On One Mechanism Forming Linear Sand Banks

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Asymmetric tidal currents (Huthnance, 1973) provide a fluid-dynamical basis for Caston's (1972) description of linear sand-bank maintenance by converging sand transport. We suppose (i) depth-uniform tidal currents, slightly inclined to the bank crest, (ii) bottom-drift, which retards the current more over the bank and (iii) a faster-than-linear increase of sand transport with current. Then over a sloping bank side the total tidal current having an upslope component and the associated onto-bank sand transport are stronger than the retarded reverse tidal current and transport coming off the bank.

Supposing that (iv) sand is more easily transported 'downhill' short-wavelength perturbations on a level sea floor are suppressed. There is a maximum bed-form growth rate at a particular wavelength (typically 250 times the water depth) and orientation (relative to the tidal currents) which probably evolve and persist during subsequent sand-bank growth. The orientation is sensitive to the (uncertain) formulation of supposition (iv), and is probably also susceptible to (for example) the trend of an adjacent coastline.

In the representative context of friction-dominated tidal currents, the banks evolve to an equilibrium profile which is flatter on top than a sinusoid owing to wind-wave erosion and the inclination to the tidal current. For a limited sand supply the banks narrow to about one-fifth of their separation; further restriction mainly reduces their height. A net sand-transport overall due to a stronger ebb tide (say) than flood, as occurs over the Norfolk Sandbanks, yields the observed steeper slope on the obliquely downstream side of the bank (as viewed by the stronger ebb current).

1. Introduction

Tidal current ridges or linear sand banks are quasi-periodic bed-forms roughly aligned with the tidal currents and occurring widely on continental shelves (Off, 1963), apparently where tidal currents are strong and sand is in good supply. They are large—up to 80 km long, typically 1 to 3 km broad and tens of metres high—usually a large fraction of water depth. Their dimensions and spacing (several kilometres) vary between examples but tend to increase together (Allen, 1968).

It is natural to enquire how such remarkable features arise and persist, particularly in the \emph{a priori} hostile conditions of strong turbulent currents in shallow seas subject to wind-wave
action. An understanding of sand-bank evolution is important to navigation (how often should they be surveyed?) and as evidence regarding (for example) their age, overall sediment transport and earlier sea levels.

Their quasi-periodicity suggests a bed-form instability, although such a phenomenon appears to have been investigated (and confirmed) only for shorter-scale cross-stream forms (Richards, 1980); Kennedy (1969) assumed irrotational flow and an unknown lag of the sediment transport behind the instantaneous current; Furnes (1974) further included the dynamical effect of stratification owing to suspended sediment. Off (1963) and Houbolt (1968) suggested, without dynamical explanation, that linear sand banks might be maintained by secondary vertical circulations in the transect plane, with up-slope currents near the sea bed and compensatory surface currents away from the banks. No such circulation has been measured in currents near linear sand banks (McCave, 1979). Caston (1972) used detailed sand-wave and current observations over the Norfolk Sandbanks (Caston & Stride, 1970). He found that the strongest tidal currents and sand transport are primarily parallel to the ridge (and clockwise around it), but on the bank itself turn progressively towards the crest in the shallower water, thereby tending to maintain the bank. The bank is typically asymmetric, with the steeper side facing obliquely towards the direction of net sand transport. These findings have been extended by Kenyon et al. (1981) to other sand banks in the southern North Sea.

This paper gives fluid-dynamical support for Caston’s (1972) description. The shallow-water equations of motion for fluid of depth $h(y)$ are formulated in section 2, together with a sediment transport formula favouring down-slope transport, thereby following Richards (1980) but omitting the transport lag assumed by Kennedy (1969) for shorter-scale bedforms. In section 3 we consider growth rates for sinusoidal bed-forms of small amplitude; in any given context the orientation and spacing exhibiting the fastest growth should predominate in time and persist in the large-amplitude features which subsequently develop. Section 4 considers the end of this process, a final equilibrium profile, particularly in the most easily analysed and apparently representative context of friction-dominated currents. We model the effects on the profile of wind-wave action, net sand-transport through the system and limited sand supply. Evolution from the initial growth phase to the final equilibrium is illustrated by examples in section 5.

There is uncertainty regarding precise algebraic forms for bottom frictional drag and in particular the sand transport as modified over a sloping bottom. Hence we have felt free to make simplified analyses in order to obtain analytical solutions where possible (checking that qualitative conclusions are robust); we do not pursue quantitative accuracy with numerical calculations using any supposedly definitive drag and transport laws.

2. Formulation

(a) Currents

The flow is essentially as described in Huthnance (1973). We consider (Figure 1) homogeneous fluid of density $\rho$ between the sea floor $z = -h(y)$ and the undisturbed surface $z = 0$, and apply the shallow-water equations commonly used for tidal currents and discussed by Heaps (1978). The depth-uniform horizontal velocity $\mathbf{u} = (u(y,t), v(y,t))$, independent of the coordinate $x$ along the banks, results from an externally imposed uniform rectilinear tidal current far from the sandbanks (or before they develop) where $h = H$(uniform). This uniform ‘far’ current is $UI(t) (\cos a, \sin a)$ where the scale factor $U$
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Figure L Definition of axes and topography.

is its peak magnitude, $I(t)$ is its time dependence ($-I \leq I \leq 1$) and $(\cos \alpha, \sin \alpha)$ is its direction, inclined at an angle $\alpha$ to the depth contours.

On the short length scales (1 km) considered here, the rise and fall of the surface plays only a small role in mass conservation, which with $u$ and $h$ independent of $x$ simplifies to

$$
 hv = HUI \sin \alpha,
$$

expressing constancy of mass flux across the parallel depth contours. Hence $v$ is known.

The momentum equation for the velocity component $u$ parallel to the depth contours is

$$
 \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} - fu + C_m |u|^{m-1} \frac{u}{h} \\
= -\rho^{-1} \frac{\partial p}{\partial x}
$$

$$
= U \cos \alpha \frac{dI}{dt} - fUI \sin \alpha + C_m U^n |I|^{m-1} \frac{I \cos \alpha}{H}
$$

since the $y$-momentum equation gives $\omega = \partial / \partial x (\partial p / \partial y)$: the pressure gradient $\partial p / \partial x$ is independent of $y$ and may be replaced by its large-$y$ value. We have included an $n$th power bottom drag

$$
 \rho C_m |u|^{n-1} u \text{ per unit area}
$$

($m = 2$ is widely favoured), and $f$ is the Coriolis parameter, $t$ time.

