Quantum phase transition in an atomic Bose gas with a Feshbach resonance

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Introduction

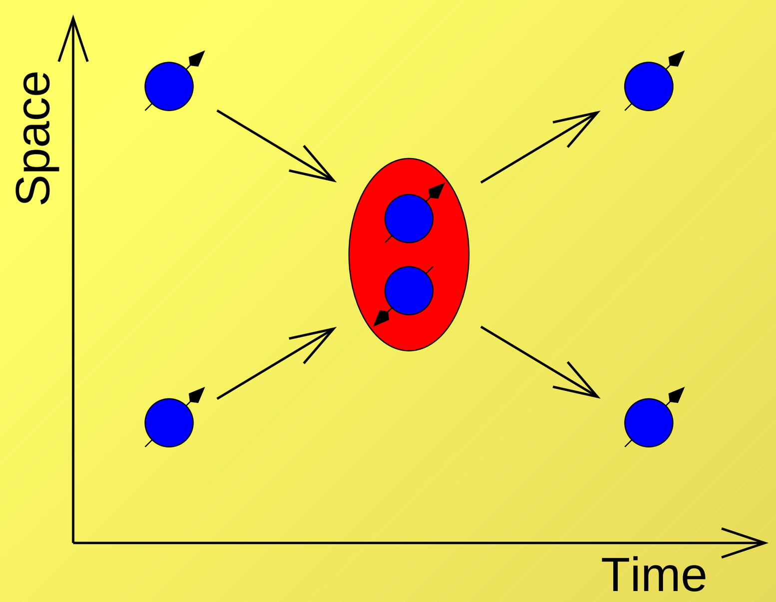
Feshbach resonances provide an experimentally accessible way to tune interactions and molecule formation in atomic gases. If used under low temperature circumstances, these techniques can lead to some interesting phenomena. For example, one can use Feshbach resonances to reach the BCS transition, or Bose-Einstein condensation of Cooper pairs, in trapped gases of fermionic atoms. Regal *et al.* were able to convert a fraction of fermionic atoms into diatomic molecules as a first step [1]. Another interesting application is the observation of coherent atom-molecule oscillations [2]. An atomic condensate is coherently coupled to a molecular condensate, which can be detected in a Ramsey experiment [3].

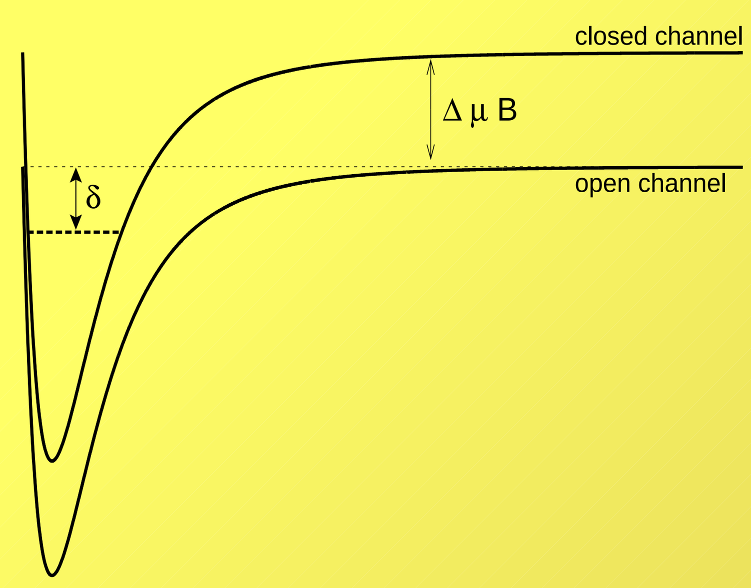
On this poster, we present work [4] that shows that near a Feshbach resonance, a quantum phase transition occurs between a phase with only a molecular Bose-Einstein condensate and a phase with both an atomic and a molecular condensate. We have shown that the transition is characterized by an Ising order parameter, and determined the phase diagram of this transition.

Feshbach resonances

The essence of a Feshbach resonance is that there is a resonance in the scattering process of two particles, that is due to a long-lived bound state. This state has a binding energy close to the energy of the incoming particle. Also, the bound state exists in another part of the quantum-mechanical Hilbert space than the part associated with the incoming particles. In the simplest case, these two parts of the Hilbert space are referred to as the closed and open channel, respectively.

