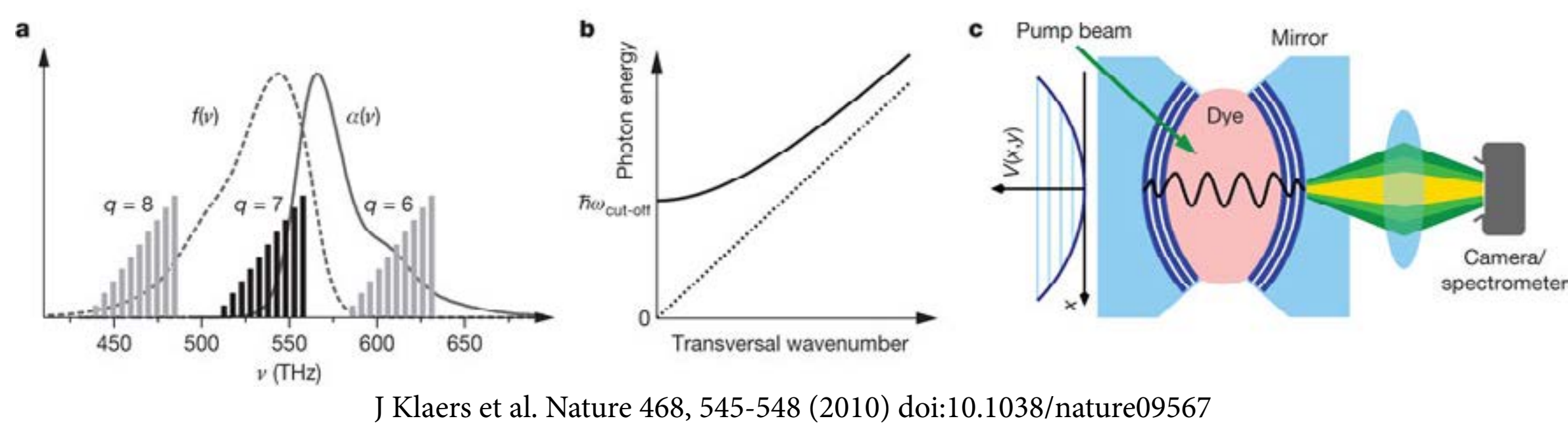


Effects of dissipation on the superfluid-Mott-Insulator transition of photons

Experiment



Photons are confined in a dye-filled optical microresonator [2]. Since the emission and absorption of photons with longitudinal mode number $q = 7$ dominates over other emission processes, the number of photons in the cavity is conserved. The photon gas is formally equivalent to a two-dimensional harmonically trapped massive Bose gas \longrightarrow Bose-Einstein condensation of photons at non-zero temperature! By periodically changing the index of refraction of the dye inside the cavity a periodic potential for the photons can be induced.

Superfluid-Mott-Insulator transition

The Bose-Hubbard model describes a Bose gas in a periodic potential with a nearest-neighbour hopping parameter t and on-site interaction strength U .

$$S = \int_0^{\hbar\beta} d\tau \left\{ \sum_i a_i^*(\tau) \left(\hbar \frac{\partial}{\partial \tau} + \epsilon_i - \mu \right) a_i(\tau) - t \sum_{\langle i,j \rangle} a_i^*(\tau) a_j(\tau) + \frac{U}{2} \sum_i a_i^*(\tau) a_i^*(\tau) a_i(\tau) a_i(\tau) \right\}$$

$t \ll U$: Mott Insulator

- Hopping is energetically unfavorable.
- Equal and integer amount of particles per site.
- Particle number fluctuations are suppressed.