Let $T$ be the tidal period and $L$ the spatial period of $h$ in $y$. We non-dimensionalize

$$
 u, v, h, t, y, f
$$

against

$$
 U, U, H, \frac{T L}{2\pi}, \frac{L}{2\pi}, \frac{2\pi}{T}
$$
and define non-dimensional parameters:

\[ C = C_n U^{m-2} \approx 0.025, \]

\[ \varepsilon = \frac{UT}{L} \] (relating the typical tidal excursion \( O(UT) \) of the water to the topographic length scale \( L \)),

\[ F = \frac{CUT}{2\pi H}. \]

Substituting for \( v \) from (2.1), (2.2) for \( u \) becomes

\[ \frac{\partial u}{\partial t} + \frac{dI}{dt} \cos a + \varepsilon l h^{-1} \sin a \frac{\partial u}{\partial y} - Fl \sin a (h^{-1} - 1) \]

\[ = F[I]^{m-1} I \cos a - Fh^{-1}[u^2 + I^2 h^{-2} \sin^2 a]^{m-1} u. \] (2.3)

This completely determines the flow given that \( u = I \cos a \) in large \( y \) or if \( h = 1 \).

This analysis for depth-uniform currents cannot represent (nor refute) the disputed secondary circulation in a vertical plane suggested by Houbolt (1968). Depth-uniformity is associated with gentle bottom slopes, suggesting that any secondary circulation should be relatively small, scaling with the slope. Vigorous tidal mixing through the vertical also inhibits secondary circulations.

(b) Sand transport

We usually assume a dimensional form

\[ \frac{S \sqrt{u^3}}{g} |u|^n (u, v + \lambda |u| \frac{dh}{dy}) \] (2.4)

for the sand transport rate (volume/unit width/time). This incorporates

(i) a dimensional scale factor \( S\sqrt{u^3}/g \) which is fixed by the given values of \( U \) and gravitational acceleration \( g \). \( S \) is a non-dimensional coefficient such that \( S\sqrt{u^3}/g \) scales the sand transport rate. The rest of (2.4) has been non-dimensionalized as above and is \( O(1) \);

(ii) a simple \((n+1)\)-power law \( |u|^{n+1} \) in the temporally and spatially varying current \( u \). Values \( n = m \geq 2 \) correspond to a \( |u|^3 \) law for bedload transport. Then the formulae of Rottner (1959) and Ackers & White (1973) agree on \( S \approx 0.65C \) for sand if \( |u| \) greatly exceeds the threshold for sediment motion, whilst Bagnold's formula as evaluated by Gadd et al. (1978) for offshore flows gives \( S \approx 2.5C \) for 450 \( \mu \) sand typical of banks. Finer sand may result in a significant suspended load and a larger value of \( n \);

(iii) a basic transport direction parallel to the flow \((u, v)\);

(iv) an enhancement factor \( 1 + \lambda dh/dy \) for the down-slope component of transport. This is an uncertain extension to two horizontal dimensions of Bagnold's (1956) factor \((1 - \lambda dh/dy)^{-1}\) for the enhancement of down-slope bed-load transport. Allowing for the scaling of \( h \) and \( y \),

\[ \lambda \approx 2\pi H[L \tan (\text{friction angle})], \]

so \( \lambda \) is small. If a large proportion of the transport is as suspended load which is not enhanced down-slope, the effective value of \( \lambda \) is reduced. Richards (1980) used the better-accepted one-dimensional form when considering transverse bed-forms.

A measure of the uncertainty in extending Bagnold's factor to flows which are not parallel to the slope is the alternative transport rate

\[ \frac{S \sqrt{u^3}}{g} |u|^n (u, v + \lambda |u| \frac{dh}{dy}) \]
which results from Bagnold's work-rate considerations if we assume that the transport is in the direction of total sea-bed stress (due to the current and a gravity component).

A sea-floor slope may further influence sand transport rates via a reduction in the threshold velocity for sediment motion by a factor

$$
(1 - \lambda v |u|^{-1} \frac{dh}{dy})
$$

if the current flows down the slope. Including this effect alone (i.e. omitting (iv) above) results in a transport rate

$$
\frac{SU^3}{g} \left[ |u| - V \left( 1 - \lambda v |u|^{-1} \frac{dh}{dy} \right) \right]^n u
$$

(2.5)

which parallels (2.4) except that the non-dimensionalized threshold velocity $V$ for sediment motion on a level bed is included explicitly (c.f. Ackers & White, 1973) and down-slope currents enhance the transport through the reduced threshold. Since the work rate concept leading to (2.4) neglects the threshold velocity $V$, it is not clear how (2.4) and (2.5) should be combined to take account of both factors. In one horizontal dimension (2.5) is barely distinguishable from (2.4) and is therefore employed to test the robustness of conclusions to the uncertain transport form, particularly as modified by the slope in two dimensions.

(c) Bed-form evolution

The depth increases if there is a net divergence of sand-transport. Thus by (2.4) conservation of sand requires

$$
(1 - \rho) \frac{\partial h}{\partial t} = \frac{SU^3 T}{g H L} \left[ \frac{\partial}{\partial x} \left( |u| \right) u + \frac{\partial}{\partial y} \left( |u| \right)^n \left( v + \lambda |v| \frac{dh}{dy} \right) \right],
$$

where the factor $SU^3 T |g H L$ results from the scaling and non-dimensionalization. Since $|u|$ and $u$ are independent of $x$, and we can substitute for $v$ from (2.1),

$$
\frac{\partial h}{\partial t} = \frac{SU^3 T}{(1 - \rho) g H L} \frac{\partial}{\partial y} \left[ |u|^2 + I h^{-2} \sin^2 \alpha \left( I h^{-1} \sin \alpha + \lambda h^{-1} I \sin \alpha \frac{\partial h}{\partial y} \right) \right],
$$

(2.6)

where $\rho$ is the porosity of the bed and $u$ is determined by (2.3). Hence the evolutionary timescale is $(1 - \rho) g H L / SU^3$. For typical values $\rho = 0.4$, $H = 30$ m, $L = 6$ km, $S = 0.0016$, $U = 1$ m s$^{-1}$, this is about 20 years. Since it arises directly from the estimated magnitude of the transport divergence, this must be regarded as a minimum time-scale. Examples in section 5 indicate that evolution is typically slower by a factor of 5 or so, i.e. it takes place over the centuries.

