Papers we love - Utrecht
QuickCheck
Wouter Swierstra
Requirements for a topic

- A paper that I love
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- A paper that is interesting to academics and developers
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- A paper that I love
- A paper that is interesting to academics and developers
- A paper that had significant impact on my own career
Haskell in 2000

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- There were approximately zero industrial users.
Haskell

A Purely Functional Language

Haskell is a general purpose, purely functional programming language. Haskell compilers are freely available for almost any computer.

There will be another Haskell Workshop this summer in Montreal. Later, a special issue of the Journal of Functional Programming will be devoted to Haskell.

The Haskell Wiki has a new address: http://haskell.org/wiki/wiki.

Site contents:

- A Short Introduction to Haskell and its Advantages
- The Haskell Bookshelf: Books and Papers about Haskell and Functional Programming
- Definition of the Language and the Standard Libraries
- Haskell in Education
- Haskell Compilers and Interpreters
- Libraries and Tools for Haskell
- Haskell in Practice: Experiences and Applications
- The Future of Haskell
- The Haskell Mailing List
- Links to People and Pages Related to Haskell and Functional Programming
- Haskell Humor

This site is maintained by John Peterson and Olaf Chitil. Suggestions and comments welcome. This web site is a service to the Haskell community; new contributions are always welcome. If you wish to add your project, compiler, paper, class, or anything else to this site please contact the authors.

Main site at Yale, USA; Mirrors at Aachen, Germany, St. Andrews, Scotland, and the UK Mirror Service

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Doaitse came back from ICFP ’00 and handed me this paper...
The problem

How do we test software?

As a running example, let’s assume we’re developing a library for implementing queues:

\[
\begin{align*}
\text{enq} & : \text{Int} \to \text{Queue} \to \text{Queue} \\
\text{deq} & : \text{Queue} \to \text{Maybe Queue} \\
\text{front} & : \text{Queue} \to \text{Maybe Int} \\
\text{empty} & : \text{Queue} \\
\text{toList} & : \text{Queue} \to \text{List Int} \\
\text{fromList} & : \text{List Int} \to \text{Queue}
\end{align*}
\]
Writing tests

Typically, we write unit tests by hand:

```haskell
testFrontEnq :: Bool
testFrontEnq =
    front (enq 4 empty) == Just 4

testDeqEmpty :: Bool
testDeqEmpty =
    deq empty == Nothing

...
Test framework

And we may want to group these tests in a list and check they are all true:

```plaintext
runTests = and [testFrontEnq, testDeqEmpty, ...]
```
Writing tests

But writing these unit tests manually has drawbacks:

- to get good coverage, we need to write many tests;
- to get good coverage, we need to think quite hard about suitable input data.
- when a test fails, the test framework won’t help us figure out why.
In our example, our tests weren’t very good:

```
testFrontEnq :: Bool
testFrontEnq =
    front (enq 4 empty) == Just 4
```

This property also holds for a stack!
In our strawman test framework, all of our tests were assertions of type \( \text{Bool} \)...

Oftentimes, there is nothing special about the constants we’re using in our tests – we’d like to abstract over them:

\[
\begin{align*}
\text{testFrontEnq} & \quad \Colon \quad \text{Queue} \rightarrow \text{Int} \rightarrow \text{Bool} \\
\text{testFrontEnq} \; q \; x & = \\
\quad \text{front (enq i q)} & \; == \; \text{Just} \; i
\end{align*}
\]

What we’d really like to test is whether or not this \textit{property} holds for our queue implementation.
QuickCheck is a Haskell test framework that lets you test these properties:

```haskell
> quickCheck testFrontEnq
Falsifiable, after 4 tests:
(Queue [1], 2)
```

It generates inputs for the `testFrontEnq` function and checks whether the property holds for the generated inputs.
We can, of course, fix our test to check that we have the desired FIFO behaviour:

```haskell
testFifo :: Queue -> Int -> Bool
testFifo q x =
  last (toList (enq i q)) == Just i
```
As a first approximation, it may help to think of the `quickCheck` function being implemented as follows:

```haskell
quickCheck :: (a -> Bool) -> IO ()
quickCheck p = go ...
  where
    go :: [a] -> IO ()
go []       = print "All tests succeed"
go (x:xs)   =
    if p x then go xs
    else print ("Falsified " ++ show x)
```

The only question still open is: how do we generate inputs for our property?
Implementing QuickCheck

As a first approximation, it may help to think of the quickCheck function being implemented as follows:

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quickCheck :: (a -> Bool) -> IO ()
quickCheck p = go ...
  where
    go :: [a] -> IO ()
go [] = print "All tests succeed"
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  if p x then go xs
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```

The only question still open is: how do we generate inputs for our property?
Generating inputs

Basically, we use Haskell’s classes (aka ad-hoc polymorphism, traits, protocols, interfaces) to define how to generate input for all the types of data that we wish to test.

- QuickCheck generates random input;
- Other libraries (SmallCheck) enumerate all inputs up to a given size;
- Other hybrid choices also exist.

The idea is the same: generate data for our properties to test if they hold.
**Using the random inputs**

Suppose we have a type class defined (pretty much) as follows:

```haskell
class Arbitrary a where
    arbitrary :: RandomGenerator -> a
```

This captures the idea of being able to generate elements of type `a` randomly.

```haskell
-- use arbitrary and a rng to generate
-- a list of elements
elements :: Arbitrary a => IO [a]
```

```haskell
quickCheck :: Arbitrary a => (a -> Bool) -> IO ()
quickCheck p = do elts <- elements;
                  go elts
```
Testing Queues

There are different ways to implement Queues in Haskell:

- the simple implementation uses lists, but may be inefficient;
- a clever implementation providing amortized $O(1)$ access.

Can we use QuickCheck to show that they’re equivalent?
Simple queues

newtype Queue = Queue [Int]

enq :: Int -> Queue -> Queue
enq x (Queue xs) = Queue (xs ++ [x])

front :: Queue -> Maybe Int
front (Queue (x:xs)) = Just x
front (Queue []) = Nothing

empty :: Queue
empty = Queue []

...
newtype Queue = Queue ([Int],[Int])

enq :: Int -> Queue -> Queue
enq x (Queue (fs,bs)) = Queue (fs, x:bs)

front :: Queue -> Maybe Int
front (Queue (x:fs,bs)) = Just x
front (Queue ([] ,bs)) = Nothing

empty :: Queue
empty = Queue ([],[])
The only interesting thing we need to do is ensure that dequeuing elements maintains the invariant that the first queue is only empty, when the entire queue is empty:

\[
\text{deq} :: \text{Queue} \rightarrow \text{Maybe Queue} \\
\text{deq} (\text{Queue} (x:fs,bs)) = \text{restore} (fs,bs) \\
\text{where} \\
\text{restore} ([],bs) = (\text{reverse} bs, []) \\
\text{restore} (fs,bs) = (fs,bs)
\]
Testing our implementation

This gives us two ways to implement the same spec:

- the reference implementation using lists;
- the more efficient version using queues.

How do we relate the two?
Using QuickCheck

We can convert between our efficient implementation $I$ and reference implementation $R$ easily enough:

$$convert :: I.Queue \rightarrow R.Queue$$

$$convert \ (I.\text{Queue} \ (fs, bs)) = R.\text{Queue} \ (fs ++ \ \text{reverse} \ bs)$$

$$\text{testEmpty} = convert \ I.\text{empty} == R.\text{empty}$$

$$\text{testEnq} \ x \ q = convert \ (I.\text{enq} \ x \ q) == R.\text{enq} \ x \ q$$

...
Using QuickCheck

We can convert between our efficient implementation \( I \) and reference implementation \( R \) easily enough:

\[
\text{convert} :: I.\text{Queue} \rightarrow R.\text{Queue} \\
\text{convert} (I.\text{Queue} (fs, bs)) = \\
R.\text{Queue} (fs ++ \text{reverse } bs)
\]

\[
\text{testEmpty} = \text{convert } I.\text{empty} == R.\text{empty} \\
\text{testEnq} \ x \ q = \text{convert } (I.\text{enq} \ x \ q) == R.\text{enq} \ x \ q \\
\ldots
\]

Yet these tests fail! We should only consider queues satisfying our invariant.
Revising our tests

\[
\text{invariant (Queue (fs,bs)) = not (null fs) || null bs}
\]

\[
\begin{align*}
\text{testEmpty} &= \text{convert I.empty == R.empty} \\
\text{testEnq } x \ q &= \\
&\quad \text{invariant } q \implies \text{convert (I.enq } x \ q) == \text{R.enq } x \ q
\end{align*}
\]

We add the precondition invariant \( q \) to our tests.

Any test data that does not satisfy the invariant is discarded.
Nested calls?

This tests our two implementations line up after one call.
But what if we want to test that they line up after many calls?

testEnqEnq x y q = invariant q ==> convert (I.enq x (I.enq y q)) == R.enq x (R.enq y q)

Aren’t we trying to automate our tests?
Code is data

We can define an explicit data type capturing the API of our Queue libraries:

