Abstract

In [3], the group of Trapani et al. performed experiments, showing the possibility of spontaneously generated X-shaped light pulses. Their formation is shown to depend on linear and nonlinear effects. The most important feature of those light bullets is their self-organising, stable spatiotemporal confinement. The claim of having them found is corroborated by various experiments as well as theoretical and numerical investigations.

1 Introduction

Laser pulses play an important role in many applications in research but also in daily life. Usually, Gaussian pulses are produced and used because they can be generated in a natural way from our diffraction optics. In vacuum, an ideally shaped Gaussian beam would show no spacial divergence, because the dispersion relation is homogeneously given by $c = \omega/k$, which sets both, phase velocity and group velocity to the speed of light $c$. In a medium, the wave vector $\vec{k}$ depends on the frequency $\omega$ which gives rise to spacial and temporal divergence of wave packets - they get broader with time. Another issue is chirping, a nonlinear effect which causes the basic frequency $\omega_0$ in a wave packet to change over its width. Very short Gaussian pulses with high intensities are not stable in shape though.

In the past years, several approaches to overcome those problems were found. Wave packets which are localized in time and space are therefore called light bullets because their movement and even interaction can be regarded similar to that of particles. Now in their paper [3], the group of Trapani, Valiulis et al. claim to have found a way to produce special light bullets in a self focusing way, which is a new feature. The underlying principles are based on both, linear and nonlinear effects. I will outline their argumentation, connected with a rough overview over recent developments and have a critical look at their methods.

2 Previous known facts and experiments

2.1 Light bullets, based on linear material properties

It has been shown, that under certain circumstances, there exist spatiotemporal localized solutions of the Maxwell equations for wave packets. Those are only based on the linear material constants and the group velocity dispersion (GVD). To show this, consider the following wave equation:

$$\hat{L}(\omega_0) E = 0$$

$$\hat{L}(\omega_0) = i\partial_z + \frac{1}{2k_0} \nabla^2_\perp - \frac{k''}{2} \partial_t^2$$

(1)

with $\nabla^2_\perp = \partial^2_r + r^{-1} \partial_r$. The equation describes propagating wave packets with cylindrical symmetry, since in the standard form of the electromagnetic wave equation, there is also $\theta$ dependence through a non-transverse part of the Laplacian $\nabla^2$. If we now assume a slowly varying envelope function $E = E(r, t, z)$, a wave packet has the standard form $E(r, t, z) \cdot \exp[i(k_0 z - \omega_0 T)]$, where we are in a barycentric frame which transforms the time to $T \mapsto t = T - k' z$. For a standard plane wave, this yields the following differential equation for the underlying dispersion relation:

$$\frac{dk}{d\omega} = \frac{k_0}{\omega_0} - \frac{\omega_0}{2} \frac{d^2 k}{d\omega^2}$$

(2)

Note that equation 1 only describes wave propagation in positive $z$ direction because of the $i\partial_z$ term. The general idea is now to concentrate on the envelope function $E(r, t, z)$. A Fourier transformation of 1 yields the following result (RHS is divergent):

$$\omega k' - \frac{k^2}{2k_0} + \frac{\omega_0^2}{2} = -\int_0^{\infty} dr \frac{ik}{2k_0 r} \exp(ikr)$$

(3)

Trapani et al. bring it to a slightly different form:

$$k' \Omega^2/2 - K^2/(2k_0) = \beta$$

(4)
This looks like fig. 1(a) when plotted. Apparently one can show that superpositions of cylindrical Bessel functions indeed give stable solutions if GVD is normal, that is $k'' > 0$. Those not further specified superpositions build the name-giving X-shape in space time, which means that there is an intensity maximum in the middle and conically decaying edges, which look a bit like an hourglass. In this context, this is remarkable as Bessel functions $J_0(x)$ decay slower than exponential functions.

