Applying Genetic Algorithms

In this exercise we will discuss a number of optimisation problems for which we can obtain solutions using Genetic Algorithms. The aim of this exercise is to practise translating these problems to a format compatible with Genetic Algorithms as described in chapter 5 of [1]: we will rewrite them to

$$\max_{x \in D} f(x).$$

Problems

Consider the following problems.

1. Given a number $k \in \mathbb{N}$, find the maximum number in $\{1, 2, \ldots, k\}$.
2. Given a smooth function $g : [a, b] \rightarrow \mathbb{R}$ with $a, b \in \mathbb{R}$, find a global maximum of $g$.
3. Given a smooth function $h : [0, 1]^n \rightarrow \mathbb{R}$ with $n \in \mathbb{N}$ a given dimension, find a global minimum of $h$.
4. Given a matrix $A \in \mathbb{R}^{m \times n}$ and a vector $b \in \text{im}(A) \subseteq \mathbb{R}^m$ in the image of $A$, find a solution $x \in \mathbb{R}^n$ to the equation $Ax = b$.
5. We have $k \in \mathbb{N}$ objects labeled by $1, \ldots, k$ with positive weights $w_1, \ldots, w_k \in [0, \infty]$. All these objects will be divided among two persons and carried from one location to another. Find out which person should take what objects for the carrying to be as fair as possible (in the sense that both persons should carry the same amount of weight as much as possible).
6. The travelling salesman problem (section 5.4.1 of [1]).
7. Charges on a sphere (section 5.4.2 of [1]) for $s = 1$ (Coloumb).
8. (Bonus) Evolving the Mona Lisa, [2].

Assignment

Please answer the following questions for each problem $P$ listed above.

(a) What would be a set $D$ describing the domain (consisting of all potential solutions) of the problem $P$?

(b) Please give a function $f$ from the domain $D$ to $\mathbb{R}$ such that a solution $x$ to equation (1) is an optimal solution to $P$.

(c) How would you represent elements from the domain $D$ as bit strings? How many bits would you need? Can you represent all points in the domain? Is this a problem?

(d) How would you locally improve (section 5.3.3) the chromosomes of your population to improve your potential solutions?

(e) Can you think of a way to reduce the domain $D$ (while retaining the full problem) by using symmetries of the problem?

References


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1Try to rewrite this as a minimisation problem: $Ax = b \iff Ax - b = 0$.
2You do not have to worry about translating between bit strings and points in $D$ for this exercise.