Stochastic Dynamics of a Trapped Bose-Einstein Condensate

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Nonequilibrium dynamics

A Bose-Einstein condensate is at nonzero temperatures described by the Langevin equation [1,2]:

$$i\hbar\frac{\partial \phi(\vec{x},t)}{\partial t} = \left[\frac{-\hbar^2\nabla^2}{2m} + V^{ext}(\vec{x}) - \mu - iR(\vec{x},t) + T^{2B}|\phi(\vec{x},t)|^2\right]\phi(\vec{x},t) + \eta(\vec{x},t)$$

Dissipation:
Condensate growth or evaporation from the thermal cloud.

Fluctuations:
Thermal fluctuations due to incoherent collisions.

The strength of the gaussian noise is determined by the Keldysh self-energy:

$$\langle \eta^*(\vec{x}',t')\eta(\vec{x},t)\rangle = \frac{i\hbar^2}{2}\Sigma^K(\vec{x},t)\,\delta(\vec{x}'-\vec{x})\,\delta(t'-t)$$

The microscopic expressions for the Keldysh self-energy and the dissipation take the form of collision integrals. However, if we assume the thermal cloud to be sufficiently close to equilibrium we can relate the strength of the fluctuations to the dissipation by means of the **fluctuation-dissipation theorem:**

$$iR(\vec{x},t) = -\frac{\beta}{4}\hbar\Sigma^K(\vec{x},t)\left[\frac{-\hbar^2\nabla^2}{2m} + V^{ext}(\vec{x}) - \mu + T^{2B}|\phi(\vec{x},t)|^2\right]$$

The fluctuation-dissipation theorem ensures relaxation of the system towards the correct physical equilibrium probability distribution.

Variational approach

For a harmonic external trapping potential, we take for the condensate wave function a gaussian variational *ansatz*:

$$|\phi(\vec{x},t)| = \sqrt{N_C(t)}\prod_j\left(\frac{1}{\pi q_j^2(t)}\right)^{1/4}\exp\left[-\frac{\vec{x}^2}{2q_j^2(t)}\right]$$

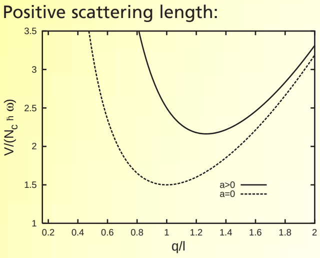
This results in Langevin equations for the variational paramaters $q_j(t)$, which are equivalent to the equations of motion for a Brownian particle with effective mass $m_{eff} = 1/2mN_C$ in a potential $V(q,N_C)$:

$$\frac{1}{2}mN_C(t)\ddot{q}_j(t) + \frac{N_C(t)\beta}{4}i\hbar^2\Sigma^K(t)\frac{\dot{q}_j(t)}{q_j^2(t)} = -\frac{\partial V(\vec{q}(t),N_C(t))}{\partial q_j} + \frac{\sqrt{2N_C(t)}}{q_j(t)}\xi_j(t)$$

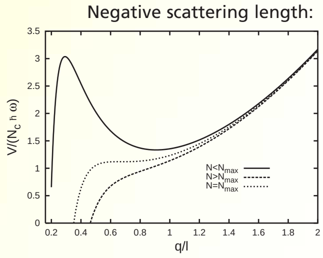
The potential $V(q,N_C)$ is given by

$$V(\vec{q},N_C) = \sum_j\left(\frac{N_C\hbar^2}{4mq_j^2} + \frac{1}{4}mN_C\omega_j^2q_j^2\right) + \frac{a\hbar^2N_C^2}{\sqrt{2\pi}\,mq_xq_yq_z}$$

Positive scattering length:



Negative scattering length:



-A metastable condensate is possible only for $N_C < N_{max} \approx 1470$ in the Gaussian approximation.

-The condensate can collapse by overcoming the macroscopic energy barrier.

