An interview with Henri Poincaré

Mathematics is the art of giving the same name to different things

It looked like a daunting and perhaps even impossible enterprise to interview the famous professor Henri Poincaré. However, it turned out to be possible on the condition that professor Poincaré was permitted to formulate the questions himself. We were happy to accept this condition. The interview is elaborated by Ferdinand Verhulst.

[We start with some questions about the foundations of mathematical thinking.]

Question: What is the nature of mathematical reasoning? Is it really deductive as is usually believed? [1, essay ‘Sur la nature de la raisonnement mathématique’]

Answer: A deep analysis shows us that it is not, that it uses to a certain measure inductive reasoning and in this way it is fruitful. Opening an arbitrary book on mathematics, we find the author announcing that he wants to generalize a known proposition. So, the mathematical method proceeds from the particular to the general and how is it then that we can call it deductive?

[Thinking about the foundations of mathematics can be interesting, but a question for the ‘mathematician at work’ is whether exploring the foundations actually improves our mathematical reasoning.]

Question: Have we achieved absolute rigour [in mathematics]? In each stage of the evolution, our predecessors believed to have achieved this. If they were wrong, are we not also wrong like them? [2, essay ‘L’intuition et la logique en mathématiques’]

Answer: We believe that in our reasoning, we don’t need intuition. The philosophers tell us that this is an illusion. Pure logic will lead us only to tautologies. This cannot create anything new, from logic alone no science can emerge. For the other thing we need, we have no other word than ‘intuition’. If one wants to take the trouble to be rigorous in today’s analysis, there are only syllogisms or an appeal to the intuition of numbers [induction], the on-
ly one which cannot deceive us. One can say that today absolute rigour has been achieved.

[You have been very productive in mathematics and in other fields, so a few questions about research strategy are of interest to us.]

Question: Pure analysis puts a great many procedures at our disposal with a guarantee for correct answers. But, from all the roads we can take, which one will lead us as quickly as possible to the goal? Who will tell us which one to choose? [2, essay ‘La science et la réalité’]

Answer: We need a faculty that shows us the goal from far away and this faculty is ‘intuition’. Logic and intuition both play a necessary part. Both are indispensable. Logic can give certainty only and is the instrument of proof, intuition is the instrument of invention.

[What can we say about the relation between mathematics, which is concerned with objects of the mind, and the empirical sciences. Some people from the physical sciences will say “mathematics is just a tool, we use as little of it as possible, most mathematics is too artificial.”]

Question: Experience is the only source of truth, only this can give us certainty. But if experience is everything, what place remains for mathematical physics? What has experimental physics to do with such a help which seems useless and perhaps even dangerous? [1, essay ‘Les hypothèses et la physique’]

Answer: The scientist must order; one makes science with facts as one makes a house with stones. A big collection of facts is no more a science than a heap of stones is a house. A collection still be the same? [4, essay ‘L’évolution des lois’]

Question: Were the laws of nature of former eras those of today? Will the laws of tomorrow still be the same? [4, essay ‘L’évolution des lois’]

Answer: If we imagine two minds similar to ours observing the universe on two occasions differing for example by millions of years, each of these minds will construct a science which will be a system of laws deduced from observed facts. It is probable that these sciences will be very different and in that sense it could be said that the laws have evolved. But however great the difference may be, it will always be possible to conceive of an intellect, of the same nature as ours but endowed with a much longer life, which will be able to complete the synthesis. To this intellect, the laws will not have changed, science will be unalterable; the scientists will merely have been imperfectly informed.

[Here is a very practical question that has to do with deterministic, but chaotic phenomena in nature. You were the first to write about this in your work on dynamical systems. Roughly 70 years after this, scientists became aware of these fundamental aspects of nature by the papers of Lorenz on an atmospheric model and by Hénon and Heiles on a galactic model. It is amazing that no scientist in physics or engineering picked this up before. The following questions and answers show your insight in the problem of the predictability of real-life phenomena.]

