

H. T. C. Stoof, K. B. Gubbels, and D. B. M.  
Dickerscheid

# Ultracold Quantum Fields

June 4, 2008

Springer



# Contents

<b>1</b>	<b>Introduction</b> .....	1
1.1	Ultracold atomic quantum gases .....	2
1.2	Outline .....	6
1.2.1	Part one .....	6
1.2.2	Part two .....	8
1.2.3	Part three .....	10
<b>2</b>	<b>Gaussian integrals</b> .....	15
2.1	The Gaussian integral over real variables .....	15
2.1.1	Generating function .....	16
2.1.2	Multi-dimensional Gaussian integral .....	18
2.2	Complex analysis .....	20
2.2.1	Differentiation and contour integrals .....	20
2.2.2	Laurent series and the residue theorem .....	22
2.3	Gaussian integrals over complex variables .....	25
2.4	Grassmann variables .....	26
2.5	Problems .....	29
<b>3</b>	<b>Quantum Mechanics</b> .....	33
3.1	Hilbert spaces .....	34
3.2	Observables .....	35
3.3	Schrödinger vs. Heisenberg picture .....	39
3.4	Bosonic harmonic oscillator .....	41
3.5	Creation and annihilation operators .....	42
3.6	Three-dimensional harmonic oscillator .....	43
3.7	Coherent states .....	45
3.8	Fermionic harmonic oscillator .....	47
3.9	Spin .....	49
3.10	Perturbation theory .....	52
3.11	Scattering theory .....	54
3.12	Many-particle quantum mechanics .....	57

3.13	Problems	57
<b>4</b>	<b>Statistical physics</b>	<b>59</b>
4.1	Legendre transformations	59
4.2	Statistical physics	61
4.2.1	Spin chain	61
4.2.2	Canonical ensemble	64
4.2.3	Grand-canonical ensemble	65
4.3	Ideal gases	67
4.3.1	Ideal Maxwell-Boltzmann gas	68
4.3.2	Ideal Bose gas: Bose-Einstein condensation	72
4.3.3	Ideal Fermi gas	77
4.4	Density matrix	81
4.5	Problems	83
<b>5</b>	<b>Path integrals</b>	<b>85</b>
5.1	Functionals and functional derivatives	85
5.2	Principle of least action	87
5.3	Phase-space representation	89
5.4	The Feynman path integral	90
5.4.1	Continuum limit and fluctuation expansion	93
5.4.2	Gel'fand-Yaglom method	95
5.5	Matrix elements and time ordering	99
5.6	Quantum-mechanical partition function	103
5.7	Expectation values	104
5.8	Hubbard-Stratonovich transformation	105
5.9	Problems	107
<b>6</b>	<b>Second quantization</b>	<b>109</b>
6.1	Many-body Hamiltonian	110
6.2	Fock space	111
6.3	Creation and annihilation operators	114
6.3.1	Second-quantized Hamiltonian	116
6.3.2	Field operators	117
6.4	Equivalence of first and second quantization	119
6.5	Coherent states	122
6.6	Problems	125
<b>7</b>	<b>Functional integrals</b>	<b>131</b>
7.1	Grand-canonical partition function	131
7.2	Ideal quantum gases	134
7.2.1	Semiclassical method	135
7.2.2	Matsubara expansion	137
7.2.3	Green's function method	142
7.3	Wick's theorem	146
7.4	Problems	149

<b>8</b>	<b>Interactions and Feynman diagrams</b>	151
8.1	Quasiparticles	152
8.1.1	The Lehmann representation	153
8.1.2	The spectral-weight function	156
8.1.3	Collective excitations	158
8.2	Perturbation theory	159
8.3	Dyson's equation	164
8.4	Hartree-Fock approximation	167
8.5	Variational approach	168
8.6	Hubbard-Stratonovich transformation	172
8.6.1	Hartree theory	172
8.6.2	Fock theory	176
8.6.3	Hartree-Fock theory for an atomic Fermi gas	178
8.7	The jellium model	182
8.7.1	Field-theory approach	184
8.7.2	Effective action	185
8.7.3	Dispersion and screened Coulomb interaction	188
8.8	Problems	190
<b>9</b>	<b>Landau theory of phase transitions</b>	193
9.1	Ising model in $d$ dimensions	194
9.2	Landau approach	198
9.3	Hubbard-Stratonovich transformation	203
9.4	Gaussian Fluctuations	205
9.5	Spontaneous symmetry breaking	208
9.6	Problems	211
<b>10</b>	<b>Atomic physics</b>	213
10.1	Atomic structure	214
10.1.1	Fine structure	215
10.1.2	Hyperfine structure	216
10.2	Zeeman effect	217
10.3	Two-body scattering in vacuum	219
10.3.1	Two-body transition matrix	220
10.3.2	Partial-wave expansion	222
10.3.3	Scattering from a square-well potential	224
10.4	Two-body scattering in a medium	227
10.5	Physical regimes	230
10.6	Problems	232
<b>11</b>	<b>Bose-Einstein condensation</b>	235
11.1	Definitions for a Bose-Einstein condensate	236
11.2	Superfluidity	238
11.2.1	Landau criterion	239
11.2.2	Superfluid density	240