Only the average of (2.6) over a tidal cycle is relevant; we denote this by an overbar ($\bar{\cdot}$).

3. Initial growth

We consider a perturbation of wavelength $L$, i.e. $h = 1 + \delta \sin y$, and work to lowest order in $\delta \ll 1$. Writing $u = I \cos \alpha + \delta \alpha(t) \cos y + \delta(t) \sin y$, $v(t) = a + ib$ satisfies

$$
F^{-1} w' \left[ |I|^{-1} q - iI \right] w = iF \left[ |I|^{-1} r \cos \alpha - jF \sin \alpha \right]
$$

(3.1)

$$
\bar{w}(2\pi) = w(0)
$$
by (2.3), determining \( u \). Here \( q \equiv m \cos^2 \alpha + \sin^2 \alpha \), \( l \equiv k \sin \alpha \) where \( k \equiv \varepsilon/F \), and \( r \equiv m \sin^2 \alpha + \cos^2 \alpha \). The depth increases according to

\[
\delta^{-1} \frac{d \delta}{dt} = \frac{S U^2}{(1-\rho)gH} F n k \left[ \cos a \sin a \left( -a \frac{\lambda}{\sin a} \right) - \frac{\lambda}{n} \sin a \right] [I]^*+1
\]

by (2.6); \( a(t) \) is determined by (3.1) and we have used \( \delta[I]^* = 0 \) (implied by (3.1) if \( I(t+\pi) = -I(t) \)).

In general, for a given context \((S, \rho, U, g, H; f, F, \lambda, m, n)\), (3.2) determines a growth rate \( \sigma_G(k, \alpha) \) appropriate to the particular spacing \( L \) (represented by the wavenumber \( k = 2\pi H/CL \)) and orientation \( \alpha \) of the low bed-form \( \delta \sin \gamma \). This is clearest if

\[
I(t) = \begin{cases} 
1 & (0 < t < \pi) \\
-1 & (\pi < t < 2\pi),
\end{cases}
\]

i.e. the imposed tidal current has a square-wave rather than a sinusoidal time-dependence; then (3.1) can be easily solved analytically and (3.2) is explicit:

\[
\sigma_G(k, \alpha) = \frac{S U^2}{(1-\rho)gH} \frac{k^2}{n^2} \left[ \cos a \sin^2 a \frac{F \cos a - f \sin a}{q^2 + l^2} \left\{ 1 - 2q \frac{\cosh \pi F q - \cos \pi F l}{\pi F (q^2 + l^2) \sinh \pi F q} \right\} \right.
\]

\[\left. - \frac{\lambda}{n} |\sin a| \right]
\]

where \( \lambda \equiv C/\tan \) (friction angle) and again \( l = k \sin \alpha \).

If (preferably) \( I(t) = \sin t \) for the oscillatory tidal current, then (3.1) must be integrated numerically to obtain \( a(t) \) for evaluating \( \sigma_G(k, \alpha) \) by (3.2). In fact the growth rates \( \sigma_G \) differ little from (3.4) apart from a reduction factor \( [\sin^*+1] \). Figure 2 shows contours of \( \sigma_G(k, \alpha) \)

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Figure 2. Contours of growth rate \( \sigma_G(k, \alpha) \) as a multiple of \( SU^2 F \sin/2(1-\rho)gH \) when \( f = \omega, F = 1, \lambda = \omega \cos \alpha, m = 2 = n \). Since \( f = \omega \) there is symmetry between positive and negative inclinations \( \alpha \).
for representative values \( f = 0, F = 1, \lambda = 0.005, m = 2 = n \). Bed-form growth (rather than decay) is indicated over a wide range of wavenumbers \( k \) and orientations \( a \), with broad maxima near \( k_M = 10, a_M = \pm 28^\circ \). This value of \( k_M \) corresponds to a spacing \( L_M \approx 250H \).

We may expect bed-forms with near-maximum growth rates to predominate eventually, i.e. sand banks should have \( L \) and \( a \) near \( L_M, a_M \). In the Norfolk Sandbanks context \( L_M \approx 71 \) km, so that there is fair agreement with observed values \( L \approx 9 \) km (Off, 1963), \( a \approx -20^\circ \).

Figure 3 illustrates the dependence of \( k_M, a_M \) and the maximum growth rate \( \sigma_{G}(k_M, a_M) \) on \( f, F, \lambda, m \) and \( n \) individually. We also show the dependence on \( E \), the ratio of minor to major axes of the ‘far’ tidal current ellipse when this is not rectilinear (negative \( E \) denotes clockwise polarization). The analysis follows (2.1)-(2.6) with

\[
\begin{align*}
I \cos a & \rightarrow I \cos a - f \sin a \\
I \sin a & \rightarrow I \sin a + f \cos a
\end{align*}
\]

throughout, where \( I(t) = \sin t, f(t) = -E \cos t \). The dependence on \( E \) is somewhat erratic but not large, except that as \( E \rightarrow \pm 1 \) (circular polarization) the context loses all directionality, so that \( a_M \) while tending to zero also becomes undefined.

Maximum growth rates generally depend only weakly on \( \lambda/n \) and \( m \). However, they increase strongly with positive Coriolis parameter \( f \) (if \( a \) is negative) and with friction \( F \). This is a direct result of tidal current asymmetry; the growth factor \( (Fr \cos a - f \sin a) \) of (3.4) appears in Huthnance (1973) as an asymmetry factor giving mean currents and a second harmonic. A rough physical description of how the current asymmetries arise from \( f \neq 0 \) and \( F \neq 0 \), respectively, is as follows. Potential vorticity conservation (the shallow-water formulation of angular momentum conservation, but not explicit in (2.1), (2.2)) with \( f \neq 0 \) tends to strengthen anticyclonic currents around shallower water. When \( F \neq 0 \), frictional drag, viz. the \( C_m \) term in (2.2), is more effective in shallower water, causing a current coming off a sandbank to have been retarded more than the reverse current approaching the bank.

Given the asymmetry \(-a/|I|^n \sin a > 0 \), (3.2) shows that growth rates also increase with \( n > 0 \). That is, the faster-than-linear sand transport increase with current is important. It gives greater on-bank transport during the half cycle of stronger \( u \) (the current component along the bank) than off-bank transport during the opposite half cycle (Figure 4), despite the symmetry of the on/off-bank current component \( v = Ih^{-1} \sin a (I(t+n) = -I(t)) \).