Question: Why have meteorologists such difficulty in predicting the weather with any certainty? [3, essay ‘Le hasard’]

Answer: We see that great disturbances are generally produced in regions where the atmosphere is in unstable equilibrium. The meteorologists see very well that the equilibrium is unstable, that a cyclone will be formed somewhere, but they are not in a position to say exactly where. A tenth of a degree more or less at any given point, and the cyclone will burst here and not there, and extends its ravages over districts it would otherwise have spared. Here, again, we find the same contrast between a very trifling cause that is inappreciable to the observer, and considerable effects, that are sometimes terrible disasters.

Question: Do these probability considerations apply outside science? [3, essay ‘Le hasard’]

Answer: It is the same in the humanities, and particularly in history. The historian is obliged to make a selection of the events in the period he is studying, and he only recounts those that seem to him the most important. Thus he contents himself with relating the most considerable events of the sixteenth century, for instance, and similarly the most remarkable facts of the seventeenth century. If the former are sufficient to explain the latter, we say that these latter conform to the laws of history. But if a great event of the seventeenth century owes its cause to a small fact of the sixteenth century that no history reports and that everyone has neglected, then we say that this event is due to chance, and so the word has the same sense as in the physical sciences; it means that small causes have produced great effects.

The greatest chance is the birth of a great man. No example can give a better understanding of the true character of chance.

[There is a special research topic that touches upon the fundamentals of mathematics, the understanding of the continuum of the real
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numbers. This topic was important in your time but it is still important.]

**Question:** What is exactly this continuum, this subject of mathematical reasoning? [1, essay ‘La grandeur mathématique et l’expérience’]

**Answer:** The continuum is nothing else than a set of elements, sequentially arranged, certainly infinite, but separated from each other. Our definition is however not complete. [...] One can ask whether the concept of mathematical continuum has not simply been derived from experience. One is forced to conclude that this idea has been created completely by the human spirit, but that experience has induced it.

**Question:** Is the creative power of the mind regarding the mathematical continuum exhausted? [1, essay ‘La grandeur mathématique et l’expérience’]

**Answer:** No, the work of Du Bois-Reymond shows this in a remarkable way. One knows that mathematicians distinguish between infinitesimal small quantities of different orders and that those of second order are not only infinitesimal small in an absolute sense, but also with respect to those of first order. It is not difficult to imagine infinitesimal small quantities of fractional or even irrational order and in this way we find again an ordering of the mathematical continuum.

[The styles and methods of proofs in mathematics have evolved since your time. With the enormous growth of both pure and applied mathematics there is now an abundance of mathematical styles, there are even computer-assisted proofs. Does it make sense to distinguish between beautiful and ugly mathematics, between elegant and graceless reasoning?]

**Question:** Mathematicians attach a great importance to the elegance of their methods and of their results, and this is not mere dilettantism. What is it that gives us the feeling of elegance in a solution or proof? [3, essay ‘L’avenir des mathématiques’]

**Answer:** It is the harmony of the different parts, their symmetry, and their happy adjustment; it is, in a word, all that introduces or-der, all that gives them unity, that enables us to obtain a clear comprehension of the whole as well as of the parts. Elegance may result from the feeling of surprise caused by the unlooked-for occurrence of objects not habitually associated. In this, again, it is fruitful, since it discloses thus relations that were until then unrecognized. Mathematics is the art of giving the same names to different things.

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[In your time, you were one of the last universal scientists, but then, scientists started to specialize in their research. Is this a danger or a necessity, or both?]

**Question:** How should one view specialization? [3, essay ‘L’avenir des mathématiques’]

**Answer:** As science develops, it becomes relatively more difficult to grasp it in its entirety. Then an attempt is made to cut it in pieces and to be satisfied with one of these pieces — in one word, to specialize. Too great a movement in this direction would constitute a serious obstacle to the progress of science. As I have said, it is by unexpected concurrences between its different parts that it can make progress. Too much specialization would prohibit these concurrences.

[You wrote many textbooks, but why are some people afraid of mathematics? To prove and invent new mathematics, one needs intuition, but to understand school mathematics one needs only some natural logic.]

**Question:** How is it that there are so many minds that are incapable of understanding
Henri Poincaré,

References