Sarma Phase in Trapped Unbalanced Fermi Gases

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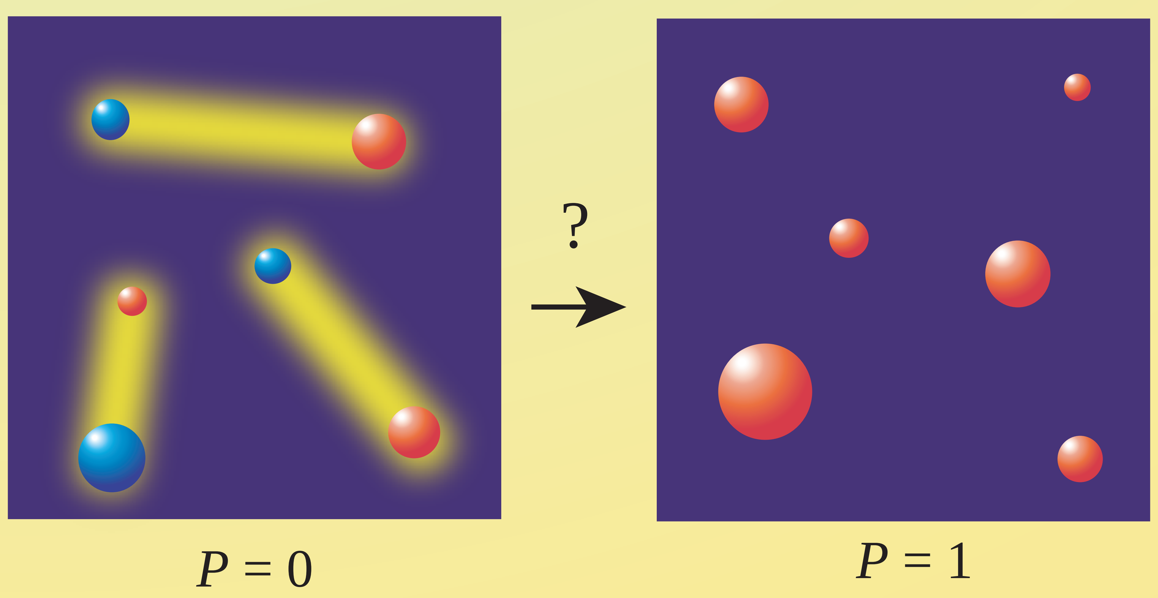
Introduction

Recently, two experimental groups have obtained full control over the polarization P of a two-component Fermi mixture, which is defined as $P = (N_+ - N_-)/(N_+ + N_-)$, where N_σ is the number of fermions in spinstate σ. This allows for the study of degenerate Fermi gases with imbalanced spin populations and tunable interactions, a topic of great interest in many areas of physics, ranging from condensed-matter physics to nuclear and astro-particle physics. In the case of attractive interactions, we know from BCS-theory at equal densities that the fermions can form Cooper-pairs, which consequently Bose-condense. Also we know that the fully polarized system is an ideal gas. The fundamental question is what happens between these two limits as a function of P, as also expressed by the figure below.



Contradictory experiments?

This fundamental question has been adressed both by the group of Wolfgang Ketterle at MIT and by the group of Randy Hulet at Rice University [1, 2, 3]. However, very surprisingly, these two outstanding groups found very contradictory results. Where the MIT experiment only reported a transition between a superfluid and a normal phase at a polarization of about 70%, the Rice-experiment seemed to observe only a transition between a non-deformed and a deformed superfluid phase at a polarization of about 9%. How can it be that two such excellent studies of the same system gave such incompatible results? This question led to an explosion of theoretical activity.