$t \gg U$: Superfluid

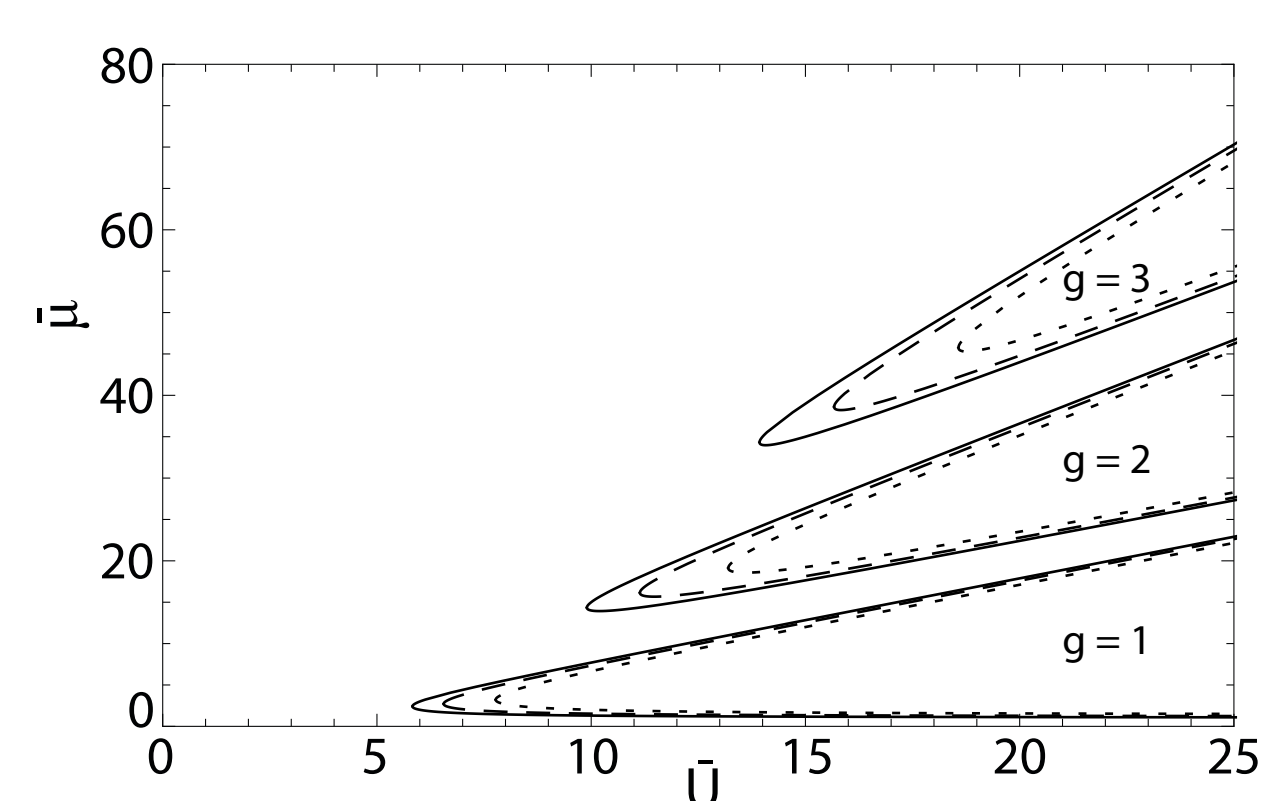
- Energy minimized by freely hopping around.
- Fluctuating number of particles per site.

Mean-Field theory

We investigate the effects of dissipation by treating the dye as an effective two level system that can absorb and emit photons.

$$S_c = \frac{g_m}{\sqrt{A\sqrt{V}}} \int_0^{\hbar\beta} d\tau \left\{ \sum_{i,p} a_i^*(\tau) b_{p,\downarrow}^*(\tau) b_{p,\uparrow}(\tau) + a_i(\tau) b_{p,\uparrow}^*(\tau) b_{p,\downarrow}(\tau) \right\}$$

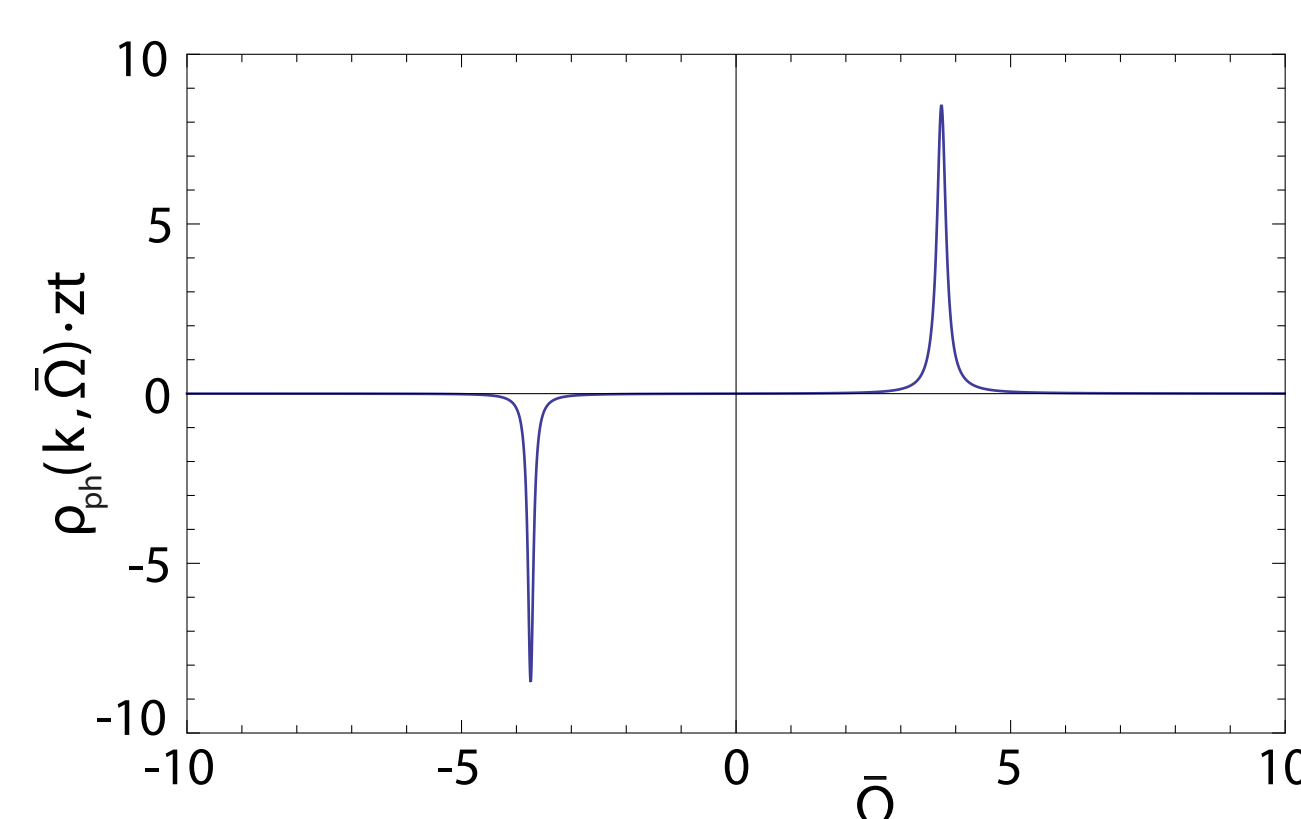
We construct a consistent mean-field theory for the superfluid order parameter $\langle a_i(\tau) \rangle$. Due to the coupling with the molecules a non-zero expectation value of $\langle a_i(\tau) \rangle$ will induce a non-zero expectation value of $\langle b_{p,\uparrow}^*(\tau) b_{p,\downarrow}(\tau) \rangle$. We solve the molecular problem exactly and perform second-order perturbation theory in the superfluid order parameter. We use the usual Landau procedure to determine the phase boundary between $\langle a_i(\tau) \rangle = 0$ and $\langle a_i(\tau) \rangle \neq 0$.