The representative value \( F = 1 \) corresponds to \( 1 \) m/sec in about \( 18 \) m water depth. From Figure 3 this appears to be near a large-\( F \) limit. A simplified analysis is then possible. By (3.1)

\[
w \approx iF \frac{1}{|I|^{n-1} \cos a - f |I|^{n-1} q - iH}
\]

representing the balance between frictional and Coriolis forces and convective acceleration of quasi-steady flow. Hence (3.2) gives

\[
\sigma_{G}(k, a) = \frac{SU^2}{(1-p)gH} F_{H^2} 
\left[ \cos a \frac{|I|^{n-1} r \cos a - f |I|^{n-1} q}{|I|^{2n-2} q^2 + l^2} |I|^{n-1} \right] \frac{\bar{\lambda}/|I|^{n+1}}{n|\sin a|} . \tag{3.5}
\]

If \( m = 2 \), this differs from the square-wave result (3.4) for large \( F \) only by an overall factor \( |I|^{n+1} \) and an effective increase of \( f \) by a factor \( |I|^{n}/|I|^{n+1} \). The similarity is because only the instantaneous magnitude and not the time-dependence of quasi-steady flow is significant.
Figure 3. (a) wavenumber $k_M$, (b) orientation $a_M$ and (c) growth rate $\sigma_0(k_M, a_M)$ as a multiple of $SU^2F\tilde{H}$ when $k_M$ and $a_M$ make $\sigma_0$ a maximum. Curves are identified by the parameter (abscissa) which varies. Otherwise $f = 0$, $F = 1$, $\tilde{\lambda} = 0.005$, $m = 2 = n$, $E = 0$. There is symmetry between $-a$, $-f$ and $a, f$. 
Linear sand banks

Ist half cycle Deep 2nd half cycle

Shallow

Deep

Figure 4. Sand-bank growth mechanism. Inclination $\alpha$ is negative as for Norfolk Sandbanks. $\rightarrow$: current, $\Rightarrow$: transport.

The alternative transport formula (2.5) requires the simple change

$$\frac{\lambda |I|^{\alpha+1}}{n|\sin \alpha|} \rightarrow V \lambda |I|^\alpha$$

in (3.5). Growth rates are usually affected little since this term is relatively small ($\lambda = 0.005$ typically).

Maximum growth occurs for $k_M$ and $\alpha_M$ such that $\partial \sigma_G/\partial k = 0 = \partial \sigma_G/\partial \alpha$; Table 1 gives a few examples. Evidently $k_M$ and $\alpha_M$ are sensitive to the drag law exponent $m$ and to the transport formula. This is because the growth rate has a rather broad maximum and depends on $k$ and $\alpha$ primarily in the single combination $l = k \sin \alpha$ as suggested by Figure 2. Thus, in the case $m = 2$ of Table 1, the growth rate for $l = l_M = 4.357$ but $\alpha = 15^\circ$ is only 10% less than the maximum at $\alpha_M = 27.8^\circ$; the growth-rate contours depict a broad ridge along $k \sin \alpha = l_M$ rather than a peak. This suggests that the sand-bank orientation $\alpha$ eventually predominating may be susceptible to external influences, e.g. the trend of an adjacent coastline. Given $\alpha$, the spacing $k^{-1}$ is somewhat better defined; a factor of 2 in $k$ brings nearly 30% reduction in growth rate.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\alpha_M$</th>
<th>$k_M$</th>
<th>$l_M = k_M \sin \alpha_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m = 2$</td>
<td>27.8°</td>
<td>9.725</td>
<td>4.537</td>
</tr>
<tr>
<td>(cf. Figure 3)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m = 1$</td>
<td>13.1°</td>
<td>12.707</td>
<td>2.875</td>
</tr>
<tr>
<td>transport (2.5)</td>
<td>12.9°</td>
<td>26.53</td>
<td>5.929</td>
</tr>
</tbody>
</table>

Table 1. Wavenumbers $k_M$ and orientations $\alpha_M$ for maximum growth according to (3.5) for friction-dominated flow. $|I| = 1$, $f = 0$, $\lambda = 0.005$, $m = 2$. 
The above examples illustrate the crucial role of preferred down-slope transport, represented by \( \lambda \), in inhibiting short-wave growth and thereby yielding a maximum growth rate at intermediate \( l \). This may be understood as follows. \( l = \varepsilon \sin \alpha / F \) compares cross-contour excursions of the water during the tidal cycle, \( \varepsilon \sin \alpha \), with the relative strength \( F \) of frictional drag. For small \( l \), current asymmetries are weak; friction dominates over the advection of flow properties to give local equilibrium between friction and the pressure gradient. For larger \( l \), a given bed-form height corresponds to a steeper slope and 'downhill' transport dominates. Intermediate \( l \) allows current asymmetries and sand-bank growth without excessive down-slope transport. If there were no preference for downhill transport, i.e. \( \lambda = 0 \), the fastest growth would occur at the shortest wavelengths \( l \rightarrow \infty \), as was also found by Richards (1980) for transverse bed-forms.

4. Final steady states

Henceforth we exploit the representativeness of friction-dominated flows with \( F \gg 1 \), \( |l| = 1, f / F = 0 \). The approximation improves in the shallower water over a growing sand bank.

If a bank has fully evolved to an equilibrium profile, then \( \partial h / \partial t = 0 \). Hence the tidally-averaged sand transport (2.4) is independent of \( y \); it is zero if \( I(t+\pi) = -I(t) \), so that

\[
[u^2 + l^2 h^{-2} \sin^2 \alpha]^{n/2} \left( I h^{-1} \sin \alpha + \dot{\lambda} h^{-1} \mid \sin \alpha \right) \frac{dh}{dy} = 0
\]

where, by (2.3),

\[
l \frac{du}{dy} + I \left[ n^2 + h^{-2} \sin^2 \alpha \right]^{m-1} u = h \cos \alpha.
\]

Taking the drag and transport law exponents as \( m = 1 \), \( n = 2 \) (without much effect on results; see below) and writing \( u/\cos \alpha = u_+ \), \( u_- \) when \( I = +1 \), \( -1 \) respectively, we have

\[
l \frac{du_+}{dy} = h - u_+, \quad l \frac{du_-}{dy} = h + u_-,
\]

\[
(2X/l)^2 l \frac{dh}{dy} \left[ \frac{1}{2}(u_+^2 + u_-^2) + h^{-2} \tan^2 \alpha \right] = u_+^2 - u_-^2 \tag{4.1}
\]

where \( X = (kl\lambda/2)^{1/4} \). Since we expect the bedforms with fastest initial growth to set the wavenumbers \( k \) and \( l \), Table 1 suggests representative values \( X = 0.332 \), \( (2X/l)^2 = 0.0214 \ll 1 \).

