Correlation Effects in Ultracold Two-Dimensional Bose Gases

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Introduction

Low-dimensional systems play a unique role in the study of many-body effects. As elucidated in the Mermin-Wagner-Hohenberg theorem, the enhanced importance of thermal fluctuations prevents a two-dimensional (2D) system with a continuous symmetry to undergo spontaneous symmetry breaking at finite temperatures. Nonetheless, the 2D XY model undergoes a special type of phase transition, known as the Berezinskii-Kosterlitz-Thouless (BKT) transition, which is characterized by the unbinding of vortex-antivortex pairs, see Fig. 1.

Figure 1 BKT transition.

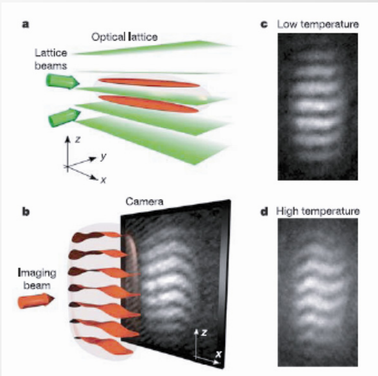


Figure 2 Observation of the BKT crossover in a trapped atomic gas [2].

For the ultracold 2D Bose gas, the absence of Bose-Einstein condensation (BEC) requires the concept of a quasicondensate to understand the existence of algebraic long range order and superfluidity [1]. Experiments for atomic gases have recently reached this interesting 2D regime to allow for the direct observation of this phenomenon in a highly controllable environment, see Fig. 2.

To describe the low-dimensional Bose gas at nonzero temperatures, the usual approach of the Bogoliubov theory is plagued with infrared divergences. However, these divergences can be shown to occur due to a spurious contribution from the condensate phase fluctuations and by removing these contributions we can arrive at a modified Popov theory, which is valid for any dimension and at all temperatures. We first study this modified Popov theory for the ultracold 2D Bose gas [3]. To improve upon the mean-field description, we next employ a renormalization group approach to take into account the quantum and thermal fluctuations more accurately, in particular in the normal phase, when the modified Popov theory reduces to Hartree-Fock theory.

Effective Interaction and Mean-Field Theory

The realization of a 2D Bose gas in experiments with atomic gases is achieved by restricting the motion of a trapped three-dimensional gas onto a plane, where the motion along the tightly confining axial direction is frozen out with the condition $k_B T, \mu \ll \hbar\omega_z$.

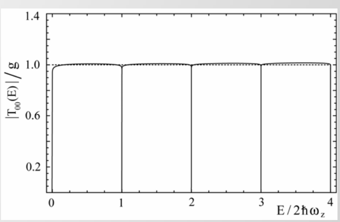
The effective interaction in the ultracold limit is in the first instance determined by the three-dimensional two-body T matrix, which after renormalization takes the form $T_{00}(E) = (2\sqrt{2\pi}\hbar^2/m)/(l/a + F(E))$.

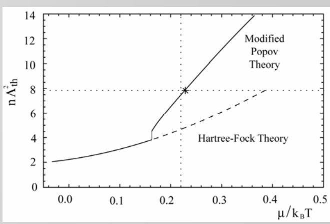
From the modified Popov theory, the equation of state for the quasicondensate is given by

$$n = n_0 + \frac{1}{V}\sum_k \left\{ \frac{\epsilon_\mathbf{k}}{2\hbar\omega_\mathbf{k}}\left[2N(\hbar\omega_\mathbf{k})+1\right] - \frac{1}{2} + \frac{n_0 T_{00}(-2\mu)}{2\epsilon_\mathbf{k}+2\mu} \right\},$$