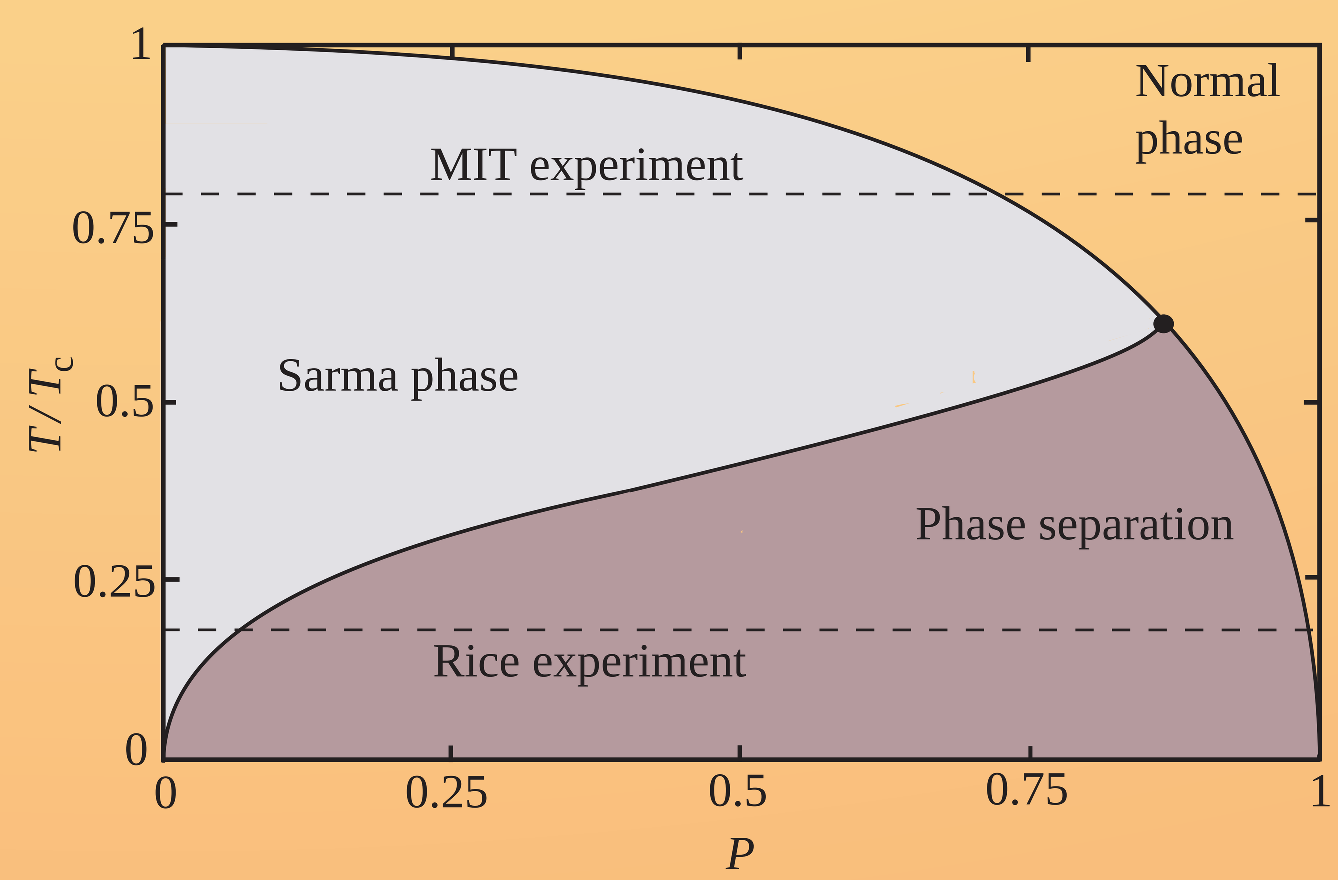
Phase diagram: the Sarma phase vs. phase separation

Our main results are summarized in the figure below, which shows the universal phase diagram of an ultracold trapped Fermi mixture in the strongly interacting regime (unitarity limit). It is obtained by minimizing the mean-field BCS thermodynamic potential Ω in the unitarity limit

$$\Omega = \sum_{\mathbf{k},\sigma=\pm} \frac{1}{2}\left(\varepsilon_{\mathbf{k}} - \mu' - \hbar\omega_{\mathbf{k}} + \frac{|\Delta|^2}{2\varepsilon_{\mathbf{k}}} - \frac{2}{\beta}\ln\left(1+e^{\beta(\sigma h - \hbar\omega_{\mathbf{k}})}\right)\right) - \frac{1}{2}\sum_{\sigma=\pm} N_\sigma \hbar\Sigma_\sigma, \quad (1)$$