### 2.2 Light Bullets, based on nonlinear effects

At high intensities, nonlinear effects have to be taken into account. It turns out, that the Kerr lensing effect can in principle also be applied to the temporal structure of a wave packet. In a barycentric wave packet, time and space are due to eq. 1 very similar because they only enter the equation through second derivatives. So the general idea of having a pulse focused in time and space by the Kerr effect is not difficult to understand, but it gets more involved very soon if one wants to look at the details. In two dimensions, it works. In three dimensions, no stable configuration has been found yet. In general, it is required to have anomalous GVD ($k'' < 0$) in this case, which is exactly the opposite case than in the linear X-wavelet regime. Another difference is, that Kerr lensing will produce bell-shaped pulses, which look quite different than X-waves. So in principle, three dimensional light bullets, based only on nonlinear effects should be possible, yet they haven’t been observed so far.

### 3 Spontaneous light pulse compression

Trapani et al. claim to have found spontaneously forming X-wave packets in a lithium triborate (LBO) $\chi^{(2)}$ crystal under nonlinear conditions. Their setup was strikingly simple, consisting of an LBO crystal and a pulsed laser with Gaussian profile. The pulses, sent to the crystal, had a central frequency in the infrared, the wavelength was $\lambda_0 = 1060\,\text{nm}$. Dimensions of the initial wave packets were $45\,\mu\text{m}$ in width and $100 - 200\,\text{fs}$ in duration (both FWHM).

By choosing the energy of the pulse, they were able to control if physics were in the linear or nonlinear regime. They found the threshold for the nonlinear regime to be at about $0.25\,\mu\text{J}$. After passing the $22\,\text{mm}$ long crystal, the pulses were compressed to a length of $20\,\text{fs}$ and a fraction of their initial diameter in the nonlinear case. Measurements were performed with usual CCD sensors, sensitive to light intensity (integral measurement). Results in fig. 2 clearly show, that diffraction is defeated and the pulse gets focused.

![Figure 1. (a) The characteristics of the Fourier transformed dispersion relation. Stable wave packets occur only on the bold lines. Those are plotted in (b) and (c).](image1)

![Figure 2. Experimental results, comparing a pulse which passed the crystal below and above the threshold of 0.25\,\mu\text{J}. Top: The pulse, after it passed the LBO crystal in the linear (left) or nonlinear (right) regime. The transverse beam profiles in the linear (white) and nonlinear (green=SH and red=FF) case. On the bottom, the temporal auto correlation is shown for the linear (red) and nonlinear (green) case.](image2)
and compressed while travelling through those 22 mm of LBO if it is in the nonlinear regime.

This is very remarkable, since (a) there was no nonlinear effect known until then which could explain this and (b) the process seems to be self organizing. From the previous two sections it is clear, that Kerr lensing cannot produce this effect because of instabilities and linear X-waves are excluded from an explanation because they do not form spontaneously but need to get prepared very carefully. Another important detail needs to be reviewed: second harmonic generation (SHG). LBO crystals show strong SHG at the mentioned frequencies and energy. So instead of having a pulse, only with the fundamental frequency (FF) corresponding to 1060 nm, there is a second important contribution. Because of GVD and the mismatch in group velocity (GVM), caused by the huge frequency difference, the SH pulse should run ahead of the FF pulse. Surprisingly, this doesn’t happen, and both frequencies “stay together” to form a uniform pulse, as can be seen in fig. 2 in the autocorrelation.

In that moment, two things are to be done: (a) show that this pulse compression has indeed X-shape and (b) find a theoretical explanation.

