ Doppler width of spectral line
- filling factor α: fraction of resolution element covered with magnetic field
- Stokes $V \sim g_l \alpha B \cos\gamma$
- Stokes $Q \sim g_l^2 \sin^2\gamma \alpha B^2 \cos 2\phi$
- Stokes $U \sim g_l^2 \sin^2\gamma \alpha B^2 \sin 2\phi$
- Warning: difficult to disentangle α and B
- Warning: difficult to determine γ
- Warning: 180 degree amiguity in azimuth

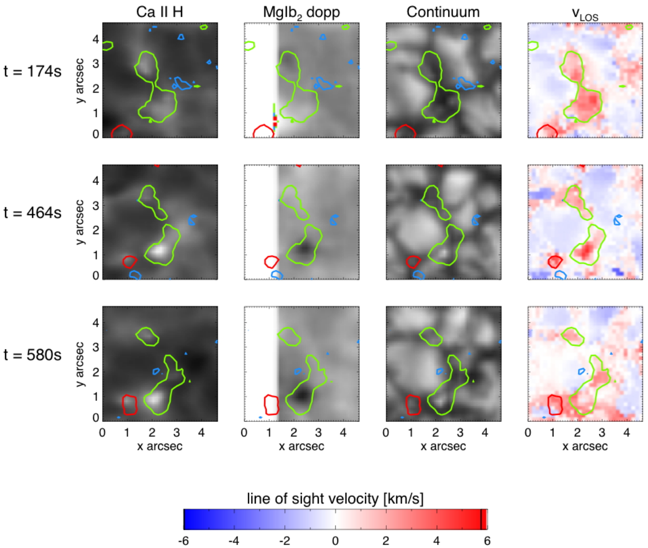
Field Strength Increase by Convective Collapse

Previous Efforts

- magnetic fields outside of sunspots much stronger than equipartition (Stenflo 1973)
- convective collapse suggested by Parker (1978)
- indirect evidence and single direct observation (Nagata et al. 2008)



Example of Convective Collapse with Hinode

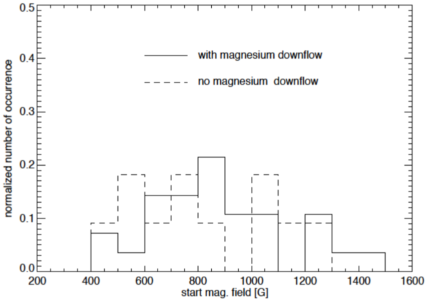


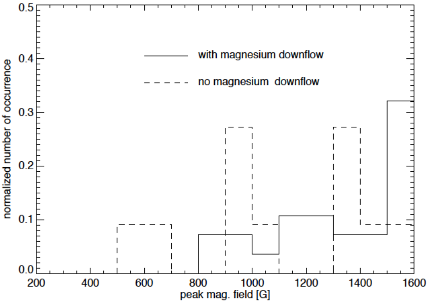
Courtesy Catherine Fischer, Utrecht University

Convective Collapse Statistics (Fischer et al. 2009)

- 49 convective collapse examples
- field strength increase coupled with downflow
- bright points develop in low photosphere and in chromosphere
- often see downflow also in high photosphere (MgI b_2)

Observed Magnetic Field Strength Increase

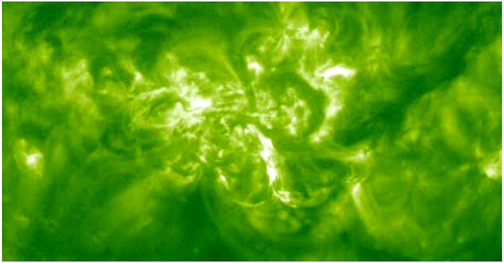




Vector Magnetic Field during an X-Class Flare

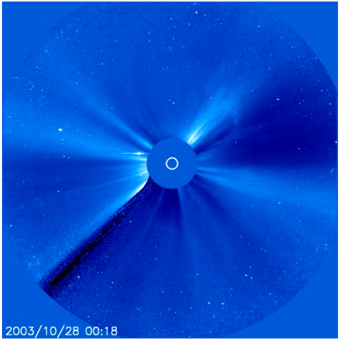
- flare: sudden release of magnetic energy in solar atmosphere
- flare, coronal mass ejection create *Space Weather*
- understand magnetic field structure and instability ⇒ forecasting
- inversion of polarized line profiles ⇒ magnetic field vector

Flares



sohowww.nascom.nasa.gov/hotshots/2003_10_28/

Coronal Mass Ejection



sohowww.nascom.nasa.gov/hotshots/2003_10_28/

X-Class Flare Observations from 13 September 2005

18 scans, about 5 min cadence, full LTE inversion of 2 Fe lines