Solutions of (4.1) (Figure 5) may be obtained numerically by directly incrementing \( u_+ \), \( u_- \) and \( h \) for small advances in \( y \), commencing at a sand-bank crest \( y = 0 \). Here \( h \) has its minimum value \( h_m \), and \( dh/dy = 0 \) implies \( u_- = -u_+(0) \) by (4.1). A search in the range \( (h_m, 1) \) is required for the correct starting value \( u_+(0) \), such that the periodic solution \( h(y) \) averages 1 (conserving sand from the original state \( h \equiv 1 \) before the banks formed).

Figure 6 indicates that solutions for typical \( (2X/l)^2 = 0.0214 \) differ little from limiting forms for small \( X/l \), which we can obtain analytically. Let \( s = y / X \),

\[
h = h_0(s) + X/l h_1(s) + \ldots
\]

\[
u_+ = 1 + X/l u_+(s) + \ldots
\]

\[
u_- = -1 + X/l u_-(s) + \ldots
\]
Then \( u_{-1} = u_{+1} = \int_0^s (h_0 - x) ds, \)

\[
\frac{dh_0}{ds} [x + h_0 \tan^2 a] + \int_0^s (h_0 - x) ds = 0
\]

(4.2)

and \( h_l(s) = 0. \)

---

Figure 5. Equilibrium bank profiles (4.1) labelled by \( \tan^4 \alpha; X/l \rightarrow \infty, \) minimum depth \( h_m = 0.1 \) or \( 0.05 \) at \( s = 0. \)

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Figure 6. Influence of model changes on equilibrium profiles \( (\tan^4 \alpha = 0.2). \)

\((- - -): \) Basic solution (4.2) with \( X/l \rightarrow \infty. \) Labelled curves: \( (2X/l)^2 = 0.194; \)

\( (2X/l)^2 = 0.0214 \) is indistinguishable from \( (2X/l)^2 = 0; \) \( m = 2; \) (2.5) sand

transport law with \( V = \frac{1}{4}. \)
For zero inclination \( a \) to the tidal current, \( \tan^2 a = 0 \) and

\[
h_0 = h_{00} + (h_m - 1) \cos \pi.
\]

(4.3)

The wavelength, \( 2\pi \) in \( s \), i.e. \( 2\pi X \) in \( y \), is usually much less than the initial wavelength, viz. \( 2\pi \) in \( y \). This shorter ‘natural’ wavelength for equilibrium profiles is found in (c) below to give narrow banks in the case of sufficiently limited sand. Otherwise, evolution calculations in section 5 below suggest that the banks would grow to the sea surface without reaching any equilibrium, if there were no wave activity [see (a) below], in this case of \( a = 0 \).

Usually, \( a \) is not zero, and shallow depths imply large transverse currents which may be expected to inhibit bank growth. If \( \tan^2 a \) is small,

\[
h_0 = h_{00} + \tan^2 a h_{01} + \ldots,
\]

where

\[
h_{01} = h_{00} + \cos \pi \left[ \int_0^s h_{01}^1 \sin \pi s \,ds - h_m^{-1} \right] - \sin \pi \int_0^s h_{10}^{-1} \cos \pi s \,ds.
\]

(4.4)

The top of the bank is flattened. Larger \( a \) or small \( h_m \) (i.e. the bank nears the surface) also broadens the bank. In extremis (4.2) becomes (for large \( h_0^2 \) \( \tan^2 a \) and small \( h_0 \))

\[
h_0^2 \tan^2 a \frac{dh_0}{ds} = s
\]

i.e.

\[
h_0 \approx h_m/(1 - s^2 h_m/2 \tan^2 a).
\]

Hence \( h_0 \) is small like \( h_m \) for \( s^2 h_m/2 \tan^2 a < 1 \), a range of approximately \( 2(2 \tan^2 a/h_m)^{1/2} \) in \( s \). The \( s \)-period must exceed this range where \( h_0 \) is small (since \( h_0 \) averages 1), and so increases without limit as \( h_m \to 0 \). Hence the initial (longer) wavelength can be matched by a flattened equilibrium bank profile reaching almost to the sea surface. Since \( h_0 \) averages 1, the extensive shallows are compensated by a deep trough between banks (Figure 5). Here \( h_0^2 \tan^2 a \) is small, so that \( h_0 \to 1 \) is sinusoidal in \( s \) by (4.2); the trough width is approximately \( \pi \) in \( s \).

If \( n = 4 \), (4.2) still holds with \( s = 2^{1/2} y / X \), suggesting little dependence of equilibrium profiles on \( n > 0 \). The value of \( m \) has no effect for small \( X / l \) if \( a = 0 \); for typical \( \tan^2 a = 0.2 \) Figure 6 shows the small difference between the solution of (4.1) and its equivalent for \( m = 2 \). For the alternative transport law (2.5),

\[
V |\sin a| h_0^{1/2} \frac{dh_0}{ds} + \int_0^s (h_0 - 1) \,ds = 0
\]

replaces (4.2); Figure 6 illustrates the reduction in horizontal scale by a factor \( (V |\sin a|)^{1/2} \), but otherwise \( a \) has no effect on the bank profile, which is always flattened on top and broadened without limit as \( h_m \to 0 \).

(a) Wind-wave action

Wind-generated surface waves increase turbulence and sand transport over the top of a bank, perhaps flattening or lowering the crest. We neglect the direct energy input \( \sim C_D \rho s^3 U_w^{1/2} (0.03 U_w) \) to the sea from the wind \( U_w \); it is less than \( C \rho s^3 U^3 \) from the tidal current in typical sand-bank contexts, except during hurricanes.
The surface-wave energy flux (energy density × group velocity) progressing over a sand bank is constant, determining the spatial dependence of the energy density. Hence the mean-square bottom current under a wave (amplitude \(a\), wavenumber \(\kappa\), frequency \(\sigma\) in water of depth \(d\); amplitude \(a_\infty\) in deep water) is (e.g. Kinsman, 1965)

\[
\frac{1}{2}(a\sigma)^2 \cosh^2 \kappa d = \frac{1}{2}(a_\infty \sigma)^2 \cosh^2 \kappa d (\tanh \kappa d + \kappa d \sech^2 \kappa d)
= \frac{1}{2} a_\infty^2 \sigma^2 \cosh (\frac{1}{4} d \sigma^2 / g + O(d^2 / g))
\approx WU^2h^{-3/2}
\]

where the water is shallowest. Here

\[
W = \frac{1}{2} \frac{a_\infty^2}{\sigma U^2} (g/H)^{3/2} \approx 0.073
\]

taking \(U = 1\ m\ s^{-1}\), \(H = 30\ m\) and \(a_\infty \sim 1.1\ m\), \(\sigma \sim 0.8\ s^{-1}\) (Kinsman, 1965, after Neumann) corresponding to a typical wind speed \(10\ m\ s^{-1}\). \(W\) increases as \((\text{wind speed})^4\), so that the long-term average may be larger: \(0.2\) (say).