4 Computational and theoretical support

The theoretical existence of nonlinear X-waves was shown by Trapani et. al in an earlier paper (see [1]), so it is obvious to start some observations in this direction. Expected were shapes like in fig. 3, which clearly show X-wave features. The aim is to find a possible scenario to explain the absence of pulse splitting due to FF and SHG, and the strong self focusing in time and space, as well as to find out about the form of the pulses. A set of two nonlinear equations is proposed, which focuses on the envelope functions and includes both frequencies and frequency mixing:

\[ \hat{L}(\omega_0)E_1 + \chi E_2 E_1^* \exp(-i \Delta k z) = 0 \]

\[ \hat{L}(2\omega_0)E_2 + \delta V \partial_t E_2 + \chi^{(2)} E_1^2 \exp(i \Delta k z) = 0 \]  

(5)

Thereby \( \delta V \) is the GVM between the FF \( \omega_0 \) and the SH \( 2\omega_0 \), and \( E_i \) are the envelope functions. In numerical experiments, they showed a very strong reshaping in the curse of time, see fig. 4. Here, the X-shape is again to be regarded as an hourglass shape in space and time. Some more detailed calculations gave support to the conjecture that this formation is spontaneous and directed (see fig. 5), and does not switch back to its initial state. Nonlinear parameters were chosen such, that they fit the properties of an LBO crystal best but nevertheless, they were chosen constant. The main conclusion to draw from those simulations is that they are able to provide verification of the compression and reshaping of Gaussian wave packets under the mentioned circumstances. It bridges the difference between linear X-waves...
and nonlinear (though: not yet observed) light bullets by showing that both contributions (linear and nonlinear) are necessary to form nonlinear X-waves by eqns. 5. There are also other arguments, pointing into the direction of an hourglass like structure of the pulses, connected with a linear stability analysis.

5 Experimental verification

Compression of Gaussian pulses in LBO is shown sufficiently by the experiments, mentioned above. If one measures the occurring divergence angle of the pulse after it leaves the crystal - which is easily done by moving the detector a little - one observes, that the beam divergence is about 2.5 times smaller than it would be expected from a Gaussian beam profile. This smaller angle of diffraction can on the other hand very well be described by Bessel-like functions, which are (as mentioned in part 2.1) substantial to get at least linear X-waves, so one can count this as support for the nonlinear X-wave thesis.

To get sure about the actual shape of the pulse, the group performed tomography experiments, based on cross correlation measurements to directly map the shape of the light bullets. The results agreed very well with those from the simulation, which verifies the approach (see fig. 6).

![Figure 6. Tomography of the pulse, measured by a cross correlation method in an BBO crystal. Shown are (a) intensity profiles of the measurement with a transverse inset at different times and (b) a comparison with simulations. Both agree very well and prove the X-shape of the light bullets.](image)

6 Conclusions and criticism

The team of Trapani, Valiulis et al. provided experiments to show that they were the first to find spontaneously generated X-shaped light pulses, called light bullets. Since stable light bullets would provide many opportunities in scanning microscopy methods or data transfer, they are in the focus of interest. The group provided convincing data showing that pulses in LBO crystals compress in space and time, and that those reshaped pulses are not of Gaussian nature because of divergence properties. They possibly have the shape of an hourglass, which makes them X-waves in nonlinear materials. With this work, they proved experimentally, what they had found out in theory earlier (see [1]) - those results are really new and promising, especially because of the self organisng features. However, later publications until this moment only rarely got back to those nonlinear X-waves, but rather concentrated on photonic solitons (see for instance the review [2]).

There are some points in the argumentation which are quite unusual. Their theory parts use a very vague way of arguing and are thus far from clear or convincing. For instance, eq. 1 was told to be the standard one, which is trivially not true, because the standard wave equation in electrodynamics is simply $\Box \vec{E} = 0$, with the d’Alembertian $\Box = c^{-2} \partial_{tt} - \nabla^2$.

It remains unclear, if their basic equation was the nonlinear Schrödinger equation or some modified electrodynamics. The performed numerical experiments are not trustworthy without any discussion of their methods. They didn’t mention, what sort of simulation they performed (e.g. FDTD, FEM, MMP, …), how they measured and how trustworthy themselves find the results. In general, the outcome of their simulations is very hard to verify because there are almost no concrete quantities given. The same holds for the explanations taken from theoretical physics or mathematics to corroborate their thesis; it is hard to tell if they are correct or how exactly they work.

References

