Combinator Parsers: From Toys to Tools

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Abstract
We develop, in a stepwise fashion, a set of parser combinators for constructing deterministic, error-correcting parsers. The only restriction on the grammar is that it is not left recursive. Extensive use is made of lazy evaluation, and the parsers constructed “analyze themselves”. Our new combinators may be used for the construction of large parsers to be used in compilers in practical use.

1 Introduction
There exist many different implementations of the basic parser combinators; some use basic functions [3], whereas others make use of a monadic formulation [4].

Parsers constructed with such conventional parser combinators have two disadvantages: when the grammar gets larger parsing gets slower and when the input is not a sentence of the language they break down. In [7] we presented a set of parser combinators that did not exhibit such shortcomings, provided the grammar had the so-called $LL(1)$ property; this property makes it possible to decide how to proceed during top-down parsing by looking at the next symbol in the input.

For many grammars an $LL(1)$ equivalent grammar may be constructed through left factoring, but unfortunately the resulting grammars often bear little resemblance to what the language designer had in mind. Extending such transformed grammars with functions for semantic processing is cumbersome and the elegance offered by combinator-based parsers is lost.

To alleviate this problem we set out to extend our previous combinators in a way that enables the use of longer look-ahead sequences. The new and completely different implementation is both efficient and deals with incorrect input sequences. The only remaining restriction is that the encoded grammar

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is neither directly nor indirectly left-recursive: something which can easily be circumvented by the use of appropriate chain-combinators; we do not consider this to be a real shortcoming since usually the use of such combinators expresses the intention of the language designer better than explicit left-recursive formulations.

The final implementation has been used in the construction of some large parsers. The additional cost for maintaining the information needed for being able to repair errors is negligible.

In Section 2 we recapitulate the conventional parser combinators and investigate where the problems mentioned above arise. In Section 3 we present different basic machinery which adds error correction; the combinators resulting from this are still very short and may be used for small grammars. In Section 4 we show how to extend the combinators with the (demand driven) computation of look-ahead information. In this process we minimize the number of times that a symbol is inspected. Finally we present some further extensions in Section 5 and conclusions in Section 7.

2 Conventional parser combinators

In Figure 1 we present the basic interface of the combinators together with a straightforward implementation. We will define new implementations and new types, but always in such a way that already constructed parsers can be reused with these new definitions with no or just minimal changes. To keep the presentation as simple as possible, we assume all inputs to be sequences of Symbols.

Parsers constructed using these combinators perform a depth-first search through all possible parse trees, and return all ways in which a parse can be found, an idea already found in [1]. Note that we have taken a truly “functional” approach in constructing the result of a sequential composition. Instead of constructing a value of the more complicated type \((b, a)\) out of the two simpler types \(b\) and \(a\), we have chosen to construct a value of a simpler type \(a\) out of the more complicated types \(b \rightarrow a\) and \(b\). Based on these basic combinators more complicated combinators can be constructed. For examples of the use of such combinators, and the definition of more complicated combinators, see [3,5,6] and the web site for our combinators².

As an example of how to construct parsers using these combinators and of what they return consider (for Symbol we take Int):

```plaintext
p = ( symbol 3
     <|> (+) <$> symbol 3 <*> symbol 4
     )

parser p [3, 4, 5]?
```

² see www.cs.uu.nl/groups/ST/Software/UU_Parsing
infixl 3 <|> -- choice combinator
infixl 4 <*> -- sequential combinator

type Symbol = ...
type Input = [Symbol]
type Parser a = Input -> [(a,Input)]

succeed :: a -> Parser a
symbol :: Symbol -> Parser Symbol
(<|>) :: Parser a -> Parser a -> Parser a
(<*>) :: Parser (b -> a) -> Parser b -> Parser a
parser :: Parser a -> Input -> Result a

infixl 3 <$> -- a derived combinator using the interface
(<$>) :: (b -> a) -> Parser b -> Parser a
f <$> p = succeed f <*> p
-- straightforward implementation
succeed v input = [(v , input)]
symbol a (b:bs) = if a == b then [(b,b)] else []
symbol a [] = []
(p <|> q) input = p input ++ q input
(p <*> q) input = [(pv qv, rest )
                | (pv , qinput) <- p input
                , (qv , rest ) <- q qinput
                ]

type Result a = Either a String
parser p s = case p s of
  [] -> Right "Erroneous input"
  ((res,rest):rs) -> Left res

Fig. 1. The basic combinators

> [(3, [4, 5]), (7, [5])]

The main shortcoming of the standard implementation is that when the input cannot be parsed the parser returns an empty list, without any indication about where in the input things are most likely to be wrong. As a consequence the combinators in this form are unusable for any input of significant size. From modern compilers we expect even more than just such an indication: the compiler should correct simple typing mistakes by deleting superfluous closing brackets, inserting missing semicolons etc. Furthermore it should do so while providing proper error messages.