The surface-wave currents are not turbulent \textit{per se}; indeed, the time-averaged Reynolds stress (horizontal velocity × vertical velocity) is identically zero for linear progressive waves. Rather, the wave currents reinforce the tidal currents in generating turbulent stresses and sediment transport (e.g. Grant & Madsen, 1979). Then

\[
|\mathbf{u}|^n \rightarrow |\mathbf{u}|^n + W^2h^{-3/2}
\]

in (2.4), if, in the spirit of Bagnold (1956), we regard \(|\mathbf{u}|^n (n = 2)\) as the stress factor and the final factor as a transport velocity unaffected on average by the oscillatory wave motion. Such a substitution supposes, reasonably, that conditions are hydrodynamically rough both with and without waves, and that bedload transport remains proportional to \((\text{stress} \times \text{current})\), at least on average over a wave period. However, \(\bar{W} \neq W\); wave currents are generally associated with greater bottom stresses than comparable tidal currents, by a factor of up to 50, which has been associated with vortex shedding over ripples on the bed (Longuet-Higgins, 1981). The factor is less for larger waves, counteracting their tendency to raise the long-term average of \(W\) above typical values. Hence we might estimate \(\bar{W} \approx 50W\). The stress factor of Bijker (1967) gives \(\bar{W} \sim 8W\) under typical conditions, but is raised to the power 3/2 (i.e. \(\bar{W} \sim 12W\) in effect) in practical application (e.g. Heathershaw & Hammond, 1979) when combined with Bagnold’s transport formula as evaluated by Gadd \textit{et al.} (1978) for offshore flows.

The effect of wind-waves on initial bank growth appears to be small, being represented by the substitution

\[
\lambda |I|^{n+1} \rightarrow \lambda(|I|^{n+1} + \bar{W}|I|)
\]

in (3.2) i.e. merely an increase in \(\lambda\), or \(\hat{\lambda}\) in (3.4) and (3.5), by a factor of about 2. Hence a slightly reduced growth rate and slightly increased inclination \(\alpha\) and spacing for maximum growth are suggested by Figure 3.

The equilibrium form is affected more. Replacing (4.2),

\[
\frac{dh_0}{ds} [1 + h_0^2 \tan^2 \alpha + W \cos^{-2} \alpha h_0^{3/2}] + \int_0^s (h_0 - 1)ds = 0.
\]
The top of the bank is flattened and broadened, as illustrated by Figure 7. The effect is closely similar to that of non-zero inclination \( \alpha \) to the tidal current. For large \( \sqrt{\frac{\alpha}{m}}l \) (and \( \alpha = 0 \))

\[
h_0^{-1/2} \approx h_m^{-1/2} - \frac{\pi^2}{4} \langle \bar{W} \rangle.
\]

The \( s \)-period must exceed the approximate range \( 2(4\langle \bar{W} \rangle h_m^{-1/2})^{1/2} \) over which \( h_0 \) is small, and therefore increases slowly but without limit as \( h_m \to 0 \).

In sufficiently strong currents, and especially beneath waves, suspended-load transport becomes significant and rapidly dominant. To the extent to which this occurs selectively in association with increased wave activity over the bank top, we expect the effect to be simulated by an increase in \( \bar{W} \). To the extent to which suspended load increases sand transport everywhere in the same proportion, the one effect is faster evolution of the banks.

(b) Asymmetry

An asymmetric 'far' tidal current

\[
I(t) = \begin{cases} 
A & (0 < t < \pi) \\
-1 & (\pi < t < 2\pi) 
\end{cases}
\]

is associated with a net sand transport in the direction of the stronger current. We consider simply large \( F \) with \( f/F = \sigma \).

The initial growth rate for comparison with (3.5) is

\[
\sigma_0(k,l) = \frac{SU^2}{(1-\sigma)gH} F n l^2 \left[ \frac{r \cos^2 a}{2} \left( \frac{A^{n+m-1}}{A^{2m-4} q^2+l^2} \right) + \frac{1}{q^2+l^2} \right] - \frac{\lambda}{n|\sin a|} \frac{1+A^{n+1}}{2}.
\]

If \( m = 2 \) this equals (3.5) apart from a factor \( (1+A^{n+1})/2 \) due to the altered current strength when \( 0 < t < \pi \). However, there is also a translation of the wave form at a \( y \)-velocity

\[
\frac{SU^2}{(1-\sigma)gH} F n l \left[ \frac{(\sin^2 a + n^{-1})}{2} A^{n+1} - \frac{q r \cos^2 a}{2} \left( \frac{A^{n+m-3}}{A^{2m-4} q^2+l^2} \right) \right].
\]
This is normally in the direction of the stronger tidal current, but is retrograde when \( m = 2 \) (for example) if \( k^2 < 2n - 4 \).

For an equilibrium profile we take \( m = 1, n = 2 \) as before. A slight asymmetry is influential, and we define \( \mu \) by

\[
A^3 = 1 + 8\mu X/l
\]

\([2X/l]^2 = 0.0214 \ll 1\). The \( y \)-independent tidally averaged sand transport is now proportional to \( A^3 - 1 \), so that in (4.1) the last equation becomes

\[
(2X/l)^2 \frac{dh}{dy} \left[ \frac{1}{2}(u_x^2 + u_z^2) + h^{-2} \tan^2 a + O(X/l) \right]
= u_x^2 - Au_x + (A^3 - 1) (h \cos^2 a - h^{-2} \tan^2 a) + O(X^2/l^2),
\]

the others holding with error \( O(X/l) \). Proceeding as before,

\[
\frac{dh_0}{ds} - 2\mu(h_0 - 1) + \int_0^s (h_0 - 1)ds = 0
\]

replaces (4.2); we have taken \( a = 0 \) to concentrate on the asymmetry represented by \( \mu \).

