Interaction effects on transport in a holographic model for strongly-coupled Dirac semimetals

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Abstract

Using techniques from the Anti-de-Sitter/Condensed Matter Theory correspondence, we formulate a bottom-up model that provides fermionic single-particle Green's functions. With it we investigate Dirac semimetals coupled to strongly correlated, critical fluctuations, modeled by composite operators in a dual conformal field theory. The corresponding spectral-weight function satisfies the zeroth-order frequency sum rule, making it experimentally measurable. We also determine the electrical conductivity of the Dirac semimetals, which is finite in the particle-hole symmetric case and consists of an interband and an intraband contribution. For low energy, it inherits the scaling of the critical field responsible for the interactions between the Dirac fermions.

Dirac semimetals

Dirac semimetals are 3+1-dimensional gapless semiconductors based on Dirac fermions. In the noninteracting case and for low energies, the Hamiltonian is the 4×4 matrix

$$H = \tau_3 \otimes \boldsymbol{\sigma} \cdot c\hbar\mathbf{k}$$

Dirac semimetals have recently been realized experimentally, i.a. in the crystal Cd_3As_2 [1-3]

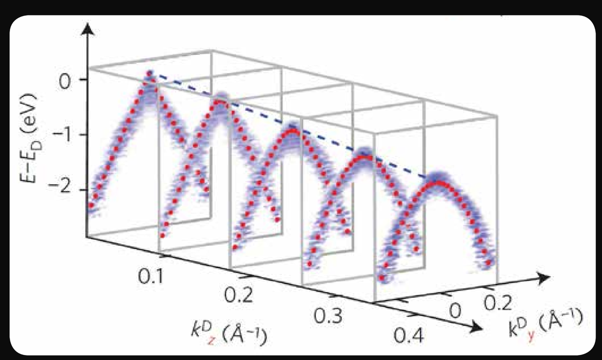


Figure from [3].

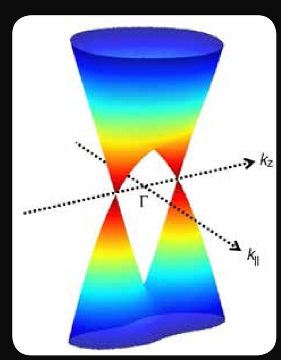


Figure from [1].

Recent theoretical work includes i.a. free systems, short-ranged interactions and Coulomb interaction [4]. We however investigate a strongly interacting Dirac semimetal based on fermions ψ coupled to a critical order parameter field $\mathcal{O}$ near a quantum critical point, modeled by a conformal field theory.

$$\mathcal{L} = \mathcal{L}_{\mathrm{CFT}}[\mathcal{O}] + \mathcal{L}_0[\bar{\psi}, \psi] + g\left(\bar{\psi}\mathcal{O} + \bar{\mathcal{O}}\psi\right)$$

If the critical point is nontrivial, the order parameter fluctuations induce strong correlations between the Dirac fermions, that are not necessarily of Coulombic nature.

Model

We apply the AdS/CMT correspondence in a bottom-up approach, to obtain a holographic Green's function describing fermionic single-particle correlations.

The gravity side is an asymptotically Anti-de-Sitter bulk space-time in 4+1 dimensions with two uncoupled probe Dirac fermions of mass $\pm M$.

Elementary fermionic excitations are described by dynamical source fields on a UV cut-off surface near the AdS boundary [5], which introduces a scale $\omega_c(M)$. In the spirit of the semiholographic approach [6], we add a kinetic term for this boundary value.

The idea is to apply the alternative quantization, and integrate over the boundary values of the two bulk fields on the UV surface, instead of fixing them to be stationary sources. The result is an effective boundary action for the dynamical source. From it, the corresponding Green's function for a massless 3+1 dimensional boundary Dirac fermion can be obtained. Each of the chiral components of this Dirac fermion is supplied by one of the bulk fermion species, which restricts -1/2 < M < 1/2. The retarded Green's function is:

$$G(k) = \frac{ck_\mu + \Sigma_\mu(k)}{\left(ck + \Sigma(k)\right)^2}\gamma^\mu\gamma^0$$

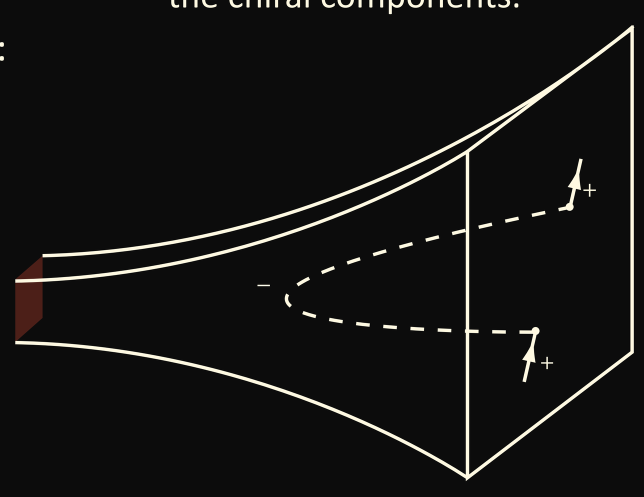
Kinetic part coming from free dynamics on UV surface.