A second disadvantage of parsers constructed in this way is that parsing gets slow when productions have many alternatives, since all alternatives are tried sequentially at each branching point, thus causing a large number of symbol comparisons. This effect becomes worse when a naive user uses the
combinators to describe very large grammars as in:

\[
\text{fold1 } (<|>) \text{ (map symbol [1..1000])}
\]

Here on the average 500 comparisons are needed in order to recognize a symbol. Such parsers may easily be implicitly constructed by the use of more complicated derived combinators, without the user actually noticing.

A further source of potential inefficiency is caused by non-determinism. When many alternatives may recognize strings with a common prefix, this prefix will be parsed several times, with usually only one of those alternatives eventually succeeding. So for highly “non-deterministic” grammars the price paid may be high, and even turn out to be exponential in the size of the input. Although it is well known how to construct deterministic automata out of non-deterministic ones, this knowledge is not used in this implementation, nor may it easily be incorporated.

We now start our description of a new implementation that solves all of the problems mentioned.

3 Error Correction

3.1 Continuation-Based Parsing

If we extend the combinators from the previous section to keep track of the farthest point in the input that was reached, the parser returns that value only after backtracking has been completed. Unfortunately, we have then lost all context information which might enable us to decide on the proper error correcting steps. So we will start by converting our combinators into a form that allows us to work on all possible alternatives concurrently, thus changing from a depth-first to a breadth-first exploration of the search space. This breadth-first approach might be seen as a way of making many parsers work in parallel, each exploring one of the possible routes to be taken.

As a first step we introduce the combinators in Figure 2, which are constructed using a continuation-based style. As we will see this will make it possible to provide information about how the parsing processes are progressing before a complete parse has been constructed. For the time being we ignore the result to be computed, and simply return a boolean value indicating whether the sentence belongs to the language or not. The continuation parameter \( r \) represents the rest of the parsing process, which is to be called when the current parser succeeds. It can be seen as encapsulating a stack of unaccounted-for symbols from the right hand sides of partially recognized productions, against which the remaining part of the input is to be matched. We have again defined a function \text{parse} that starts the parsing process. Its continuation parameter is the function \text{null}, which checks whether the input has indeed been consumed totally when the stack of pending symbols has been depleted.
type Result a = Bool
type Parser = (Input -> Bool) -> (Input -> Bool)
succeed = \ r input -> r input
symbol a = \ r input -> case input of
  (b:bs) -> a == b && r bs
  []       -> False
p <*> q   = \ r input -> p (q r) input
p <|> q   = \ r input -> p r input || q r input

parse p input = p null input -- null checks for end of input

Fig. 2. The continuation-based combinators

3.2 Parsing histories

An essential design decision now is not just to return a final result, but to combine this with the parsing history, thus enabling us to trace the parsing steps that led to this result. We consider two different kinds of parsing steps:

Ok steps, that represent the successful recognition of an input symbol

Fail steps, that represent a corrective step during the parsing process; such a step corresponds either to the insertion into or the deletion of a symbol from the input stream

data Steps result = Ok (Steps result)
  | Fail (Steps result)
  | Stop result

getresult :: Steps result -> result
getresult (Ok l)  = getresult l
getresult (Fail l) = getresult l
getresult (Stop v) = v

For the combination of the result and its parsing history we do not simply take a cartesian product, since this pair can only be constructed after having reached the end of the parsing process and thus having access to the final result. Instead, we introduced a more intricate data type, which allows us to start producing tracing information before parsing has completed. Ideally, one would like to select the result with the fewest Fail steps, i.e., that sequence that corresponds to the one with a minimal editing distance to the original input. Unfortunately this will be a very costly operation, since it implies that at all possible positions in the input all possible corrective steps have to be taken into consideration. Suppose e.g. that an unmatched then symbol is encountered, and that we want to find the optimal place to insert the missing if symbol. In this case there may be many points where it might be inserted, and many of those points are equivalent with respect to editing distance to
some correct input.

To prevent a combinatorial explosion we take a greedy approach, giving preference to the parsing with the longest prefix of \texttt{Ok} steps. So we define an ordering between the \texttt{Steps}, based on longest successful prefixes of \texttt{Ok} steps:

\[
\text{best} :: \text{Steps rslt} \to \text{Steps rslt} \to \text{Steps rslt}
\]

\[
\begin{align*}
\text{best} @ (\text{Ok}\ l) & \ 'best' (\text{Ok}\ r) = \text{Ok} \ (l \ 'best' \ r) \\
\text{best} @ (\text{Fail}\ l) & \ 'best' (\text{Fail}\ r) = \text{Fail} \ (l \ 'best' \ r) \\
\text{best} @ (\text{Stop}\ _) & \ 'best' (\text{Fail}\ _) = 1 \\
\text{best} @ (\text{Stop}\ _) & \ 'best' (\text{Ok}\ _) = 1 \\
\text{best} @ (\text{Ok}\ _) & \ 'best' (\text{Stop}\ _) = 1 \\
\text{best} @ (\text{Fail}\ _) & \ 'best' (\text{Stop}\ _) = r
\end{align*}
\]

There is an essential observation to be made here: when there is no preference between two sequences based on their first step, we postpone the decision about which of the operands to return, while still returning information about the first step in the selected result.

