Renormalization Group Theory for the Imbalanced Fermi Gas

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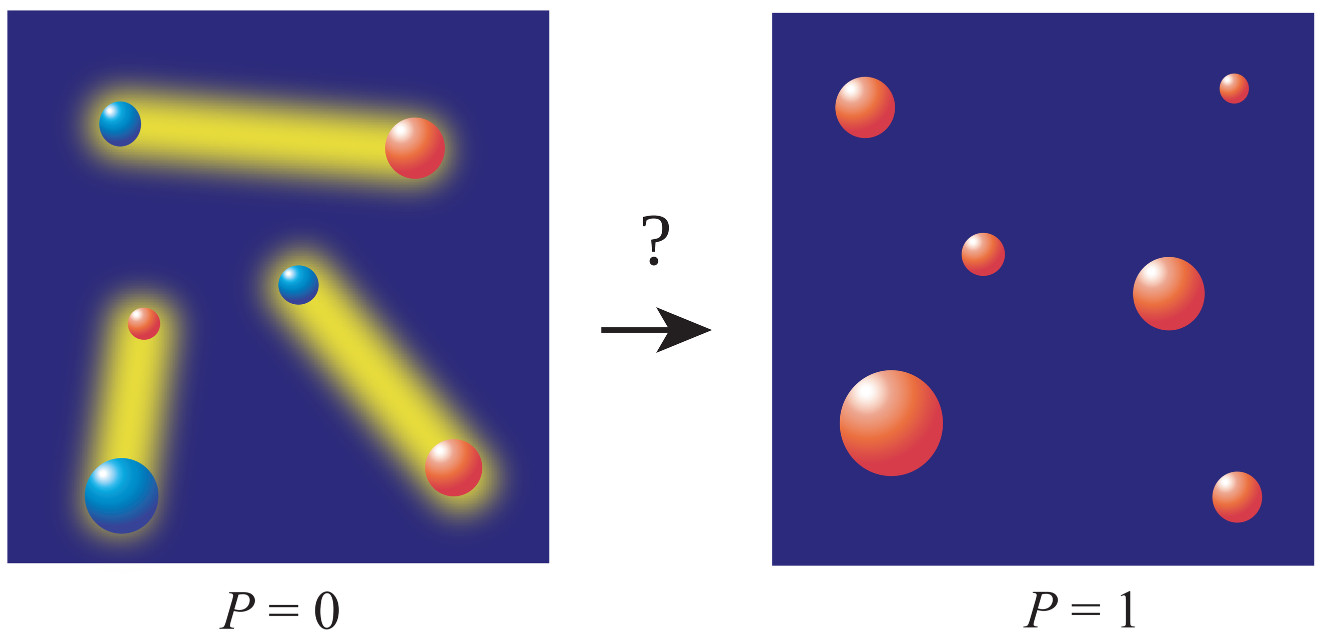
Introduction

Ultracold atomic Fermi mixtures are the dream of every physicist due to the amount of experimental control that is realizable. The interaction strength, the trapping potential and the density of particles n_α in each spin state α can be tuned at will. For two states, pairing is optimal for equal particle number and strong interaction. This leads to a record-high superfluid critical temperature on the order of the Fermi temperature. Two experimental groups have studied the mixture as a function of polarization $P = (n_\uparrow - n_\downarrow)/(n_\uparrow + n_\downarrow)$, relevant for astrophysics, condensed and nuclear matter. Since equally spinning particles do not interact at low temperatures, the fully polarized system is ideal. A transition from superfluid to normal is thus induced by varying polarization.



