Groep 6

1 a $yy' - x = 0 \qquad y(0) = 5$

$\frac{dy}{dx} \cdot y = x$

$yy' = x$

$y\,dy = x\,dx$

$$\int_0^x \tilde{x}\,dx = \int_5^y \tilde{y}\,dy$$

$$\left[\tfrac{1}{2}\tilde{x}^2\right]_0^x = \left[\tfrac{1}{2}\tilde{y}^2\right]_5^y$$

$\frac{1}{2}x^2 - 0 = \frac{1}{2}y^2 - 12\frac{1}{2}$

$\frac{1}{2}x^2 - \frac{1}{2}y^2 + 12\frac{1}{2} = 0$

$x^2 - y^2 + 25 = 0$

$y^2 = x^2 + 25$

$y = \sqrt{x^2+25}$

Controle:

$yy' - x = 0 \qquad y = \sqrt{x^2+25}$

$y' = \frac{1}{2}(x^2+25)^{-\frac{1}{2}} \cdot 2x$

$= \frac{2x}{2\sqrt{x^2+25}} = \frac{x}{\sqrt{x^2+25}}$

$yy' - x = \sqrt{x^2+25} \cdot \frac{x}{\sqrt{x^2+25}} - x = x - x = 0$

Klopt

B $y'' - 4y = 0 \qquad y(0) = 4 \qquad y'(0) = -2$

$k^2 - 4 = 0$

$k^2 = 4$

$k = 2$ of $k = -2 \qquad k_1 = 2 \qquad k_2 = -2$

$D = b^2 - 4ac = 0 - 4 \cdot 1 \cdot -4 = 16 > 0$

$D > 0$ dus de algemene oplossing is:

$y(x) = Ae^{k_1 x} + Be^{k_2 x} = Ae^{2x} + Be^{-2x}$

$y(0) = 4$

$4 = Ae^0 + Be^0 \rightarrow A + B = 4$

$y'(x) = 2Ae^{2x} - 2Be^{-2x}$

$y'(0) = -2$

$-2 = 2Ae^0 - 2Be^0 \rightarrow 2A - 2B = -2$
$A = 4 - B$
$A - B = -1$
$(4-B) - B = -1 \rightarrow 4 - 2B = -1$
$B = \frac{5}{2} = 2\frac{1}{2}$
$A = 4 - B = 1\frac{1}{2}$
$y(x) = 1\frac{1}{2}e^{2x} + 2\frac{1}{2}e^{-2x}$
controle:
$y(x) = 1\frac{1}{2}e^{2x} + 2\frac{1}{2}e^{-2x}$
$y'(x) = 3e^{2x} - 5e^{-2x}$
$y''(x) = 6e^{2x} + 10e^{-2x}$
$y'' - 4y = 0$
$6e^{2x} + 10e^{-2x} - 4(1\frac{1}{2}e^{2x} + 2\frac{1}{2}e^{-2x}) = 0$
$6e^{2x} + 10e^{-2x} - 6e^{2x} - 10e^{-2x} = 0$
klopt

C $4y'' + 16y' + 17y = 0 \qquad y(0) = 4 \qquad y'(0) = -2$
$4k^2 + 16k + 17 = 0$
$D = b^2 - 4ac = 16^2 - 4 \cdot 4 \cdot 17 = -16 < 0$
$D < 0$ dus de algemene oplossing is:
$y(x) = e^{px} \cdot (A\cos\omega x + B\sin\omega x)$ met
$p = \frac{-b}{2a}$ en $\omega = \sqrt{4ac - b^2}/2a$
$p = -2$ en $\omega = \frac{1}{2}$
$y(x) = e^{-2x} \cdot (A\cos\frac{1}{2}x + B\sin\frac{1}{2}x)$
$y'(x) = -2e^{-2x} \cdot (A\cos\frac{1}{2}x + B\sin\frac{1}{2}x) + e^{-2x} \cdot (-\frac{1}{2}A\sin\frac{1}{2}x + \frac{1}{2}\cos\frac{1}{2}x)$
$y'(x) = -2y(x) + e^{-2x} \cdot (-\frac{1}{2}A\sin\frac{1}{2}x + \frac{1}{2}\cos\frac{1}{2}x)$
$y(0) = 4 = e^{-2 \cdot 0} \cdot (A\cos\frac{1}{2} \cdot 0 + B\sin\frac{1}{2} \cdot 0) = A$
$y'(0) = -2A + e^{-2 \cdot 0}(-\frac{1}{2}A\sin\frac{1}{2} \cdot 0 + \frac{1}{2}\cos\frac{1}{2} \cdot 0) = -2$
$= -2A + \frac{1}{2}B = -2$
$A = 4$ dus $B = 12$
oplossing:
$y(x) = e^{-2x}(4\cos\frac{1}{2}x + 12\sin\frac{1}{2}x)$