3.3 Error-correcting steps

Let us now discuss the possible error-correcting steps. We have to take such a step when the next symbol in the input is different from any symbol we expect or when we expect at least one more symbol and the input is exhausted. We consider two possible correcting steps:

- pretend that the symbol was there anyway, which is equivalent to \textit{inserting} it in the input stream
- \textit{delete} the current input symbol, and try again to see whether the expected symbol is present

In both of these cases we report a \texttt{Fail} step. If we add this error recovery to the combinators defined before, we get the code in Figure 3. Note that if any input left at the end of the parsing process is left it is deleted, resulting in a number of failing steps (\texttt{Fail(Fail(... (Stop True)))}). This may seem superfluous, but is needed to indicate that not all input was consumed. The operator \texttt{||}, that was used before to find out whether at a branching point at least one of the alternatives finally led to success, has been replaced by the \texttt{best} operator which selects the “best” result. It is here that the change from a depth-first to a breadth-first approach is made: the function \texttt{||} only returns a result after at least its first operand has been completely evaluated, whereas the function \texttt{best} returns its result in an incremental way. It is the function \texttt{getresult} at the top level that is actually driving the computation by repeatedly asking for the constructor at the head of the \texttt{Steps} value.
type Result a = Steps Bool

symbol a = \ r input -> case input of (b:bs) -> if a == b then Ok(r bs)
    {- insert the symbol a -}
    else Fail (r input 'best'
    {- delete the symbol b -}
    symbol a r bs
    )
    {- insert the symbol a -}

succeed = \ r input -> r input

p <|> q = \ r input -> p r input 'best' q r input
p <*> q = \ r input -> p (q r) input

parse p = getresult . p (foldr (const Fail) (Stop True))

Fig. 3. Error correcting parsers

3.4 Computing semantic results

The combinators as just defined are quite useless, because the added error correction makes the parser always return \textbf{True}. We now have to add two more things:

(i) the computation of a result, like done in the original combinators
(ii) the generation of error messages, indicating what corrective steps were taken.

Both these components can be handled by accumulating the results computed thus far in extra arguments to the parsing functions.

3.4.1 Computing a result

Top-down parsers maintain two kinds of stacks:

- one for keeping track of what still is to be recognized (here represented by the continuation)
- one for storing “pending” elements, that is, elements of the right hand side of productions that have been recognized and are waiting to be used in a reduction (which in our case amounts to the application of the first element to the second).

Note that our parsers (or grammars if you prefer), although this may not be realized at first sight, are in a normal form in which each right-hand side alternative has length at most 2: each occurrence of a $\langle\star\rangle$ combinator introduces an (anonymous) non-terminal. If the length of a right hand side is larger than 2, the left-associativity of $\langle\star\rangle$ determines how normalization is defined. So
there is an element pending on the stack for each recognized left operand of some `<*>` parser whose right hand side part has not been recognised yet.

We decide to represent the stack of pending elements with a function too, since it may contain elements of very different types. The types of the stack containing the reduced items and of the continuation now become:

```haskell
type Stack a b = a -> b
type Future b result = b -> Input -> Steps result
```

Together this gives us the following new definition of the type `Parser`:

```haskell
type Parser a =
  forall b result .
  Future b result -- the continuation
  -> Stack a b -- the stack of pending values
  -> Input
  -> Steps result
```

This is a special type that is not allowed by the Haskell98 standard, since it contains type variables `b` and `result` that are not arguments of the type `Parser`. By quantifying with the `forall` construct we indicate that the type of the parser does not depend on these type variables, and it is only through passing functions that we link the type to its environment. This extension is now accepted by most Haskell compilers. So the parser that recognises a value of type `a` combines this value with the stack of previously found values which will result in a new stack of type `b`, which in its turn is passed to the continuation as the new stack.

The interesting combinator here is the one taking care of sequential composition which now becomes:

```haskell
((p <+> q) r stack input = p (q r) (stack.) input
```

When `pv` is the value computed by the parser `p` and `qv` the value computed by the parser `q`, the value passed on to `r` will be:

```haskell
(((stack .) pv) qv) = (stack. pv) qv = stack (pv qv)
```

which is exactly what we would expect.

Finally we have to adapt the function `parse` such it transforms the constructed result to the desired result and initializes the stack (`id`):

```haskell
parse p
  = getresult
    ( p (\ st inp -> foldr (const Fail) (Stop st) inp)
      id
    )
```

We will not give the new versions of the other combinators here, since they will show up in almost the same form in Figure 4.
3.4.2 Error reporting

Note that at the point where we introduce the error-correcting steps we cannot be sure whether these corrections will actually be on the chosen path in the search tree, and so we cannot directly add the error messages to the result: keep in mind that it is a fundamental property of our strategy that we may produce information about the result without actually having made a choice yet. Including error messages with the Fail constructors would force us to prematurely take a decision about which path to choose. So we decide to pass the error messages in an accumulating parameter too, only to be included in the result at the end of the parsing process. In order to make it possible for users to generate their own error messages (say in their own language) we return the error messages in the form of a data structure, which we make an instance of Show (see Figure 4, in which also the previous modifications have been included).