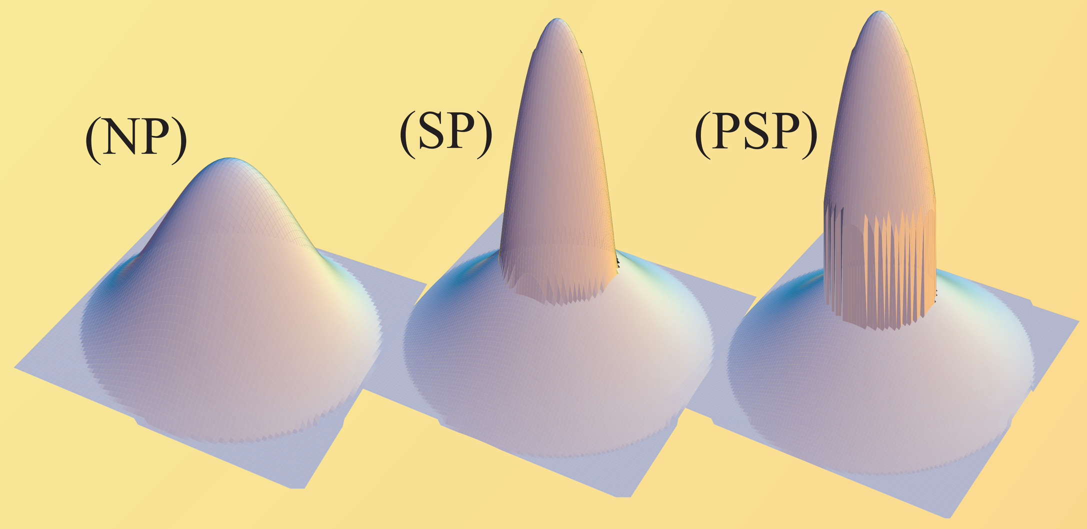
where the (renormalized) chemical potentials are given by $\mu_\sigma = \mu'_\sigma - \hbar\Sigma_\sigma$, their average: $\mu' = (\mu'_+ + \mu'_-)/2$, the difference $h = (\mu'_+ - \mu'_-)/2$, and $(\hbar\omega_{\mathbf{k}})^2 = (\varepsilon_{\mathbf{k}} - \mu')^2 + |\Delta|^2$ with Δ the BCS order parameter. The selfenergy

$$\hbar\Sigma_\sigma = \frac{3\pi^2\hbar^2}{m}(\beta_{\mathrm{MC}} - \beta_{\mathrm{BCS}})\sqrt{\frac{\hbar^2}{m}\frac{1+\beta_{\mathrm{BCS}}}{2\mu'}}\, n_{-\sigma} \quad (2)$$



is constructed such that the (renormalized) chemical potential obeys the exact relation $\mu' = (1+\beta_{MC})\varepsilon_F$ with $\beta_{MC} \approx -0.58$ as obtained from Monte Carlo simulations, rather than $\mu' = (1+\beta_{BCS})\varepsilon_F$ with $\beta_{BCS} \approx -0.41$ from BCS-theory. To account for the trap, we use the local density approximation, absorbing the trapping potential in the chemical potential, which becomes a function of position. Locally, the theory is then still homogeneous.

The phase diagram consists of three different phases. In the normal phase (NP) the gas is normal throughout the trap. In the two other phases there is a shell structure, i.e. a superfluid core surrounded by normal gas. In the Sarma phase (SP) the superfluid core is separated from the normal gas by a *second-order* transition, whereas in the phase separated phase (PSP) we have a *first-order* transition as a function of position in the trap. The three phases are depicted in the figure below.