Experiments by MIT and Rice University

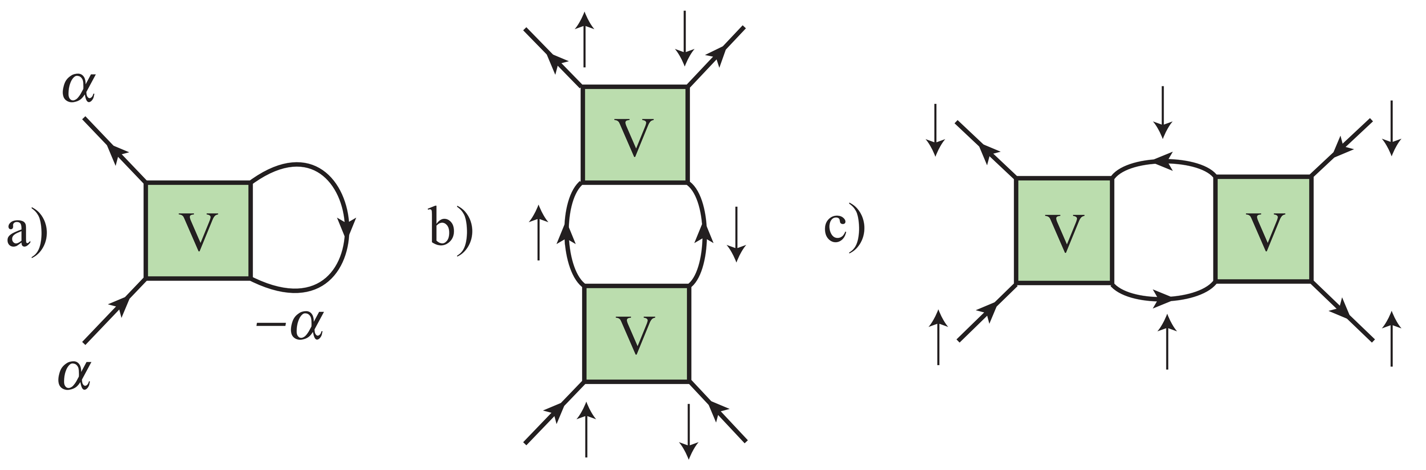
The results by Rice [1] and MIT [2] caused much debate, since some of them agreed and some not. All fit to a phase diagram with superfluid-normal transitions of second and first order, connected by a tricritical point [3]. At low temperatures, MIT found a critical polarization of 0.3 as expected from Monte-Carlo (MC) simulations and the local-density approximation (LDA). LDA applies for shallow traps, i.e. locally flat, so that the gas behaves homogeneously. As a result, MIT could map out the homogeneous phase diagram via local measurements in the trap [2] (see last figure). For Rice, LDA does not hold, leading to different physics.

Renormalization group (RG) theory for fermions

Starting point for a theoretical description is the microscopic action S,

$$S = \int_0^{\hbar\beta} d\tau \int d\mathbf{x} \left\{ \sum_{\alpha=\uparrow,\downarrow} \phi^*_\alpha \left(\hbar\frac{\partial}{\partial\tau} - \frac{\hbar^2\nabla^2}{2m} - \mu_\alpha \right) \phi_\alpha + V\phi^*_\uparrow\phi^*_\downarrow\phi_\downarrow\phi_\uparrow \right\},$$

where the fields $\phi_\alpha = \phi_\alpha(\mathbf{x},\tau)$ describe fermionic particles, $\beta = 1/k_{\mathrm{B}}T$ is the inverse temperature, m the mass, μ_α the chemical potential for spin state α and V the interaction strength. Perturbation theory fails to describe strong interactions quantitatively, so we use renormalization. Integrating out modes in a momentum shell Λ of infinitesimal width $d\Lambda$, gives the Feynman diagrams



Only one-loop diagrams survive, since loops add a factor $d\Lambda$. The contributions of the diagrams are absorbed into flowing couplings. Diagram a) is a selfenergy; b) is a ladder diagram and describes scattering in a gas, it yields

$$d\Xi(q^2, i\omega_m) = \int_\Lambda \frac{d\mathbf{k}}{(2\pi)^3} \frac{1 - N_\uparrow(\epsilon_\mathbf{k}) - N_\downarrow(\epsilon_{\mathbf{q}-\mathbf{k}})}{i\hbar\omega_m - \epsilon_\mathbf{k} - \epsilon_{\mathbf{q}-\mathbf{k}} + \mu_\uparrow + \mu_\downarrow}$$

with $\hbar\mathbf{q}$ and ω_m the center-of-mass momentum and frequency, $\epsilon_\mathbf{k} = \hbar^2k^2/2m$ the kinetic energy, and $N_\alpha(\epsilon_\mathbf{k}) = 1/(e^{\beta(\epsilon_\mathbf{k}-\mu_\alpha)} + 1)$ the Fermi distribution; c) is a bubble diagram and describes screening by particle-hole excitations. The interaction is expanded as $V^{-1}_{\mathbf{q},m} = V^{-1}_{\mathbf{0},0} - Z^{-1}_q q^2 + Z^{-1}_\omega i\hbar\omega_m$, where

$$dZ_q^{-1} = \left.\frac{\partial d\Xi(q^2,\omega)}{\partial q^2}\right|_{q=\omega=0} \quad \text{and} \quad dZ_\omega^{-1} = -\left.\frac{\partial d\Xi(q^2,\omega)}{\partial\hbar\omega}\right|_{q=\omega=0}.$$