4 Introducing Look-ahead

In the previous section we have solved the first of the problems mentioned, i.e. we made sure that a result will always be returned, together with a message about what error correcting steps were taken. In this section we solve the two remaining problems (backtracking and sequential selection), which both have to do with the low efficiency, in one sweep.

Thus far the parsers were all defined as functions about which we cannot easily get any further information. An example of such useful information is the set of symbols that may be recognized as first-symbols by a parser, or whether the parser may recognize an empty sequence. Since we cannot obtain this information from the parser itself we decide to compute this information separately, and to tuple this information with a parser that is constructed using this information.

4.1 Tries

To see what such information might look like we first introduce yet another formulation of the basic combinators: we construct a trie-structure representing all possible sentences in the language of the represented grammar (see Figure 5). This is exactly what we need for parsing: all sentences of the language are grouped by their common prefix in the trie structure. Thus it becomes possible, once the structure has been constructed, to parse the language in linear time.

For a while we forget again about computing results and error messages. Each node in the trie represents the tails of sentences with a common prefix, which in its turn is represented by the path to the root in the overall structure representing the language. A Choice node represents the non-empty tails by a mapping of the possible next symbols to the tries representing their
type Parser a =
  forall b result .
    Future b result
  -> Stack a b
  -> Errs
  -> Input
  -> Steps result

data Errors = Deleted Symbol String Errors
  | Inserted Symbol String Errors
  | Notused String

instance Show Errors where
  show (Deleted s pos e)    = "deleted " ++ show s ++ " before " ++ pos ++ show e
  show (Inserted s pos e)   = "inserted " ++ show s ++ " before " ++ pos ++ show e
  show (NotUsed "")        = ""
  show (NotUsed pos)       = "symbols starting at " ++ pos ++ " were discarded "

eof     = " end of input"

distance ss    = if null ss then eof else show (head ss)

symbol a
  = let pr = \ r st e input ->
    case input of
      (b:bs) ->
        if a == b
        then Ok (r (st s) e bs)
        else Fail((pr r st (e.Deleted b (position bs)) bs)
                     'best'
                      (r (st a) (e . Inserted a (show b))input))
        []     -> Fail (r (st a) (e . Inserted a eof) input)
    in pr

succeed v    = \ r stack errors input
  -> r (stack v) errors input

p <|> q        = \ r stack errors input
  -> p r stack errors input
  'best'

q r stack errors input

p <*> q       = \ r stack errors input
  -> p (q r) (stack.) errors input

parse p input
  = getresult ( p (\ v errors inp
                      -> foldr (const Fail)
                       (Stop (v, errors.position inp)) inp
                      ) id id input 10
                 )

Fig. 4. Correcting and error reporting combinators
type Parser = Sents
data Sents = Choice [(Symbol, Sents)]
            | Sents :|: Sents -- left is Choice, right is End
            | End

combine xss@(x@(s,ct):xs) yss@(y@(s',ct'):ys)
    = case compare s s' of
        LT -> x:combine xs yss
        GT -> y:combine xss ys
        EQ -> (s, ct <|> ct'):combine xs ys
combine [] cs' = cs'
combine cs [] = cs

symbol a = Choice [(a, End)]
succeed = End

l@(Choice as) <|> r
    = case r
        of (Choice bs) -> Choice (combine as bs)
            (p :|: q ) -> (l <|> p) :|: q
            End       -> l          :|: End
l <|> r@(Choice _) = r <|> l
_ <|> _ = error "ambiguous grammar"

Choice cs <*> q = Choice [(s, h <*> q)| (s, h) <- cs ]
l :|: r <*> q = (l <*> q) <|> (r <*> q)
End <*> q = q

parse :: Parser -> Input -> Bool
parse (Choice cs) (a:as) = or [ parse f as| (s, f) <- cs, s==a]
parse (p :|: q ) inp = parse p inp || parse q inp
parse End [] = True
parse _ _ = False

Fig. 5. Representing all possible sentences.

corresponding tails. An End node represents the end of a sentence. The :|:
nodes corresponds to nodes that are both a Choice node (stored in the left
operand) and an End node (stored in its right operand) 3. Notice that the
language $ab|ac$ is represented by:

$$\text{Choice } [(\text{'a'}, \text{Choice } [(\text{'b'}, \text{End}), (\text{'c'}, \text{End})])]$$
in which the common prefix has been factored out. In this way the cost

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3 We could have encoded this using a slightly different structure, but this would have
resulted in a more elaborate program text later.
associated with the backtracking of the parser has now been moved to the construction of the Sents structure.

For the language $a^*b|a^*c$ constructing the trie is a non-terminating process. Fortunately lazy evaluation takes care of this problem, and the merging process only proceeds far enough for recognising the current sentence.

A shortcoming of this approach, however, is that it introduces a tremendous amount of copying, because of the way sequential composition has been modelled. We do not only use the structure to make the decision process deterministic, but also to represent the stack of symbols still to be recognized. Furthermore we may have succeeded in parsing in linear time, but this is only possible because we have shifted the work to the construction of the trie structure. In the sequel we will show how we can construct the equivalent to the trie-structure by combining precomputed trie-structure building blocks.