while in the normal state it is given by the Hartree-Fock theory

$$n = \frac{1}{V}\sum_\mathbf{k} N(\epsilon_\mathbf{k} + \hbar\Sigma - \mu).$$



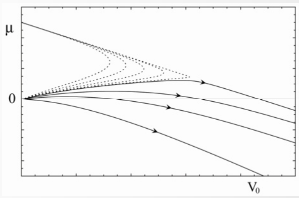


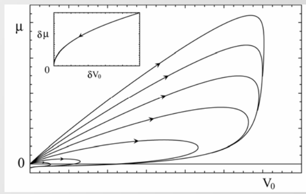
Due to its mean-field nature, the BKT physics can only be captured by complementing it with an additional renormalization group analysis on the Sine-Gordon model that takes into account vortices explicitly giving the condition for the BKT transition $n_0\Lambda_{th}^2 = 6.65$. We thus observe that above this critical temperature, the loss of superfluidity does not immediately leads to the destruction of the quasicondensate.

Renormalization Group Theory

To improve upon the density discontinuity close to the BKT transition, which is an artefact of the mean-field theory, we employ a renormalization group (RG) approach. Due to the ultracold limit of the Bose gas, the parameters which are important in determining the various nonuniversal properties are the flowing chemical potential μ and the flowing two-body interaction strength V_0. The flow equations in the symmetry-broken phase, for example, are given by

$$\begin{aligned}
\frac{d\mu}{dt} &= 2\mu - \frac{\Lambda^2}{2\pi}V_0\left[\frac{2\epsilon_\Lambda^3 + 6\mu\epsilon_\Lambda^2 + \mu^3}{2\hbar^3\omega_\Lambda^3}[2N(\hbar\omega_\Lambda)+1] - 1\right.\\
&\quad \left. + \frac{\mu(2\epsilon_\Lambda+\mu)^2}{\hbar^2\omega_\Lambda^2}\beta N(\hbar\omega_\Lambda)[N(\hbar\omega_\Lambda)+1]\right],\\
\frac{dV_0}{dt} &= -\frac{\Lambda^2}{2\pi}V_0^2\left[\frac{(\epsilon_\Lambda-\mu)^2}{2\hbar^3\omega_\Lambda^3}[2N(\hbar\omega_\Lambda)+1]\right.\\
&\quad \left. + \frac{(2\epsilon_\Lambda+\mu)^2}{\hbar^2\omega_\Lambda^2}\beta N(\hbar\omega_\Lambda)[N(\hbar\omega_\Lambda)+1]\right].
\end{aligned}$$

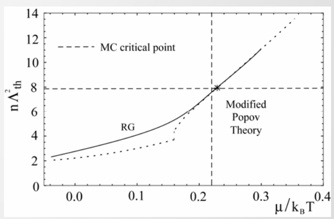


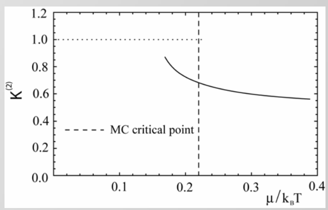


RG Equation of State and Correlation Effects

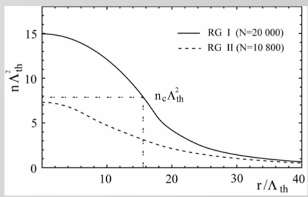
We then evaluate the one-point Green's function by integrating the momentum shell by shell to obtain the RG density curve. We find an important correction for the Hartree-Fock theory as the BKT critical point is approached, but it connects smoothly with the quasicondensate density curve already above the critical temperature. It thus resolves the artificial discontinuity and shows agreement in the quasicondensate phase.

Furthermore, the density-density correlator $K^{(2)}$ and the three-body recombination rate can be easily calculated from the modified Popov theory, which can serve as observables for effects beyond Hartree-Fock theory.





Finally, to compare with experiments for a 2D Bose gas in a harmonic trap, we evaluate the density profile within the local-density approximation. The temperature for the BKT transition, relative to the ideal gas BEC temperature in a trap is given by $T_{BKT} \simeq 0.81 T_{BEC}$ for $l/a \simeq 33.3$.



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