Putting everything together, we obtain the following RG equations,

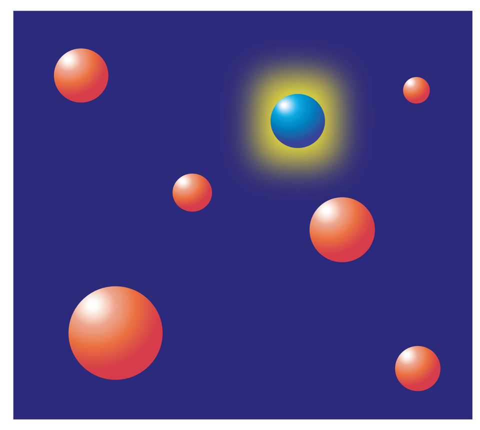
$$\frac{dV^{-1}_{\mathbf{0},0}}{d\Lambda} = \frac{\Lambda^2}{2\pi^2}\left\{\frac{1 - N_\uparrow(\epsilon_\Lambda) - N_\downarrow(\epsilon_\Lambda)}{2\epsilon_\Lambda - \mu_\uparrow - \mu_\downarrow} - \frac{N_\uparrow(\epsilon_\Lambda) - N_\downarrow(\epsilon_\Lambda)}{\mu_\uparrow - \mu_\downarrow}\right\},$$

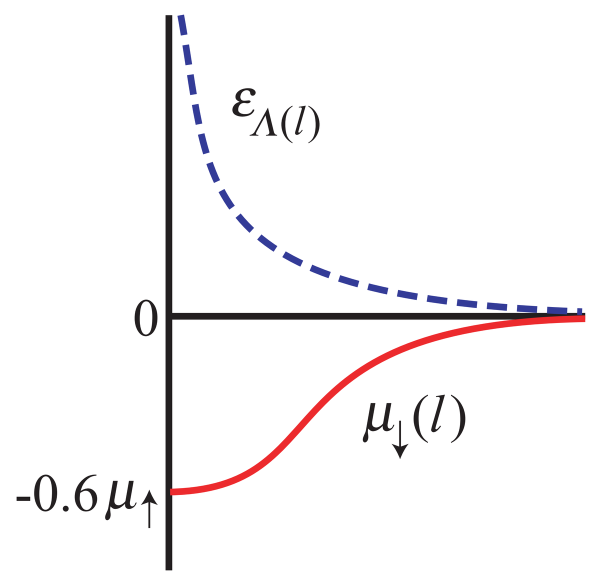
$$\frac{d\mu_\alpha}{d\Lambda} = \frac{\Lambda^2}{2\pi^2}\frac{N_{-\alpha}(\epsilon_\Lambda) + N_{\mathrm{B}}(\epsilon_\Lambda)}{-V^{-1}_{\mathbf{0},0} + Z^{-1}_q\Lambda^2 - Z^{-1}_\omega(\epsilon_\Lambda - \mu_{-\alpha})},$$

where $N_{\mathrm{B}}(\epsilon_\Lambda) = 1/(e^{\beta Z_\omega(-V^{-1}_{0,0} + Z_q\Lambda^2)} - 1)$. Note that the RG equations generate infinitely many Feynman diagrams, showing the nonperturbative nature.

Extremely imbalanced case

First, we apply the RG to one spin-down particle in a sea of spin-up particles at zero temperature. We consider the unitarity limit, where the scattering length diverges and the experiments are done. The equations are simplified, e.g. because $N_\downarrow$ is zero and $\mu_\uparrow$ is not changing.





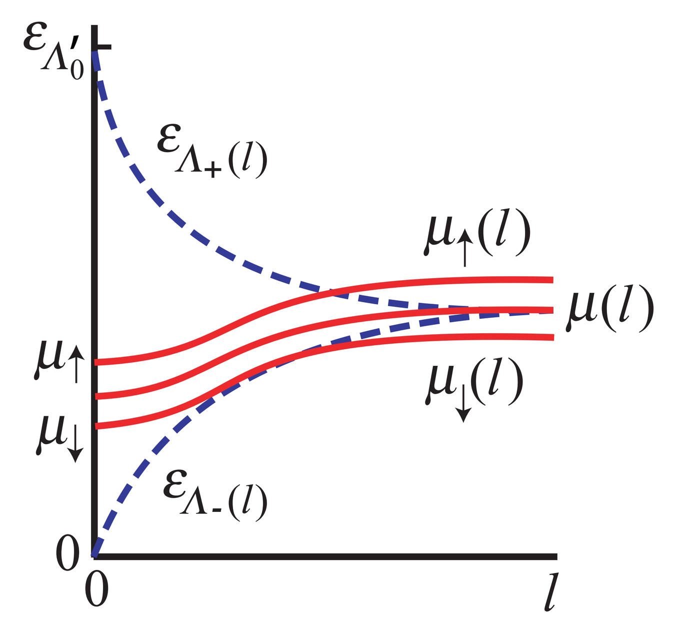
It turns out that we can use $\Lambda(l) = \Lambda_0 e^{-l}$ to integrate out all momentum shells. If $\mu_\downarrow(0) = -0.6\,\mu_\uparrow$, then $\mu_\downarrow$ renormalizes to $\mu_\downarrow(\infty) = 0$ due to the strong attractive interactions. The flow is depicted on the left. The selfenergy of the spin-down fermion is thus $-0.6\mu_\uparrow$. Since our results agree with MC calculations, the RG is suited to tackle the unitarity limit.