### 4.2 Merging the different approaches

Compare the two different approaches taken:

- continuation-based parsers that compute a value, and work on all alternatives in parallel
- the parser that interprets a trie data structure, and inspects each symbol in the input only once

In our final solution we will merge these two approaches. We will compute Sents fragments on which we base the decision how to proceed with parsing, and use the continuation based parsers to actually accept the symbols. Since the information represented in the new data structure closely resembles the information stored in a state of an $LR(0)$ automaton, we will use that terminology. So instead of building the complete Sents structure, we will construct a similar structure which may be used to select the parser to continue with.

Before proceeding, let us consider the following grammar fragment:

```haskell
succeed (\ x y -> x) <*> symbol 'a' <*> symbol 'b'
<|>

succeed (\ x y -> y) <*> symbol 'a' <*> symbol 'c'
```

The problem that arises here is what to do with the parsers preceding the respective symbol ‘a’ occurrences: we can only decide which one to take after a symbol ‘b’ or ‘c’ has been encountered, because only then we will have a definite answer about which alternative to take. This problem is solved by pushing such actions inside the trie structure to a point where the merging of the different alternatives has stopped: at that point we are again working on a single alternative and can safely perform the postponed computations. As a result of this the actual parsing and the computation of the result may run out of phase.

We now discuss the full version of the structure we have defined to do the bookkeeping of our look-ahead information (see the definition in Figure 6), and
data Look p = Shift p [(Symbol, Look p)]
  | (Look p) :+: (Look p)
  | Reduce p
  | Found p (Look p)

merge_ch :: Look p -> Look p -> Look sp
l@(Shift p pcs) 'merge_ch' right
  = case right of
      Shift q qcs -> Shift (p 'bestp' q) (combine pcs qcs)
      (left :+: right) -> (l 'merge_ch' left) :+: right
      (Reduce p)     -> l :+: (Reduce p)
      (Found _ c)    -> l 'merge_ch' c
  Found c 'merge_ch' r
  = c 'merge_ch' r
l 'merge_ch' r@(Shift _ _) = r 'merge_ch' l
_ 'merge_ch' _ = error "ambiguous grammar"

(P p) 'bestp' (P q)
  = P (\ r st e input -> p r st e input 'best' q r st e input )

Fig. 6. Computing the look-ahead information

the actions to be taken once a decision has been made. The p components
of the Look structure are the parsing functions that actually accept symbols
and do the semantic processing. We will discuss the different alternatives in
reverse order:

Found p (Look p) this alternative indicates that the only possible parser
that applies at this point is the first component of this construct. It corre-
sponds to a non-merged node in the trie structure. As such it marks also
the end of a trie fragment out of which the original try structure may be re-
constructed without any merging needed. The (Look p) part is used when
the structure is merged with further alternatives. This is the case when that
other alternative contains a path similar to the path that leads to this
Found node. As one can see in the function ‘merge_ch’ this Found constructor is
removed when the structure is merged with other alternatives.

Reduce p this indicates that in using the look-ahead information we have
encountered all symbols in the right hand side of a production (we have
reached a reduce state in LR terminology), and that the parser p is the
parser to be used.

( :+: ) this corresponds to the situation where we either may continue with
using further symbols to make a decision, or we will have to use information
about the followers of this nonterminal. This will be the only place where we
continue with a possibly non-deterministic parsing process. It corresponds
to a shift-reduce conflict in an LR(0) automaton.
Shift p [(s, Look p)] this corresponds to a shift state, in which we need at least one more symbol in order to decide how to proceed. The parser p contained in this alternative is the error correcting parser to be called when the next input symbol is not a key in the next table of this Shift point, and thus no symbol can be shifted without taking corrective steps.

Before giving the description about how to construct such a Look-ahead structure we will first explain how they are going to be used (see Figure 7).

In order to minimize the interpretive overhead associated with inspecting the look-ahead data structures, we pair each such structure with the function that given

(i) uses the input sequence to locate the parsing function to be called
(ii) and then calls this function with the input sequence.

In a proper Haskell implementation this implies that the constructors used in the look-ahead structure are being “compiled away” as a form of partial or stages evaluation. Note that the functions constructed in this way (of type Realparser) will be the real parsers to be called: the look-ahead structures merely play an intermediate role in their construction, and may be discarded as soon as the functions have been constructed. The first argument to a Realparser is the continuation, the second one the accumulated stack, the third one the error messages accumulated thus far, the fourth one its input sequence, and its result will finally be a Steps sequence, containing the parsing result at the very end.

The function mkparser interprets a Look structure and pairs it with its corresponding Realparser. The function mkparser constructs a function choose that is used in the resulting Realparser to select (choose input) a Realparser p) making use of the current input. Once selected this parser p is then called (p r s t e input). So the function choose, that is the result of the homomorphism over the Look structure, has type Input -> Realparser a.

We will now discuss how this selection process takes place, again taking the alternatives into account from bottom to top:

Found no further selection is needed so we return the function that, given the rest of the input, returns the parser contained in this alternative.

Reduce this alternative can be dealt with just as the Found alternative; no further symbols of the input have to be inspected.