Phase diagram for the Bose-Hubbard model with the photon-molecule interaction parameter γ . From top to bottom the three contributions are for respectively 3, 2 and 1 particle per site. The solid, dashed and dotted line are for $\gamma = 0$, $\gamma = 1/2$ and $\gamma = 1$.

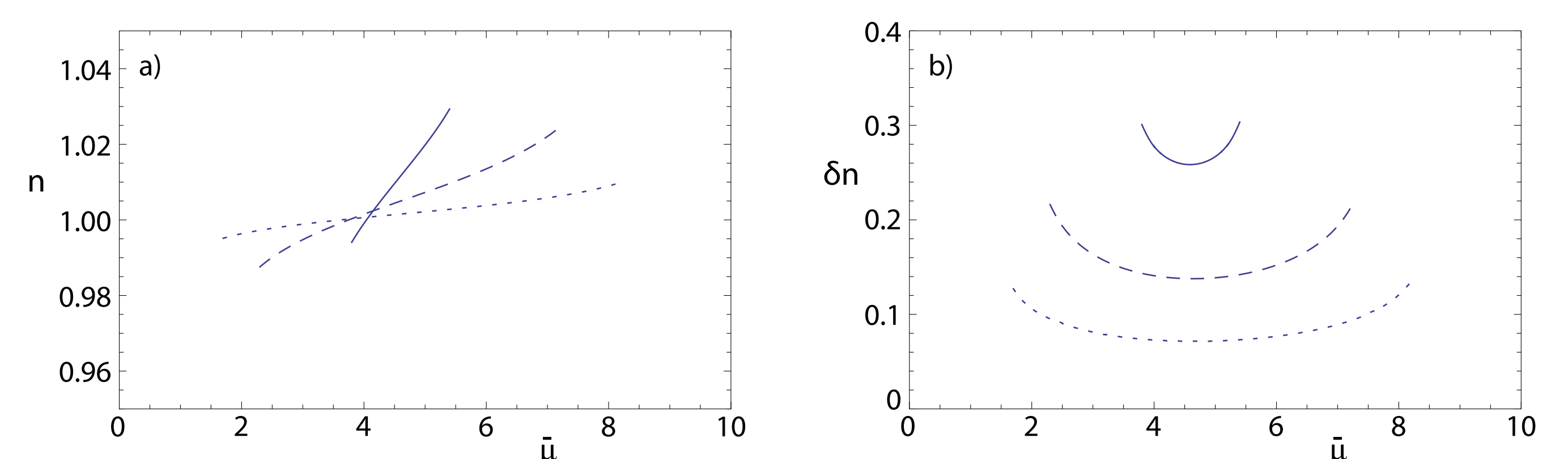
Beyond Mean-Field theory

We integrate out the molecules and perform second-order perturbation theory in the coupling constant g_m to consider Gaussian fluctuations of the photons inside the Mott lobes.



Inside the Mott lobes the excitations acquire a finite lifetime due to the coupling to the dye.

The number of photons varies inside the Mott lobes and the number fluctuations increase as the value of the damping parameter α increases.



Due to the photon-molecule coupling there are even number fluctuations are zero temperature \longrightarrow Mott Insulator does not exist!

Conclusions

- The photon-molecule coupling decreases the size of the Mott lobes.
- The excitations inside the Mott lobes acquire a finite lifetime.
- There are non-zero number fluctuations at zero temperature and therefore the true Mott Insulator is absent.

References

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