Two incoming atoms can form a long-lived bound state (a molecule) during a collision. The two incoming atoms are said to be in the open state, while the bound atoms have a different hyperfine state, and are said to be in the closed state. Due to their different hyperfine state, the two channels have a different Zeeman shift $\Delta\mu B$ in a magnetic field.





This is an illustration of a Feshbach resonance for alkali atoms. The upper potential curve corresponds to the closed-channel interaction potential $V_{\downarrow\downarrow}(\mathbf{x}-\mathbf{x}')$ that contains the bound state responsible for the Feshbach resonance, indicated by the dashed line. The lower potential curve corresponds to the open-channel interaction potential $V_{\uparrow\uparrow}(\mathbf{x}-\mathbf{x}')$.

Atoms and molecules

The coupling between atoms and molecules in the gas is provided by an interaction energy that is proportional to $g\int d\mathbf{x}\ \left(\psi^\dagger_{\rm m}(\mathbf{x})\psi_{\rm a}(\mathbf{x})\psi_{\rm a}(\mathbf{x}) + \psi^\dagger_{\rm a}(\mathbf{x})\psi^\dagger_{\rm a}(\mathbf{x})\psi_{\rm m}(\mathbf{x})\right)$, where $\psi_{\rm a}(\mathbf{x})$ and $\psi_{\rm m}(\mathbf{x})$ annihilate an atom and a molecule at position $\mathbf{x}$, respectively. This implies that if the gas contains an atomic Bose-Einstein condensate, and therefore has a nonzero value of $\langle\psi_{\rm a}(\mathbf{x})\rangle$, the gas must necessarily also contain a molecular Bose-Einstein condensate and have a nonzero value of $\langle\psi_{\rm m}(\mathbf{x})\rangle$. However, the reverse is not true and it is possible for the gas to contain only a molecular Bose-Einstein condensate. Thus

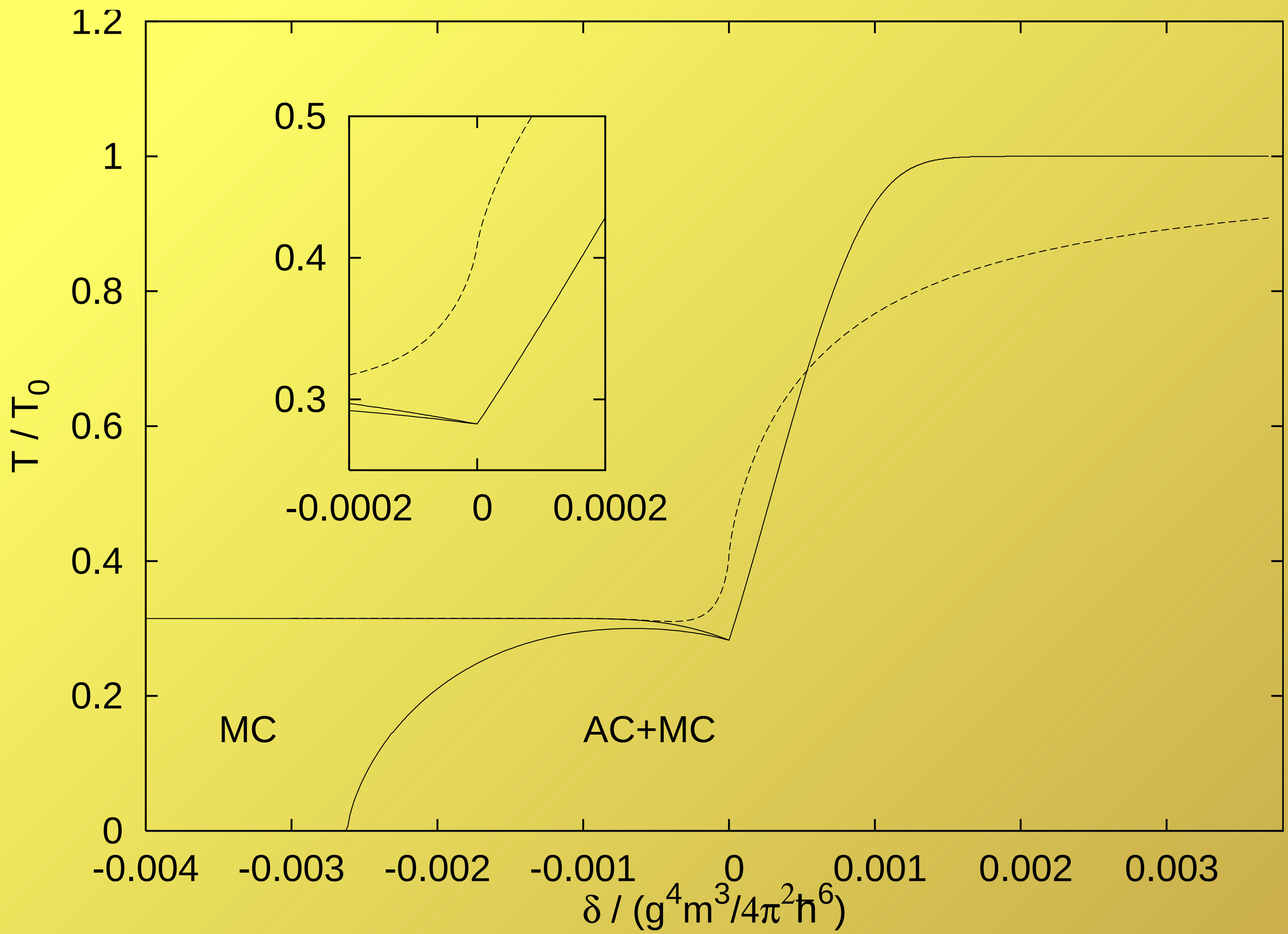
$$\langle\psi_{\rm a}(\mathbf{x})\rangle \neq 0 \Rightarrow \langle\psi_{\rm m}(\mathbf{x})\rangle \neq 0$$