Figure 8. Equilibrium bank profiles (4.6) for asymmetric currents, labelled by \( \mu \). The stronger current is into the page, net sand transport to the right.

Figure 8 sketches some solutions, which are easily found in terms of sines and cosines (if \( |\mu| < 1 = \text{critical damping} \)) and exponentials. As \( |\mu| \) increases from 0 to 1, the solutions are sinusoidal in \( s(1 - \mu^2)^{1/2} \) (i.e. the natural wavelength increases with \( |\mu| \)) but with a growth factor \( \exp(\mu s) \). Hence the steeper bank side faces in the direction of net transport. In practice, limited sand supply and other factors prevent a continued increase as \( \exp(\mu s) \) between one bank and the next; (4.6) is best regarded as applying separately to individual banks. In this spirit the non-periodic solutions for \( |\mu| \gg 1 \) are also relevant.
(c) Limited sand

This discussion is motivated by the observation that between the Norfolk Sandbanks the sea floor is composed of gravel (Caston, 1972). Hence we suppose that the depth \( h \) is limited to a maximum \( h_M \); at greater depths the bed material is immobile. The derivation of (4.2) holds under the same conditions as previously wherever \( h < h_M \). Hence the equilibrium state consists of periodic banks, whose profiles satisfy (4.2), rising above a level floor at the depth \( h_M \). Figure 9 shows some examples.

\[
\text{Figure 9. Equilibrium bank profiles (4.2) with limited sand, labelled by } \tan^\circ \alpha. \quad X = 0.8 \text{ (large) for clarity. Maximum depth } h_M = 1.2, 1.4 \text{ or } 1.8. \text{ One period is shown.}
\]

Sand conservation constrains the height of the bank so that \( h \) averages 1 overall:

\[
1 = \frac{X}{\pi} \left( \left( \frac{\pi}{X} - s_M \right) h_M + \int_0^{s_M} h_0 \, ds \right) \quad (4.7)
\]

considering one half-wavelength from mid-trough \( t = \pi/X \) to crest \( t = 0 \) via the foot of the bank \( t = s_M \) where

\[
h_0(s_M) = h_M. \quad (4.8)
\]

The solution \( h_0 \) of (4.2) depends on the depth \( h_m \) at the crest. Hence (4.7) and (4.8) determine the bank half-breadth \( s_M \) and crest depth \( h_m \) in terms of \( X \) and the sand available \( h_M - 1 \).

If \( \alpha = 0 \), \( h_0 \) is given by (4.3). Then \( s_M \) and the crest height : trough depth ratio \( (1 - h_m)/(h_M - 1) \) do not depend on the amount of sand available. For example, if \( X = 0.332 \), then

\[
\text{bank breadth/spacing } = s_M X/\pi \approx 0.18, \quad (1 - h_m)/(h_M - 1) \approx 7.83.
\]

The profiles differ only in vertical scale, provided \( h_M > 0 \) which restricts consideration to \( h_M - 1 < 0.128 \). If more sand than this is available, the evolution calculations in the following
section 5 suggest that the sandbanks would grow to reach the surface without achieving equilibrium, if there were no wind–wave activity, in this case where $a = 0$.

Usually, $a$ is not zero. For small $\tan^2 a$, one can expand about the $a = 0$ solution using (4.4). After considerable algebra, (4.7) and (4.8) yield (for $X = 0.332$)

$\text{bank breadth/spacing} = s_M X/\pi \simeq 0.18 + \left\{ \begin{array}{ll}
0.225 r^{-1/2} \tan^2 a & (r \to 0: \text{high banks}) \\
0.098 \tan^2 a & (r \to 1: \text{low banks})
\end{array} \right.$

where $r = h_n/h_M$ is crest : trough water-depth ratio.

5. Evolution

We continue to simplify the analysis using the same approximation as in section 4: $F \gg 1$, $|I| = 1$, $|F' = 0$ with $m = 1$, $n = 2$. Retaining the $\partial h/\partial t$ term in the evolution equation (2.6), and carrying out the analysis leading to (4.2),

$$\frac{\partial h}{\partial t} = \frac{\partial}{\partial y} \left\{ h^{-1} \int_{0}^{y} (h-1)dy + X^2 h^{-1}(1+h^{-2} \tan^2 a) \frac{\partial h}{\partial y} \right\}. \quad (5.1)$$

Here $t$ denotes evolutionary time in units $(1-\beta) g H L k/2SU^3 \cos^2 a$ [e.g. 135 years for the typical values of section 2(c)], and $X = (k' \lambda/2)^{1/2} = 0.332$ (say) as in section 4. A coarser approximation $(1/I)^2 \to 0$ (replacing $(X/I)^2 \to 0$ of section 4) is made at small times $t$; the error is represented by the difference between the curves $(2X/I)^2 = 0, 0.194 (= 0.0214/0.332^2)$ in Figure 6.

Solutions of (5.1) were obtained numerically. If the depth did not become very small anywhere, e.g. $h \gtrsim 0.1 \tan a$, then a simple forward time-stepping scheme sufficed if the time step $\Delta t$ was small enough;

$$\Delta t < \frac{1}{4}(\Delta y/X)^2 \min \left[ h/(1+h^{-2} \tan^2 a) \right]$$

for a grid scale $\Delta y$ (e.g. Richtmyer & Morton, 1967). If the depth $h$ becomes very small, this time step may be prohibitively short. Then an implicit scheme was used, $\partial h/\partial y$ (only) being represented at time $t + \Delta t$. The resulting linear equations for $h(t + \Delta t, y)$ $(y = 0, \Delta y, 2\Delta y, \ldots)$ in terms of $h(t, y)$ are rapidly solved since the coupling matrix is tridiagonal (Richtmyer & Morton, 1967). Sand is conserved identically by (5.1):

$$\frac{\partial}{\partial t} \int_{0}^{2\pi} hdy = 0,$$

since $\partial h/\partial y = 0$ at the crests $y = 0, 2\pi$ and initially $\int_{0}^{2\pi} (h-1)dy = 0$; the finite difference analogue also conserved sand (except for truncation errors during computation).