2 a De verandering van temperatuur ten opzichte van tijd is afhankelijk van het temperatuurverschil tussen het object en de omgeving, vermenigvuldigd met een constante. Deze is negatief, ~~om~~ waardoor de verandering positief wordt. Het object wordt warmer

b $\frac{dT}{T-A} = -k\,dt$

$$\int_{-18}^{-9} \frac{1}{T-A}\,dT = \int_{8}^{9} -k\,dt$$

$$\Big[\ln|(T-A)|\Big]_{-18}^{-9} = \Big[-kt\Big]_8^9$$

$$\ln|-9-20| - \ln|-18-20| = -k\cdot 9 + k\cdot 8$$

$$\ln\frac{29}{38} = -k$$

$$k = -\ln\frac{29}{38}$$

c $$\Big[\ln|(T-A)|\Big]_{-18}^{0} = \Big[+\ln\frac{29}{38}t\Big]_8^t$$

$$\ln|0-20| - \ln|-18-20| = \ln\frac{29}{38}t - 8\ln\frac{29}{38}$$

$$\ln 20 - \ln 38 = \ln\frac{29}{38}t - 8\ln\frac{29}{38}$$

$$\ln\frac{29}{38}t = \ln\frac{20}{38} + 8\ln\frac{29}{38}$$

$$t = \frac{\ln\frac{20}{38}}{\ln\frac{29}{38}} + \frac{8\ln\frac{29}{38}}{\ln\frac{29}{38}} = 8 + 2{,}37 = 10{,}37$$

$\frac{37}{100} \quad \frac{22}{60}$

10:22

10,37 is niet in minuten.

Dit is namelijk $\frac{37}{100}$ uur. Hoeveel van de 60 is dit? → $\frac{37 \cdot 60}{100} = 22$.

Het wordt dus 10:22

d $\frac{dT}{dt} = -k(T-A)$

$$\frac{dT}{T-A} = -k\,dt$$

$$\frac{1}{T-A}\,dT = -k\,dt$$

$$\int_{-18}^{0} \frac{1}{T-A}\,dT = \int_{8}^{9} -k\,dt$$

$$\left[\ln|T-A|\right]_{-18}^{0} = \left[-kt\right]_{8}^{9}$$

$$\ln|0-A| - \ln|-18-A| = \ln\frac{29}{38}$$

$$\cancel{|0-A| - |-18-A|} = \frac{29}{38}$$

$$\frac{|-A|}{|-18-A|} = \frac{29}{38}$$

$$\cancel{\frac{-A}{-18-A} = \frac{29}{38}} \quad -A = \frac{29}{38}(-18-A)$$

$$\cancel{-A = \frac{522}{38} - \frac{29}{38}A}$$

$$\cancel{\frac{29}{38}A} = -\frac{522}{38} \to A =$$

$$\frac{\frac{1}{A}(-A)}{\frac{1}{A}(-18-A)} = \frac{1}{\frac{18}{A}+1} = \frac{29}{38}$$

$$38 = 29\left(\frac{18}{A}+1\right)$$

$$\frac{38}{29} = \frac{18}{A}+1$$

$$\frac{38}{29}-1 = \frac{18}{A}$$

$$A = \frac{18}{\frac{38}{29}-1} = 58°C$$

3a) $\dot{u} = 4 - u^2$

$\dot{u} = 0$

$4 - u^2 = 0$

$-u^2 = -4$

$u^2 = 4$

$u = \sqrt{4} \quad \vee \quad u = -\sqrt{4}$

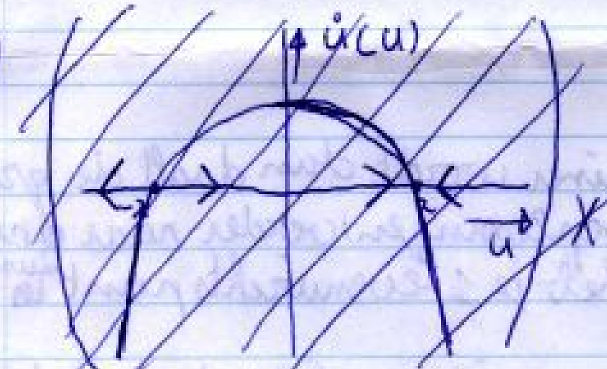
$u = 2 \quad \vee \quad u = -2$

karakter:

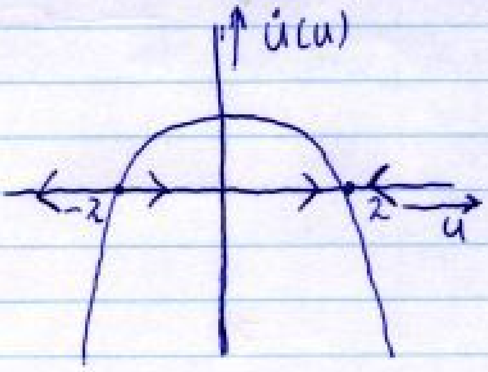
$\dot{u} = 4 - u^2$

$\ddot{u} = -2u$

$\ddot{u}(2) = -4$ | $\ddot{u}(-2) = 4$
$\ddot{u}(2) < 0$ | $\ddot{u}(-2) > 0$
dus stabiel | dus instabiel

b) 

$\dot{u}(u)$



c) $\dot{u} = \beta - u^2$

als je parameter 4 neemt krijg je 

als je parameter 2 neemt krijg je

$\dot{u} = 2 - u^2$

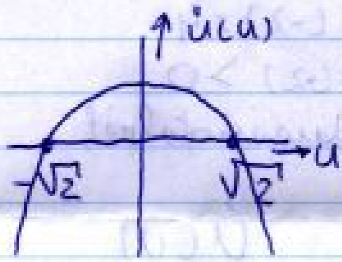
$\dot{u} = 0$

$2 - u^2 = 0$

$-u^2 = -2$

$u^2 = 2$

$u = \sqrt{2} \quad \vee \quad u = -\sqrt{2}$



als de parameter β kleiner wordt dan daalt de grafiek
omdat de evenwichtsoplossingen verder naar ~~d~~ nul gaan
en bij $\beta = 0$ dan heb je 1 evenwichtspunt dus ~~is~~ semi-
stabiel
en bij $\beta < 0$ dan is er geen snijpunt met de u-as