(:|:) in this case we return a parser that is going to choose dynamically between the two possible alternatives: either we reduce by calling the parser contained in this alternative (p), or we continue with the parser located by using further look-ahead information (css).

Shift this alternative is the most interesting one. We are dealing with the case where we have to inspect the next input symbol. Since performing a linear search here may be very expensive, we first construct a binary search tree out of the table, and partially evaluate the function pFind with respect
to this constructed search tree. The returned function is now used in the constructed function for the continuation of the selection process (and if this fails returns the error correcting parser \( p \)).

We give the code for the additional functions in Figure 9. They speak for themselves, and are not really important for understanding the overall selection process. We have assured that this searching function is only computed once and is included in the \texttt{Realparser}. The interpretation overhead associated with all the table stuff is thus only performed once. In this respect our combinators really function as a \textit{parser generator}. Having come thus far we can now describe how the \texttt{Look} structures can be constructed for the different basic combinators. The code for the basic combinators is given in Figure 8.

The code for the \(<|>\) combinator is quite straightforward: we construct the trie structure by merging the two trie structures, as described before, and invoke \texttt{mkparser} in order to tuple it with its associated \texttt{Realparser}. The code for the \(<>\) combinator is a bit more involved:

\textbf{Found} we replace the parser already present, and which may be selected without further look-ahead, with the sequential composition (‘\texttt{fwby}’) of the two \texttt{Realparsers}.

\textbf{Reduce} apparently further information about the followers of this node is available from the context (viz. the right hand side operand of the sequential composition), and we use this to provide the badly needed information about what symbols may follow. So we replace this \texttt{Reduce} node with the trie structure of the right hand side parser, but with all element in it replaced by a parser prefixed with the reducing parser from the original left hand side node\(^4\).

\( (:\mid:) \) we merge both alternatives.

\textbf{Shift} we update the error correcting parser, and postfix all parsers contained in the choice structure with the fact that they are followed by the second parser.

This definition seems to be horrendously costly, but again we are saved by lazy evaluation. Keep in mind that these \texttt{Look} structures are only being used in the function \texttt{mkparser}, and are only inspected for the branches until a \texttt{Found} or \texttt{Reduce} node is reached. If the grammar is \textit{LL}(1) this will only be one step! As soon as \texttt{mkparser} has done its job the whole structure may be garbage collected.

In the code for \texttt{symbol} we see two local functions:

\begin{itemize}
  \item \texttt{pr}: the original error correcting parser
  \item \texttt{pr'}: this function is only called when a function constructed by the aforementioned \texttt{choose} has indeed discovered that the expected symbol is present.
\end{itemize}

\(^4\) Strictly speaking this is only needed when the reduce node actually is the right operand of a \( :\mid:\) construct, indicating the existence of a now resolvable shift-reduce conflict.
newtype RealParser a
  = P (forall b result . (b -> Errs -> Input -> Steps result) -> (a -> b) -> Errs -> Input -> Steps result)

data Parser a = Parser (Look (RealParser a)) (RealParser a)
mkparser :: Look a -> Parser a

cata_Look (sem_Shift, sem_Or, sem_Reduce, sem_Found)
  = let r = \ c -> case c of
          (Shift p csr)
            -> sem_Shift p [(s,r ch) | (s, ch) <- csr]
            (left :|: right )
            -> sem_Or (r left) (r right)
            (Reduce p )
            -> sem_Reduce p
            (Found p cs )
            -> sem_Found p (r cs)
in r

map_Look f = cata_Look ( \ qp csr -> Shift (f qp) csr , \ left right -> left :|: right , \ qp -> Reduce (f qp) , \ qp csr -> Found (f qp) csr )

mkparser cs
  = let choose
      = cata_Look
        (\ p css
          -> let locfind = pFind (tab2tree css)
              in \ inp -> case inp of
                      [] -> p
                      (s:ss) -> case locfind s of
                                      Just cp -> (cp ss)
                                      Nothing -> p
          ,\ css p -> (p ‘bestp’). css
          ,\ p -> const p
          ,\ p cs -> const p
        ) cs
      in Parser cs (P (\ r st e input -> let (P p) = choose input in p r st e input))

Fig. 7. Constructing parsers out of Look ahead structures
symbol a
  = let pr = ... as before
      pr' = (\ r -> \ st e (s:ss) -> Ok (r (st s) e ss ))
    in mkparser (Found (P pr)
                   (Shift (P pr)
                    [(a, (Reduce (P pr'))) ] ))

succeed v
  = mkparser(End (P (\ r -> \ st e input -> r (st v) e input)))

(Parser cp _) <|> (Parser cq _) = mkparser (cp ‘merge_ch’ cq)

(Parser cp _) <*> ~(Parser csq qp)
  = mkparser$cata_Look ( \ pp csr -> Shift (pp ‘fwby’ qp) csr
                        , \ csr rp -> csr ‘merge_look’ rp
                        , \ pp -> map_Look (pp ‘fwby’) csq
                        , \ pp csr -> Found (pp ‘fwby’ qp) csr
                        ) cp
  where (P p) ‘fwby’ (P q) = P (\ r st e i -> p (q r) (st.) e i)

Fig. 8. The final basic combinators

In this case the check that it is present and the error correcting behavior
 can be skipped. So all we have to do is to take a single symbol from the
 input, incorporate it into the result, and continue parsing. We record the
 successful step by adding an OK-step.

