

Exercises, day 1**Exercise 1.1.** Let

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}.$$

- (a) What is the Range of A ?
- (b) What is the Null space of A ?
- (c) What is the rank of A ?
- (d) What are the eigenvalues of A ?

Exercise 1.2. Let Q be a square orthonormal matrix (i.e., the columns of Q are mutually orthogonal and normalized).

- (a) Prove that Q preserves the 2-norm of vectors.
- (b) Prove that the 2-norm condition number of Q equals one.
- (c) Is this also the case for the p -norm condition number for values of $p \in [1, \infty], p \neq 2$?
- (d) Also prove that orthogonal transformations preserve the 2-norm of matrices: $\|QA\|_2 = \|A\|_2$ and $\|BQ\|_2 = \|B\|_2$ with A and B matrices of matching dimension.

Exercise 1.3. Show that for p -norms and matrices A and B of matching dimension

$$\|AB\|_p \leq \|A\|_p \|B\|_p.$$

Exercise 1.4. Prove that

- (a) the Frobenius norm a norm is,
- (b) $\|A\|_F = \|A^T\|_F$ for all matrices A ,
- (c) $\|AB\|_F \leq \|A\|_2 \|B\|_F$ for all matrices A, B of matching dimensions,
- (d) $\|A\|_2 \leq \|A\|_F$ for all matrices A .

Exercise 1.5. Prove that induced norms are matrix-norms.**Exercise 1.6.** Show that $fl(AB) = AB + E$ with $|E| \leq nu|A||B| + O(u^2)$