The Langevin equations for $q_j(t)$ are coupled to a stochastic rate equation for the number of atoms in the condensate:

$$\frac{dN_C(t)}{dt} = -\frac{\beta}{2}i\Sigma^K(t)[\mu_C(t)-\mu]N_C(t) + 2\sqrt{N_C(t)}\,\eta(t)$$

The parameter $\mu_C(t)$ is the chemical potential of the condensate in the gaussian approximation. The strength of the noise $\xi_j(t)$ and $\eta(t)$ is such that the condensate relaxes to the correct physical equilibrium probability distribution.

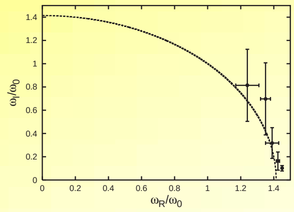
References
[1] H.T.C. Stoof, Phys. Rev. Lett. **78**, 768 (1997).
[2] H.T.C. Stoof, J. Low. Temp. Phys. **114**, 11 (1999).
[3] D.S. Jin, M.R. Matthews, J.R. Ensher, C.E. Wieman, and E.A. Cornell, Phys.Rev. Lett. **78**, 764 (1997).
[4] J.M. Gerton, D. Strekalov, I. Prodan, R.G. Hulet, Nature **408**, 692 (2000).

Applications

1. Collective modes

Using the Langevin equations for the variational parameters and the stochastic rate equation for the number of atoms in the condensate we calculate the frequencies and collisional damping rates of the low-lying collective excitations as a function of temperature. Using these equations we find that the complex frequencies of the quadrupole mode lie on a circle in the complex ω-plane:

$$|\omega_{quad}| \cong \sqrt{2}\,\omega_r$$

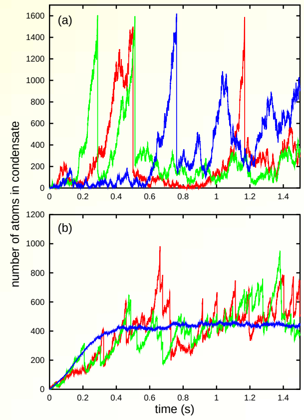


The complex ω-plane.

where ω_r is the radial frequency of the trapping potential. Plotting the experimental datapoints of Jin *et al.*[3] in the complex frequency plane shows that they lie indeed on a circle.

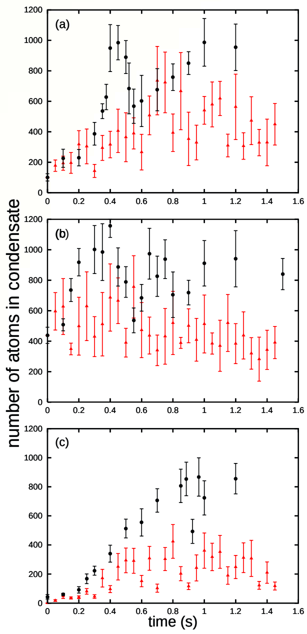
2. Condensate growth and collapse

Using our Langevin equations and the stochastic rate equation for the number of atoms in the condensate we describe the growth of a ^7Li condensate, for the experimental conditions as realized in the experiments of Gerton *et al.* [4]. We model the collapse by putting the number of atoms equal to *200±40* once a collapse is initiated. The solutions of the stochastic equations show that the average is strongly dependent on the number of individual runs one averages over:



(a) Growth collapse curves of a ^7Li condensate, and (b) their averages. The colored lines in (a) display the number of condensate atoms for the solutions of the stochastic equations for different realizations of the noise. In (b) the red line corresponds to an average over 5 realizations, the green line to 10, and the blue line to an average over 1000 realizations of the noise.

To simulate the experiments one has to obtain each datapoint by averaging over different runs, with the same initial conditions.



Simulations of the experiments performed by Gerton *et al.*[4]. The results of the simulations are denoted by red triangles, the experimental data is shown as black circles. In (a) and (b) each data-point of the simulations is an average over 5 runs, as in the experiments. For (c) 10 runs per point were done. The errorbars denote the uncertainty in the mean.

Conclusions

The variational solution of the Langevin field equation that describes the nonequilibrium dynamics of a Bose-Einstein condensate is a powerful tool for studying many interesting problems. Apart from the applications discussed here, the treatment of the dissipative dynamics of topological excitations such as vortices and skyrmions becomes also feasible within this method.