11.3	Field-theory approach	241
11.3.1	Bogoliubov theory and the Gross-Pitaevskii equation	243
11.3.2	Dyson equation	245
11.3.3	Quasiparticle dispersion	246
11.4	Thermodynamic potential for bosons	248
11.5	Bogoliubov-de Gennes equation	251
11.6	Popov theory	253
11.7	Hydrodynamic-like approach	255
11.7.1	Time-dependent Gross-Pitaevskii equation	256
11.7.2	Collective modes	258
11.8	Rotating Bose-Einstein condensates	259
11.9	Attractive interactions	262
11.9.1	Effective action	264
11.9.2	Breathing mode	266
11.9.3	Metastability of the condensate	268
11.10	Problems	269
<b>12</b>	<b>Condensation of fermionic pairs</b>	<b>273</b>
12.1	Introduction	273
12.2	Thouless criterion	274
12.3	Hubbard-Stratonovich transformation	276
12.4	Bardeen-Cooper-Schrieffer theory	278
12.5	Critical temperature	280
12.6	Gap equation	283
12.7	Thermodynamic potential for fermions	286
12.8	The BEC-BCS crossover	288
12.8.1	Theoretical results	289
12.8.2	Comparison with experiment	292
12.9	Problems	294
<b>13</b>	<b>Symmetries and symmetry breaking</b>	<b>299</b>
13.1	Effective actions	300
13.2	Noether's theorem	303
13.3	Ward identities	305
13.3.1	Hugenholtz-Pines theorem	309
13.3.2	Bragg scattering	310
13.4	RF spectroscopy	313
13.4.1	Second-order perturbation theory	317
13.4.2	Ladder summations	319
13.4.3	Absence of clock shift	320
13.4.4	Absence of vertex corrections	324
13.5	Phase diffusion	325
13.6	Problems	328

<b>14 Renormalization group theory</b> .....	329
14.1 The renormalization-group transformation .....	330
14.1.1 Scaling .....	333
14.1.2 Interactions .....	335
14.2 Quantum effects .....	340
14.2.1 Interactions .....	343
14.2.2 Nonuniversal quantities .....	346
14.3 Renormalization group for fermions .....	347
14.3.1 Renormalization-group equations .....	348
14.3.2 Extremely-imbalanced case .....	350
14.3.3 Homogeneous phase diagram .....	352
14.4 Problems .....	354
<b>15 Low-dimensional systems</b> .....	359
15.1 Modified Popov Theory .....	360
15.1.1 Phase fluctuations .....	360
15.1.2 Many-body T matrix .....	362
15.1.3 Long-wavelength physics .....	363
15.2 Comparison with Popov Theory .....	364
15.2.1 One dimension .....	364
15.2.2 Two dimensions .....	366
15.2.3 Three dimensions .....	370
15.3 Vortices in two dimensions .....	371
15.4 Kosterlitz-Thouless Phase Transition .....	373
15.5 Trapped Bose Gases .....	377
15.5.1 Density profiles .....	379
15.5.2 Phase fluctuations .....	381
15.5.3 Comparison with exact results .....	385
15.6 Problems .....	389
<b>16 Optical lattices</b> .....	391
16.1 Optical lattices .....	392
16.2 Coupling between atoms and light .....	393
16.2.1 Two-level approximation .....	393
16.2.2 Fine structure .....	395
16.3 Band structure .....	397
16.4 Hubbard models .....	397
16.5 Superfluid-Mott insulator transition .....	400
16.5.1 Bogoliubov approximation .....	402
16.5.2 Decoupling approximation .....	407
16.5.3 Hubbard-Stratonovich transformation .....	413
16.6 Fluctuations .....	418
16.6.1 Mott insulator .....	420
16.6.2 Superfluid phase .....	421
16.7 Bragg spectroscopy .....	423

16.8 Problems .....	428
<b>17 Feshbach resonances .....</b>	<b>431</b>
17.1 Example of a Feshbach resonance .....	432
17.2 Bare atom-molecule theory .....	439
17.3 Ladder summations .....	447
17.4 Effective atom-molecule theory .....	453
17.4.1 Scattering properties .....	454
17.4.2 Bound-state energy .....	455
17.4.3 Molecular density of states .....	456
17.5 Popov theory for the Bose-Einstein condensed phase .....	458
17.6 Experiments .....	462
17.7 Josephson frequency .....	466
17.8 Problems .....	471
<b>References .....</b>	<b>473</b>
<b>Index .....</b>	<b>479</b>