Self-energy, depends on M. Comes from numerically solving classical equation of motion in curved background, shown below for one of the chiral components.

For $T=0$, analytical expression for self-energy:

$$\Sigma_\mu(k, T=0) = \left(\frac{ck}{\omega_c}\right)^{2M-1} ck_\mu$$

In the case $M = 1/2$, which requires a separate computation, the self-energy scales linearly with logarithmic corrections.

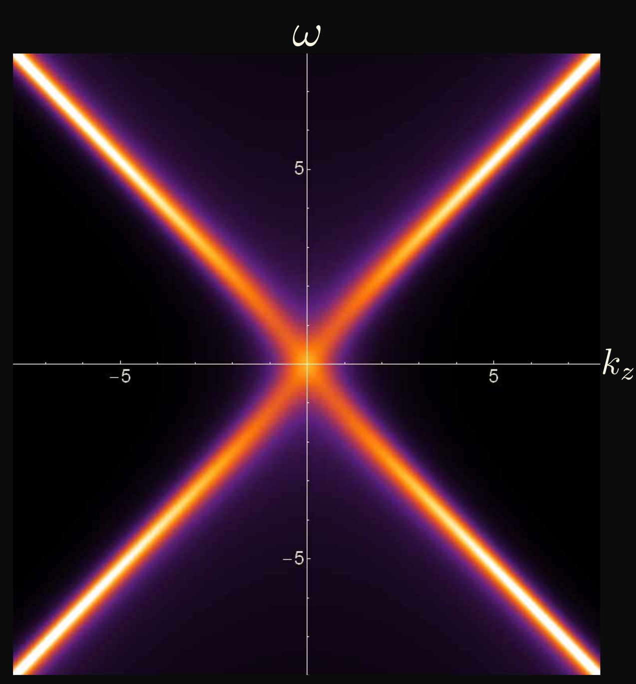


This setup can be interpreted as coupling the Dirac semimetal to a critical strongly-interacting system of collective degrees of freedom, modeled by the composite operators in the dual conformal field theory.

Spectrum

The Green's function describes a gapless, particle-hole symmetric, fermionic many-body system that behaves as a Dirac semimetal. It is strongly interacting in the infrared and free in the ultraviolet. The spectral-weight function, which is experimentally observable in condensed-matter systems, is given by

$$A(\mathbf{k}, \omega) = \frac{1}{\pi}\mathrm{Im}\, G(\mathbf{k}, \omega^+)$$



Left: density plot of the spectrum for T>0 and M=1/4. Even at zero chemical potential, interactions lead to nontrivial dynamics: many-body correlations due to the possibility of creating particle-hole pairs.

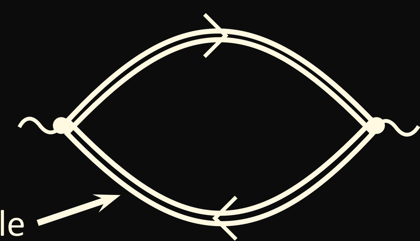
For T=0 the peaks in the spectral-weight function are smeared out inside the lightcone. A nonzero temperature breaks Lorentz invariance and has an additional smearing-out effect, this time also outside of the lightcone.

Conductivity

The fermionic contribution to the electrical conductivity is computed in linear-response theory, by minimally coupling the 3+1 dimensional Dirac fermions to a small electric field. The Kubo formula:

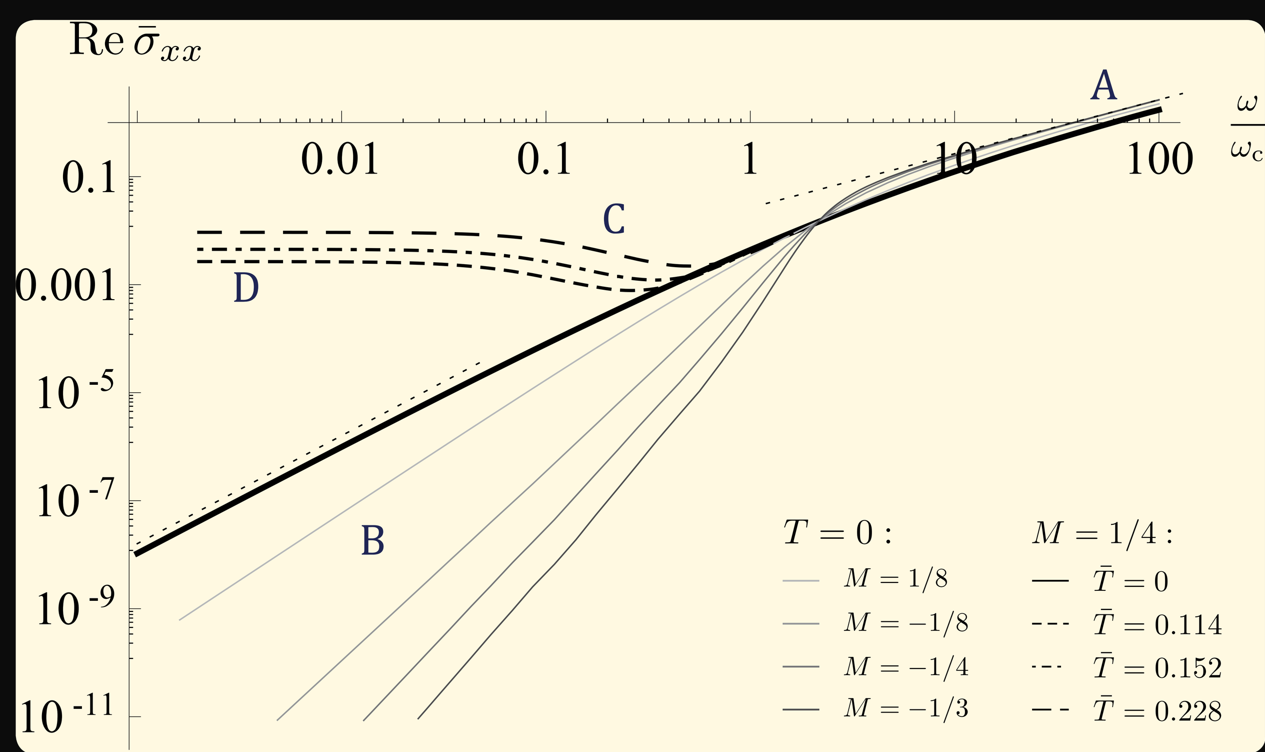
$$\sigma^{\mu\nu}(\mathbf{q}, \omega) = \frac{i\Pi^{\mu\nu}(\mathbf{q}, \omega^+)}{\omega}$$

Current-current correlation tensor, calculated from bubble diagram with dressed propagators



The dc conductivity is finite because of particle-hole symmetry: electric fields do not couple to the center-of-mass motion, and interactions lead to current relaxation in finite time. Also due to particle-hole symmetry, vertex corrections are not crucial for obtaining the qualitative behavior of the conductivity.

The total conductivity $\mathrm{Re}\,\bar{\sigma}_{xx} = (\hbar c/e^2\omega_c)\mathrm{Re}\,\sigma_{xx}$ has two contributions, corresponding to interband and intraband transitions. $T=0$



A: For $T=0$ and high frequency, it approaches the free result

$$\mathrm{Re}\,\sigma_{xx}(\mathbf{0}, \omega) = \frac{e^2|\omega|}{12\pi\hbar c}$$

B: For $T=0$, low frequency (solid lines for various M) there is a crossover to the infrared behavior. The self- energy dominates and the conductivity vanishes as

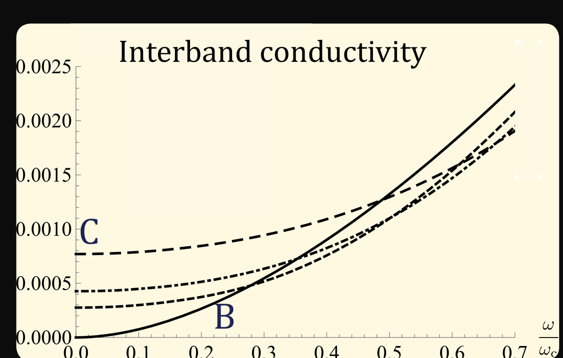
$$\mathrm{Re}\,\sigma_{xx}(\mathbf{0}, \omega) \sim \frac{e^2\omega_c}{12\pi\hbar c}\left(\frac{|\omega|}{\omega_c}\right)^{3-4M}$$

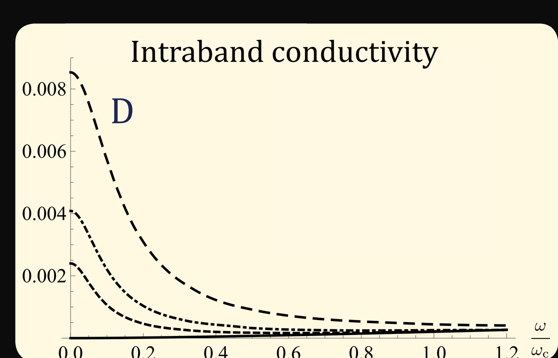
So it inherits the scaling of the critical field responsible for the interactions between the Dirac fermions.

C: For $T>0$ (dashed lines for various T) and vanishing frequency, the interband conductivity becomes a T-dependent constant:

$$\mathrm{Re}\,\sigma_{xx}(\mathbf{0}, \omega) \sim \frac{e^2\omega_c}{12\pi\hbar c}\left(\frac{k_B T}{\hbar\omega_c}\right)^{3-4M}$$

D: For $T>0$ and vanishing frequency, there is an upturn in the intraband conductivity, a Drude peak.





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Background images: Cd_3As_2 unit cell, figure adapted from [1]; Earth, credit: N. Kuring NASA GSFC, using data from the VIIRS instrument aboard Suomi NPP; Large Magellanic Cloud region near the Tarantula Nebula, credit: ESO.