Three phases

So we expect two possible symmetry-broken phases. In the **normal phase**, the gas is invariant under the phase transition

$$\psi_{\rm a}(\mathbf{x}) \to e^{i\theta}\psi_{\rm a}(\mathbf{x}), \qquad (1)$$
$$\psi_{\rm m}(\mathbf{x}) \to e^{2i\theta}\psi_{\rm m}(\mathbf{x}) \qquad (2)$$

If the gas contains an **atomic and a molecular condensate** (AC+MC), this $U(1)$ symmetry is completely broken. Then, there can also be a **molecular condensate** (MC), where a residual discrete symmetry remains, because $\langle\psi_{\rm m}(\mathbf{x})\rangle \to \langle\psi_{\rm m}(\mathbf{x})\rangle$ for $\theta = \pi$. This phase therefore only breaks the $U(1)/Z_2$ symmetry spontaneously. Between the AC+MC and MC phases there must exist an Ising-like transition, breaking the residual Z_2 symmetry. The full phase diagram is shown in this figure.



For large positive detuning δ, molecules have an energy that is far higher than the treshold of the two-atom continuum. Our gas consists of atoms only, and the critical temperature for an ideal gas at which Bose-Einstein condensation takes place is

$$T = T_0 = (2\pi\hbar^2/mk_{\rm B})(n/\zeta(3/2))^{2/3} \qquad (\delta \gg 0)$$

For large negative detuning, the molecular energy lies far below the treshold of the two-atom continuum. We expect the gas to consist solely of stable molecules, that condense at a temperature

$$T = T_0/2^{5/3}$$

Finite lifetime effects

In the initial calculations we have not included the effects of the finite lifetime of the molecules at positive δ, or the rogue-dissociation process (molecular dissociation into thermal atoms) [5] for negative δ. Especially for positive detuning, this makes a big difference, as it decreases the average energy of the bound state significantly, making them more accessible for the system. For the critical T for BEC, these effects are included in the dashed line in the figure. In the future we would like to include the effects of rogue dissociation on the critical line of the Ising transition.

References

[1] C.A.Regal, C.Ticknor, J.L.Bohn, and D.S.Jin, Nature **424**, 47 (2003).

[2] E.A.Donley, N.R.Claussen, S.T.Thompson, and C.E.Wieman, Nature **417**, 529 (2002).

[3] R.A.Duine and H.T.C.Stoof, cond-mat/0312254.

[4] M.W.J. Romans, R.A. Duine, Subir Sachdev, H.T.C. Stoof, cond-mat/0312446.

[5] M.Mackie, K.-A.Suominen, and J.Javanainen, Phys. Rev. Lett. **89**, 180403 (2002).