```haskell
data QAPI = Enq Int QAPI
    | Front (Maybe Int -> QAPI)
    | Deq (Maybe Queue -> QAPI)
    | ...
```
Code is data

We can define an explicit data type capturing the API of our Queue libraries:

```haskell
data QAPI = Enq Int QAPI
  | Front (Maybe Int -> QAPI)
  | Deq (Maybe Queue -> QAPI)
  | ...
```

And then interpret this data type as a sequence of commands on queues:

```haskell
evaluate :: QAPI -> Queue -> Maybe Queue
evaluate (Enq x c) q = evaluate c (enq x q)
evaluate (Front c) q = evaluate (c (front q)) q
evaluate (Deq c) q = evaluate (c (deq q)) (deq q)
...```
But if we have a data type representing commands on queues:

```haskell
data QAPI = Enq Int QAPI
  | Front (Maybe Int -> QAPI)
  | Deq (Maybe Queue -> QAPI)
  | ...
```

Why not generate random series of commands?

```haskell
instance Arbitrary QAPI where
  arbitrary = ...
```
Testing reference and implementation

And test that these two APIs produce the same results

referenceTest : QAPI -> I.Queue -> Bool
referenceTest cmds q =
  I.evaluate cmds q ==
  R.evaluate cmds (convert q)
What are our tests?

- *Unit tests* test that a property holds for certain values;
- Using QuickCheck, we can test that a property holds for *many* values – QuickCheck is generating unit tests.
- We can even use QuickCheck to generate a series of API calls – QuickCheck is generating completely new tests!
Given an API, what are the properties that hold of its functions?

That seems like an impossible problem to solve automatically…

Yet using QuickCheck you can get pretty close.
Suppose we have the API for a handful of list functions:

(++): [a] -> [a] -> [a]
(++) : a -> [a] -> [a]
[] : [a]
reverse : [a] -> [a]

What terms can we build using these functions?
Enumerating terms

Given some values $x$ and $y$, we can enumerate all possible terms built using these functions:

- $[]$
- $[] \; ++ \; []$
- $(x : [])$
- $(x : y : [])$
- `reverse (x : [])`
- ...
Enumerating terms

Given some values $x$ and $y$, we can enumerate all possible terms built using these functions:

```
[]
[] ++ []
(x : [])
(x : y : [])
reverse (x : [])
...
```

As a first approximation, assume that all these terms are equal.
Start testing!

Now choose random values for $x$ and $y$ – which terms can you distinguish?

For example, $\text{reverse} \ (x : [] )$ and $x : y : []$ are different for any choice of $x$ and $y$. 
Start testing!

Now choose random values for x and y – which terms can you distinguish?

For example, reverse (x : []) and x:y:[] are different for any choice of x and y.

But no choice of x, can distinguish reverse (x : []) and x : [] – hence these must be equal!
QuickSpec

To work well, QuickSpec does quite some work to remove duplicate equations and present a ‘minimal’ set of equations. But for many APIs it manages to compute sensible properties from scratch.
QuickCheck in practice

- Functions to classify the data that is generated;
- Functions to define random data generators;
- Controlling size of data generated and number of tests;
- ‘Shrinking’ of counterexamples to facilitate diagnosis.
What makes QuickCheck work?

If you take a step back, what is it that makes QuickCheck work so well?

- Purity – all functions are known to be free of side-effects;
- Types – the types of our properties drive the generation of random inputs.
What makes QuickCheck work?

If you take a step back, what is it that makes QuickCheck work so well?

- Purity – all functions are known to be free of side-effects;
- Types – the types of our properties drive the generation of random inputs.

This is playing exactly to the strengths of Haskell!
QuickCheck has been ported to 35+ different programming languages.

It forces you to think of *specifications* rather than unit tests. Bugs may still show up in the specification, the random data generator, or code under test – each requires separate diagnosis.
Advantages

- The underlying ideas of QuickCheck are very widely applicable to different languages and systems.
- It is particularly good at spotting interactions that conventional test cases miss.
- QuickCheck makes diagnosis simple by shrinking inputs.
- QuickCheck makes it easier to achieve much better test coverage.

It’s been used with great success in many different projects.
Why the love?

A simple idea, implemented in about 200 loc. (The original code was included in an appendix of the paper.)

It plays to Haskell’s strengths: types and purity.

Changes the way we think about testing.

Useful in practice.

Another useful piece of kit in the formal reasoning toolkit.
Questions?