If we do not make use of look-ahead information we have to apply the function
pr, and this is the function that is contained in the first Found construct.
When this parser is merged with other parsers, this wrapper is removed and
the parser will only be called after it has been decided that it will succeed:
hence the occurrences of pr’ in the rest of the text. Only when the test
somehow fails to find a proper look-ahead we use the old pr again in order to
incorporate corrective steps.

For the sake of completeness we incorporate the additional functions used
in Figure 9.

5 Extensions

In the full set of combinators, we have included some further extensions.

Computation of a full look-ahead may be costly, e.g. when the choice
structures that are computed become very large, and are not used very often.
In such cases one may want to use a non-deterministic approach. For this
purpose dynamic versions are also available that have the efficiency of the
backtracking approach given before.

We also note that the process of passing a value and error messages around
data BinSearchTree a v
  = Node (BinSearchTree a v) a v (BinSearchTree a v)
  | Nil

tab2tree tab = tree
where
  (tree,[]) = s12bst (length tab) (tab)
s12bst 0 list = (Nil , list)
s12bst 1 ((a,v):rest) = (Node Nil a v Nil, rest)
s12bst n list
  = let 
     l1 = (n - 1) ‘div’ 2 ; rl = n - 1 - l1
     (lt,(a,v):list1) = s12bst l1 list
     (rt, list2) = s12bst rl list1 
    in (Node lt a v rt, list2)

pFind Nil = \i -> Nothing
pFind (Node left r v right) = let findleft = pFind left
                               findright= pFind right
                               in \i -> case compare i r
                                 of EQ -> Just v
                                      LT -> findleft i
                                      GT -> findright i

Fig. 9. The additional function for Braun tree construction and inspection

can be extended to incorporate the accumulation of any further needed information; examples of such kinds of information are the name of the file being parsed, a line number, an environment in which to locate specific identifiers, etc. In that case the state should at least be able to store error messages, and recognized symbols. Extra combinators have been introduced in the library to update the state.

The production version of our combinators contains numerous further small improvements. As an example of such a subtle improvement consider the code of the function choose. Once it cannot locate the next symbol in the shift table it resorts to the error correcting version. This parser will try all alternatives, and compare all those results. But we know for sure that the first step of each result will be a Fail and thus that the first step of the selected result will be too. So instead of first finding out what is the best way to fail, and only then reporting that parsing failed, it is better to immediately report a fail step and to remove the first fail step from the actual result; it is quite likely that we are dealing with a shift/reduce conflict and have gone over to the dynamic comparison of the two alternatives, and that, since we fail, the other alternative will succeed. Since the fact that repair is possible may be
discovered during the selection in a choice structure we have to make sure that no backtracking is taking place over this prefix if we want to guarantee that all correct symbols are examined only once. In the production version this adds one further accumulating parameter to the construction of the actual parsers.

Sometimes, once an erroneous situation has been detected, a better correction can be produced by taking not only the future but also some of the past into account. We have produced versions of our combinators that do so, and which base the decision about how to proceed by comparing sequences of parsing steps, instead of taking the greedy approach, and looking just ahead far enough to see a difference. By making different choices for the function \textbf{best} we can incorporate different error correction strategies.

In a sequential composition we always incorporate the call to the second parser in the trie structure of the first one. In general it is undecidable in our approach whether this is really needed for getting a deterministic parser; it may be used to resolve a shift/reduce conflict, but computing whether such a conflict may occur in the (possibly infinite) trie structure may lead to a non-terminating computation. An example of this we have seen before with the grammar for the language \(a^*b|a^*c\); in our approach it is not possible to discover that, when building the trie structure, we have reached a situation equivalent to the root after taking the ‘a’ branch. We may however take an approximate approach, in which we try to find out whether we can be sure it is not needed to create an updated version of the trie structure. As an example of such a situation consider:

\[
\text{number} = \text{fold1} \ (<|>) \ (\text{map} \ \text{symbol} \ [1..1000]) \\
\text{plus} = \text{succeed} \ (+) \ <*> \ \text{number} <*> \ \text{number}
\]

In this case an (unneeded) copy is made of the first \text{number} parser, in order to incorporate the call to +, and then once more a copy of this structure is made in order to incorporate the second \text{number} parser. Since we can immediately see that a \text{number} cannot be empty, and all alternatives are disjoint, and leading in one step to a \text{Found} node, we can postpone the updating process, and create a parser using the ‘fwby’ operator immediately.

Another problem that occurs is that the number of different possible error corrections may get quite high, and worse, that they are all equivalent. If an operator is missing between two operands there are usually quite a number of candidates to be inserted, all resulting in a single failing step. In the approach given this would imply that the rest of the input is parsed once for each of these different possible corrections, trying to find out whether there really is no difference. In contrast to generalised LR parsing we do not have access to an explicit representation of the grammar, and especially we cannot compare the functions that represent the different states, and as a consequence we cannot discover that we are comparing many parallell but equivalent parses. In the library there are facilities for limiting such indefinite comparisons by specifying different insertion and deletion costs for symbols and by limiting
the distance over which such comparisons are being made. In this way fine-
tuning of the error correction process is possible without interfering with the
rest of the parsing process.

