# Quantum phases in an optical lattice

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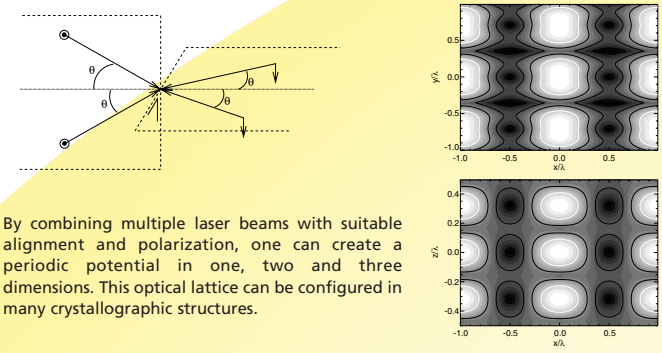
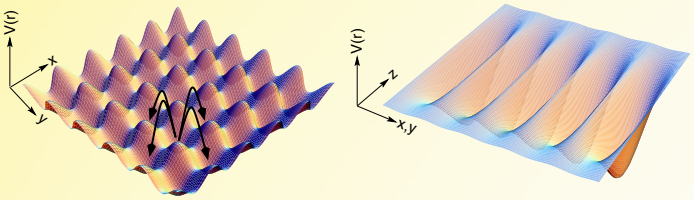
## Optical lattices




By combining multiple laser beams with suitable alignment and polarization, one can create a periodic potential in one, two and three dimensions. This optical lattice can be configured in many crystallographic structures.

## Bose Einstein condensation in one lattice site

In three dimensional lattices, the filling factor is of the order of one particle per site. This means we can assume that these particles are all in the harmonic oscillator ground state of the lattice sites. However in one or two dimensional lattices the filling factor is much higher, which means we have to calculate the properties of a Bose gas in each individual lattice site. Given the enormous aspect ratio of the trapping potential of one well, we have to treat these systems as low dimensional Bose gases.




## The Bose Hubbard Model

The Bose Hubbard model described a lattice Bose gas with a nearest neighbour hopping parameter t, an onsite interaction U and a chemical potential μ

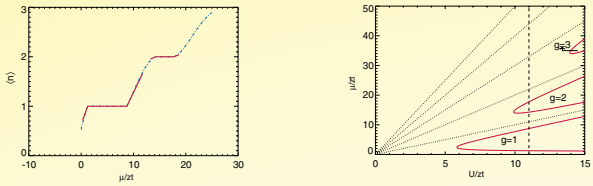
$$H = -t\sum_{\langle i,j\rangle} b_i^\dagger b_j + \frac{1}{2}U\sum_i b_i^\dagger b_i^\dagger b_i b_i - \mu\sum_i b_i^\dagger b_i$$

Introducing a real mean field order parameter $\psi = \langle b_i^\dagger\rangle = \langle b_i\rangle$ , we can reduce this to an onsite Hamiltonian

$$H_i = -zt\psi\left(b_i^\dagger + b_i\right) + \frac{1}{2}Un_i(n_i - 1) - \mu n_i$$

This model predicts a quantum phase transition from a superfluid phase to a Mott insulator phase. The superfluid phase is characterized by the fact that there is phase correlation between the sites, but no particle number correlation. The insulator phase has zero number fluctuations and lacks the phase correlations. An intuitive picture for this transition is the following. If we start with a lattice with an integer number of particles at each site and add one particle, the system does not care where we place this particle since the interaction energy at each site is the same. This means that in this case, the particle can move around freely. On the other hand, if we would try to move a particle in the original situation, this would cause an energy increase or decrease depending of the values of the hopping and interaction parameters. Now we can understand that if the system has a large interaction parameter, the number fluctuations due to hopping from one site to another will be suppressed.

## Perturbation theory



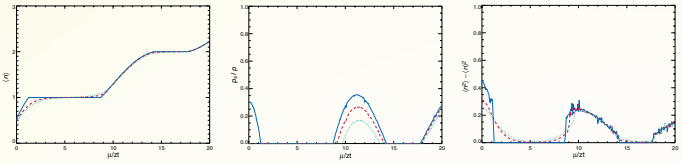

The onsite Bose Hubbard Hamiltonian can be numerically diagonalized with arbitrary precision. This yields the Mott insulator phase transition. To find an analytical expression for the location of the transition in the phase diagram, we have to resort to second order perturbation theory in the order parameter. For the quadratic term in the order parameter we find

$$H^{(2)} = \psi^2\left(1 + \frac{nzt}{U(n-1)-\mu} + \frac{(n+1)zt}{\mu - Un}\right)$$

By equating this term to zero, we find the Mott insulating lobes in the phase diagram shown below. Furthermore, using fourth order perturbation theory, we can find the density as a function of the chemical potential, which yields a typical density profile as shown below. We can also locate the tips of the lobes as shown in the phase diagram. These tips give us the critical value of the interaction for each value of the filling factor.

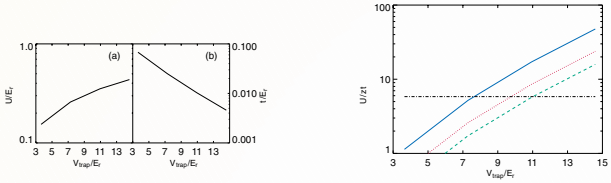
$$\left(\frac{U}{zt}\right)_{\text{crit}} = 2n + 1 + \sqrt{(2n+1)^2 - 1}$$

## Finite temperature




If we want to determine the finite temperature properties of bosonic atoms in an optical lattice we have to take into account to properties of the onsite wave function as a function of temperature and the properties of the Hubbard model. The change of the wave function with increasing temperature reflects itself only in a change of the interaction and tunneling parameters, which results in a shift in the phase diagram. The effect of a finite temperature in the Hubbard model is much more profound, as the Mott insulator transition is a real quantum phase transition and therefore it exists only at zero temperature. Above are plots of the density profile, the superfluid order parameter and the number fluctuations, calculated at T=0.1zt, 0.8zt and 1.0zt. It can be seen that the Mott insulator appears only at very low temperature, but that there is still a clear reduction in number fluctuations at higher temperatures.

## Experimental realization




For the connection between theory and experiment, we have to calculate the hopping and interaction parameters. We calculate the interaction parameter by integrating the density squared times a delta function interaction potential in terms of the triplet s-wave scattering length. The hopping parameter is calculated as a tight binding integral between two neighbouring lattice site. In both cases, we need to determine first the condensate wave function at each site. In three dimensions, the filling factor is at most a few atoms, so we can use the harmonic oscillator ground state. In lower dimensions, we determine the condensate wave function in the Thomas-Fermi limit.

The result of the calculation for a typical three dimensional lattice loaded with sodium atoms is plotted above as a function of the potential depth. Since the critical parameter is the interaction divided by the hopping, we also show a plot of this quantity.

### References

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