The data-parallel iterative solution of the finite element discretization of stream-function models for global ocean circulation

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Introduction

Islands and Continents lead to "global" conditions in ocean circulation simulation

This destroys structure in the problem

$\rightarrow$ indirect addressing
$\rightarrow$ hampers vectorization/parallelization

Goal of the research:

- Develop discretization+solution method
- that attains the structure of the problem
- so that indirect addressing is minimized
- and data-parallelization is facilitated

Stream function-vorticity model

$$-r\nabla^2\psi - \beta\frac{\partial\psi}{\partial x} - A\nabla^2\zeta = \nabla\times\mathbf{F},$$
$$-\nabla^2\psi + \zeta = 0$$

- ψ: stream function,
- ζ: vorticity,
- r: bottom friction,
- β: Coriolis parameter
- A: viscosity
- $\mathbf{F} = \frac{\tau}{\rho H}$, force field, with
  - τ: Wind force,
  - ρ: Water density,
  - H: Average water depth.

Stream function-vorticity model

Velocities can be computed from ψ:

$$u = \frac{\partial\psi}{\partial y},\ v = -\frac{\partial\psi}{\partial x},\ \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}.$$

Boundary conditions: No slip.

$$\psi = C_k,\ \ \frac{\partial\psi}{\partial n} = 0 \quad \text{on coast}$$

Water level cont. around an island:

$$\oint_{\Gamma_k} A\frac{\partial\zeta}{\partial n}\,ds = -\oint_{\Gamma_k}\mathbf{F}\cdot\mathbf{s}\,ds.$$

Condition needed to determine C_k's.

Slip condition on North pole: $\psi=\zeta=0$

Discretization

The PDE's are expressed in spherical coordinates $\longrightarrow$ rectangular domain

Step 1: Define a structured grid on the rectangular domain, disregarding continents.

Step 2: Discretize the PDE's everywhere, using the FEM, $\longrightarrow$ structured matrix

Step 3: Define Master and Slave nodes on the continents. Contributions to Slave nodes, e.g. of the wind forcing, must be lumped to values in the Master node.

Step 4: Solve the following system of equations:

$$\mathbf{P^T K P x} = \mathbf{P^T f}. \tag{1}$$

The system matrix $\mathbf{P^T K P}$

$\mathbf{P}$ is defined by

For "sea-points":

$$\mathbf{P_j} = \mathbf{e_j} \quad \forall\, \mathbf{j} \text{ in the sea.}$$

For "land-points":

$$\mathbf{P_k} = \Sigma\, \mathbf{e_j} \quad \forall\, \mathbf{j} \text{ on same island.}$$

$\mathbf{K}$ is highly nonsymmetric but has nice diagonal structure

Note that $\mathbf{P^T K P}$ is unstructured

BiCGstab(ℓ)

BiCGstab(ℓ) is Krylov subspace method, a combination of BiCG with GMRES.

BiCGstab(ℓ) is both fast and robust.

Method is composed of:

- Matvecs
- Preconditioning operations
- Inner products (BLAS)
- Vector updates (BLAS)

Optimized BLAS routines are available on most computers

Polynomial Preconditioner

Solve preconditioned system

$$P_k(\mathbf{P^T K P})\mathbf{P^T K P x} = P_k(\mathbf{P^T K P})\mathbf{P^T f}$$

with P_k pol. of degree k and $tP_k(t) \approx 1$

Polynomial can determined from spectral information of Bi-CG process

Evaluation of Pol.Prec. using Horner's rule

Only matvecs and vector updates required

The matvec $\mathbf{y} = \mathbf{P^T K P x}$

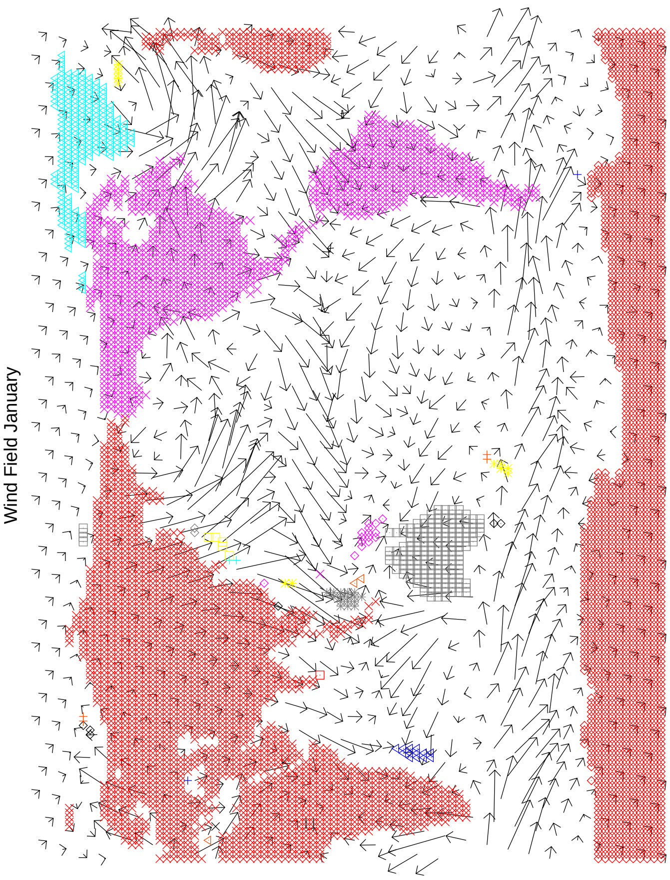
The matvec can be performed in three steps:

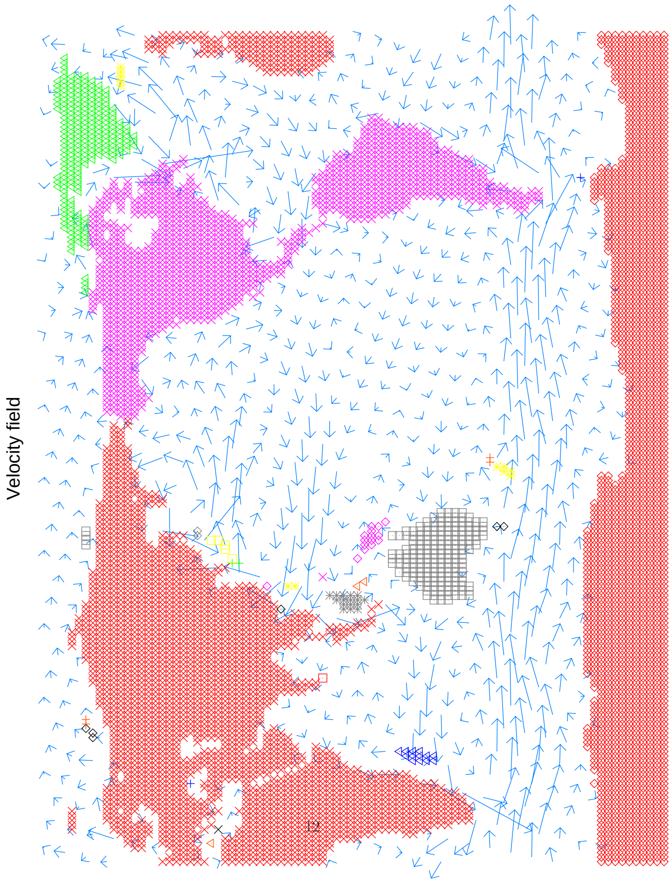
1. Give all Slave nodes the value of the "master" node: $\bar{\mathbf{x}} = \mathbf{P x}$ (Scatter)
2. Multiply: $\bar{\mathbf{y}} = \mathbf{K}\bar{\mathbf{x}}$
   (Structured matrix-vector product).
3. Add up the values of the stream-function on each continent: $\mathbf{y} = \mathbf{P^T}\bar{\mathbf{y}}$
   (Gather+Update)

Step 2 is highly data-parallel, but the gather and update operations are not

Example

- 1^o resolution
- Average windfield 1970-1980 January
- Ocean depth $500m$
- Water density $\rho = 1000kg/m^3$
- Earth radius $R = 6.4 \cdot 10^6$
- Coriolis param. $\beta = 2.3 \cdot 10^{-11} \cos\theta/ms$
- Lateral viscosity $A = 500m^2/s$
- $r = 5 \cdot 10^{-6}s^{-1}$





Numerical results

System size 130 000, Machine: Cray C90

Degree Pol.Prec.	# Iterations	# Matvecs
0	2254	18032
1	548	8768
3	248	7936
5	179	8592

Iter. and matvecs for BiCGstab(4) plus PP.

- Vector operation reduced by factor 10
- Time without Pol. Prec.: 1 018 s.
- Time with Pol. Prec. degree 3: 316 s.

Data-parallel results

Peak performance Cray C90 $\approx$ 1 Gflops

Our Performance $\approx$ 0.1 Glops

Why?

Performance structured matvec 400 Mflops

This is optimal (three loads, one store)

This operations takes 25 % of the time

But

Performance gather operation 1 Mflops!

This operations takes 60 % ot the time

Concluding remarks

- Discretization and solution procedure exploit structure in grid
- Combination of BiCGstab(ℓ) and Pol.Prec. works well
- Gather operation remains bottle neck for data parallel performance