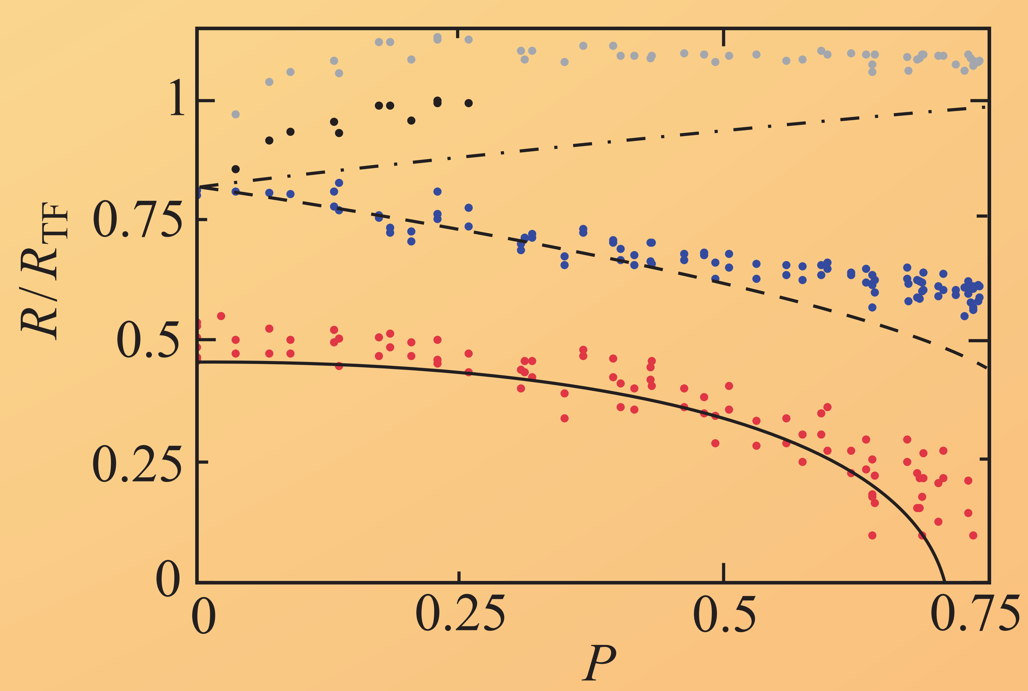


Note that in the SP, Δ will become arbitrarily small. Therefore, when h is nonzero (finite P), we have at least a shell where $h > \Delta$, yielding gapless superfluidity (and explaining the name Sarma phase).

Comparison with MIT-experiment

Our phase diagram immediately gives rise to a natural explanation for the differences in observations between Rice and MIT. We think that the measurements at MIT were performed above the tricritical point and showed the transition between the SP and the NP. The experiment at Rice was done below this point and observed the transition between the SP and the PSP. The first-order nature of the PSP gives rise to surface tension leading to deformations as observed at Rice. At higher temperatures the Rice group obtains very similar results to those reported by MIT [4], in full agreement with our phase diagram.

We can also perform a more quantitative comparison with the MIT experiment. To this end we calculate the condensate radius (solid line), the size of the cloud of the minority (dashed line) and the majority spin species (dash-dotted line) as a function of the polarization P. These can be compared with the data from MIT (swarm of dots). In general, the agreement is very good.



Conclusions and outlook

We have calculated the universal phase diagram for a trapped Fermi mixture with a population imbalance in the unitarity limit. This immediately gives rise to a natural explanation for the differences seen in the experiments performed at MIT and Rice. Further comparison of our theory with the MIT experiment gives very good agreement. Our remaining problem is that fluctuations are not properly taken into acount, leading to high absolute temperatures. More advance methods have to be used to overcome this problem.

References

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[2] M. W. Zwierlein *et al.*, Science **311**, 492 (2006)
[3] M. W. Zwierlein *et al.*, Nature **442**, 54 (2006)
[4] G. B. Partridge *et al.*, Phys. Rev. Lett. **97**, 190407 (2006)
[5] K. B. Gubbels *et al.*, Phys. Rev. Lett. **97**, 210402 (2006)