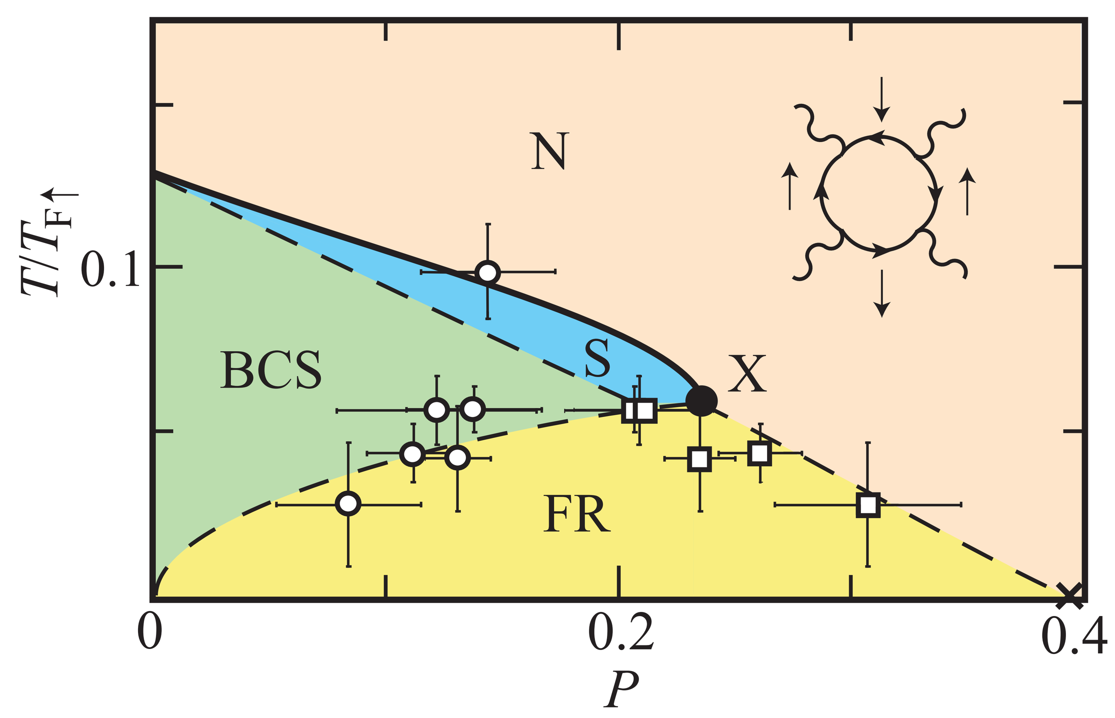
Homogeneous phase diagram

For the transition to the superfluid state, we use the critical condition $V^{-1}_{\mathbf{0},0} = 0$. It means that the many-body T matrix diverges, i.e. a new bound state enters the system. For weak interaction, selfenergies are neglible and μ_α does not flow. We find a reduction of the critical temperature with $1/e$ due to screening (Gor'kov correction). For strong interaction, selfenergy effects are important and we use

$$\Lambda_\pm(l) = \left(\Lambda'_\pm - \sqrt{\frac{2m\mu}{\hbar^2}}\right)e^{-l} + \sqrt{\frac{2m\mu(l)}{\hbar^2}}$$

to flow to the average Fermi level as shown. In the balanced case, we find $T_c = 0.13T_{\mathrm{F}}$ and $\mu = 0.55\epsilon_{\mathrm{F}}$ [4] in agreement with MC.





We can also determine the tricritical point (X), where second-order transitions to the superfluid phase turn to first order. It is at $P_{c3} = 0.24$ and $T_{c3} = 0.06T_{\mathrm{F},\uparrow}$ [4]. In the phase diagram above, the data at the boundaries are by MIT [2], while the solid line is the calculated second-order line. Also shown is the Feyman diagram that determines the tricritical point. At non-zero P the transition is into the gapless superfluid Sarma (S) phase [3]. The dashed lines for the BCS superfluid and the forbidden region (FR) are a guide to the eye.

References

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