Figure 10(a) illustrates the case of $a = 0$ with unlimited sand for which there is no equilibrium profile; in the absence of wave activity the bank grows until it reaches the sea surface. However, if either $a \neq 0$ or the sand is sufficiently restricted so that there is an equilibrium profile, then the bank appears to evolve towards that equilibrium fairly directly [Figure 10(b), (c)]. This involves up to three successive stages which merge into each other: (i) the initial sinusoidal perturbation amplifies; (ii) if sand is limited, exposure of the underlying level $h = h_M$ is followed by lateral contraction and vertical growth of the bank; (iii) if $a \neq 0$, the bank profile 'squares-up' with the top levelling and broadening and the sides steepening; with unlimited sand there is more broadening and the trough deepens. Significant changes take place during unit intervals in $t$ (e.g. 135 years).
Figure 10. Bank evolution according to (5.1), labelled by evolutionary time $\tau$. $h = 1 - 0.01 \cos \gamma$ at $\tau = 0$. Equilibrium profile (---). $X = 0.8$ (large) for clarity. Unlimited sand: (a) $a = 0$, (b) $\tan^2 a = 0.2$. Limited sand: (c) $a = 0$, $h_M = 1.4$. 
6. Discussion

The model is simple in that few physical 'extras' are involved; any model must include a description of the currents as modified by the sandbanks, and a formulation of sand transport and conservation. Depth-independent currents together with a direct \( |u|^n+1 \) sand transport formula suffice to form and maintain sandbanks if \( n>0 \). The one addition is a preference for 'downhill' sand transport—widely accepted although the formulation in two horizontal dimensions is uncertain. Its inclusion leads to a spacing and orientation for fastest bank growth, and evolution to an equilibrium profile, which is flattened on top by wind–wave action and asymmetric if the ebb and flood tidal currents are unequal (the steeper side faces in the sense of the net sand transport). Such plausible consequences and qualitative agreement with observed bank forms suggest that down-slope transport preference is an important influence.

We have omitted various other possible bank-forming mechanisms. Our depth-averaged current neither represents nor disproves the secondary vertical circulation proposed by Off (1963) and Houholt (1968). However, such a circulation might perhaps be regarded as unnecessary, and over small bottom slopes is probably weak, with less effect than the primary distortions of the depth-averaged current. Internal tides associated with temperature-, salinity- or sediment-stratification are not modelled. They may well give rise to bed forms in an appropriate context, e.g. Cartwright (1959). In a shorter-scale context, Kennedy (1969) considered a lag of sediment transport behind the instantaneous current. This may occur in changing tidal currents, where turbulence is thought to depend on the water's previous 'experience'. However, such hysteresis is apparently reduced near the sea floor (Bowden & Ferguson, 1980) where most sand transport occurs. For sand banks, moreover, zero lag already gives growth; the small lag (a few degrees in phase) expected in the context of banks kilometre(s) across seems unlikely to have much effect. It has been omitted. Differential bed roughness has also been omitted, although it may be another means of initiating sand-bank growth. (Indeed, our simple \( |u|^n+1 \) transport with wind-wave modification is the only account taken of changes in bed roughness as the current varies.) In suitable contexts, these various neglected factors may well influence or cause sand-bank growth.

The model treated only parallel depth contours. Hence it has no bearing on Caston's (1972) suggestions that a finite-length bank tends to elongate and that three banks may form from one as meanders grow and break. However, these suggestions might usefully be investigated using a two-dimensional analogue of the simple quasi-steady flow assumed in sections 4 and 5. Then the problem reduces to one evolution equation for the bottom topography, the integration over a tidal cycle becoming trivial.

The sand-bank growth mechanism in the model parallels findings from observed currents and sand waves by Caston (1972) and Kenyon et al. (1981). Figure 4 shows the currents turning towards the crest as they rise onto the bank. This controls the sediment transport since the current coming off the bank during the reverse tidal half-cycle is weaker, having been slowed by bottom friction in the shallows on top of the bank. Hence the net sand transport is onto the bank. The mechanism applies generally in polarized tidal currents, although the degenerate rectilinear case has been emphasized here.

The initial growth phase in sand-bank evolution tends to select the fastest growing bedforms from among all the various wavelengths and orientations making up an uneven sea floor. The model does define a wavelength and orientation of fastest growth, but not closely; there is some scope for the influence of coastline alignment or imposed spacing in a gulf (for example). Since the 'chosen' wavelength is basically proportional to the water depth, and
persists in the spacing of well developed sandbanks, the model predicts (with considerable scatter) bank spacing \( \approx 250 \times \text{mean water depth} \).

Allen (1968) found that Off's bank heights increased roughly as \((\text{spacing})^{1/2}\). Compared with the relevant mean water depth when the banks formed initially, bank heights may be (i) increased because present (interglacial) water depths are relatively large, (ii) increased by measuring from the bottom of the trough which is below mean water depth, (iii) decreased by wind-wave action, (iv) decreased if there is insufficient sand for the bank to rise near to the surface. Factor (iii) is stronger in shallow water, but apparently not important since some banks rise close to the sea surface. Factor (i) is relatively important in shallow water and factor (iv) in deep water; both reduce the rate at which bank height increases with spacing. Hence 'bank height \( \propto (\text{spacing})^{1/2} \) is consistent with the model's 'depth \( \approx 0.004 \times \text{spacing} \)'. However, factors (i) to (iv) primarily introduce scatter rather than a systematic trend. Even without the extraordinarily close and steep banks on Georges shoal, Off's (1963) values are well scattered:

\[
\text{bank height : spacing} = 0.0038 \pm 0.0020 \text{ (standard deviation)}
\]
despite small ranges of values; almost all spacings are between 1 and 10 miles and heights between 10 and 100 ft (respectively, 1.6–16 km, 3–30 m). At some individual locations (e.g. the Gulf of Korea), the banks appear to increase their spacing with increasing water depth along their length; this may lead to some complexity with banks bending and interleaving.

Model equilibrium profiles depend strongly on the amount of sand available, less sand decreasing the bank width to about one-fifth of the spacing and then lowering the height. Amongst Off's (1963) examples, only the Gulf of Cambay has banks narrower than one-fifth of the spacing, which is unusually large. Otherwise there is a full range of bank width : spacing ratios.

With typical evolution times measured in centuries, the large sand banks considered here change reasonably slowly from a navigational and surveying viewpoint, but rapidly enough to represent a 'snapshot' of the mean sea level, tidal and sedimentological conditions when the banks formed. Those conditions determine their orientation, spacing and core composition. Relatively rapid evolution also implies that the overall size and profile are in equilibrium under present conditions. The composition of the bulk of a bank should reflect some of the changes in conditions since its formation.

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