Finally we have included basic parsers for ranges of symbols, thus making
the combinators also quite usable for describing lexers [2].

By defining additional combinators that extend the basic machinery we
may deal with ambiguous grammars too. As an example consider a simple
lexer defined as follows:

\[
\begin{align*}
\text{many } p & = \text{let manyp} = \text{many } p \\
& \quad \text{in } \ \text{succeed } (:) \ast p \ast \ast \text{manyp} \\
& \quad \langle | \rangle \ \text{succeed } [] \\
\text{letter} & = 'a' \ 'upto' \ 'z' \\
\text{identifier} & = (:) \ast \ast \text{letter} \ast \ast \text{many letter} \\
\text{token} & [ \ ] = \text{succeed } [] \\
\text{token} & (s:ss) = \text{succeed } (:) \ast \ast \text{symbol } s \ast \ast \text{token } ss \\
\text{lexsym} & = \text{identifier} \langle | \rangle \ \text{token } "if" \langle | \rangle \ldots
\end{align*}
\]

In this case we have two problems:

(i) we want to use a greedy approach when recognising identifiers by taking
    as many letters as possible into account
(ii) the string "if" should be parsed as a token and not as an identifier

We may solve this problem by introducing a new combinator:

\[
\begin{align*}
\text{step } v \ n \\
& = \text{mkparser} \\
& \quad \langle \ \text{End } (P \ (\ \& \ r \ \text{st } e \ \text{input} \rightarrow \text{Step } n \ (r \ (\text{st } v) \ e \ \text{input}) \\
& \quad ) \ ) \ )
\end{align*}
\]

and changing the code above into:

\[
\begin{align*}
\text{many } n \ p & = \text{let manyp} = \text{many } p \\
& \quad \text{in } \ \text{succeed } (:) \ast p \ast \ast \text{manyp} \\
& \quad \langle | \rangle \ \text{succeed } [] \ n \\
\text{identifier} & = (:) \ast \ast \text{letter} \ast \ast \text{many } 2 \ \text{letter} \\
\text{token} & [ \ ] = \text{step } [ ] 1 \\
\text{token} & (s:ss) = \text{succeed } (:) \ast \ast \text{symbol } s \ast \ast \text{token } ss \\
\end{align*}
\]

If we have seen the text "if", and there are no further steps possible with
cost 0 (e.g. further letters), then we choose the \text{token} alternative since 1 < 2.

6 Efficiency

It is beyond the scope of this paper to provide a detailed discussion about the
efficiency and space usage of the parsers constructed in this way. We notice
however the following:
• it follows directly from the construction process that the number of constructed parsers can be equivalent to the number of $LR(0)$ states in a bottom-up parser, so there is no space leak to be expected from that; this is however only the case if the programmer has performed left factorisation by hand. If not the system will perform the left factorisation, but since it cannot identify equality of parsers it will do so by merging two identical copies of the same look-ahead information.

• the number of times a symbol is inspected may be more than once: where a bottom-up parser would have a shift-reduce conflict we basically pursue both paths until a difference is found; if the grammar is $LR(1)$ this implies we are inspecting one symbol twice in such a situation. We feel that in all practical circumstances this is to be preferred over the construction of full $LR(1)$ parsers, where the number of states easily explodes. It is the relatively low number of states of the $LR(0)$ automaton, and thus of our approach, that makes the $LALR(1)$ handled by Yacc to be preferred over $LR(1)$. Besides that has our approach the advantage of smoothly handling also longer look-aheads when needed. The lazy evaluation takes nicely care of the parallel pursuit for success that would be a nightmare to encode in C or Java.

• using the decision trees makes that at each choice point we have a complexity that is logarithmic in the number of possible next symbols. The simple parser combinators all are linear in that number, a fact that really starts to hurt when dealing with productions that have really many alternatives.

7 Conclusions

We have shown that it is possible to analyze grammars and construct parsers, that are both efficient and correct errors. The overhead for the error correction is only a few reductions per symbol in the absence of errors: adding the `Ok` step and removing it again.

When comparing the current approach with the one for the $LL(1)$ grammars we see we have now included all look-ahead information in one single data structure, thus getting a more uniform approach. Furthermore the decision about whether to insert or delete a symbol is done in a more local way, using precise look-ahead, especially in the case of follower symbols. This in general makes parsers continue longer in a more satisfactory way; previously a parser might prematurely decide to insert a sequence of symbols to complete a program, and throw away the rest of the input as being not needed.

When comparing the parsers coded using our library with those written in Yacc, the inputs look much nicer. Reduce-reduce conflict resulting from the incorporation of semantic actions do not occur, since always a sufficient context is taken into account. We conclude by referring to the title of this paper by claiming that finally parser combinators have reached the adult state.
and are the tool to be chosen when one wishes to construct a parser: already they were nice, but now they have become useful.

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References


