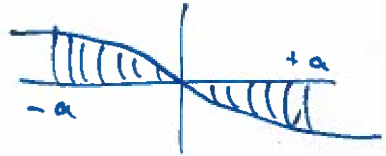
23/10/18 (1)

$\int_{-\infty}^{+\infty}$? (vraag van vorige week donderdag)

Zij f een oneven fie $\qquad f(-x) = -f(x)$

dan geldt $\int_{-a}^{a} f(x)\,dx = 0$ voor $a > 0$.



maar niet noodzakelijk: $\int_{-\infty}^{+\infty} f(x)\,dx = 0$.

want dit zijn twee limieten, dus splitsen: $\int_{-\infty}^{0} f(x)dx + \int_{0}^{\infty} f(x)dx$

<u>Vb 1</u> Neem $f(x) = x$ dit is een oneven fie.

$\int_{-\infty}^{0} x\,dx = \frac{1}{2}x^2 \Big|_{x=-\infty}^{x=0} = -\infty$ Dus divergent en $\int_{0}^{\infty} x\,dx = \frac{1}{2}x^2\Big|_{x=0}^{+\infty} = +\infty$ ook divergent

Je kunt niks zeggen over de Som $-\infty + \infty$

<u>Vb 2</u> Neem $f(x) = e^{-|x|}$

$\int_{-\infty}^{0} e^{-|x|}dx = \int_{0}^{\infty} e^{-x}dx = -e^{-x}\Big|_{x=0}^{x=\infty} = 0 - -1 = +1$ dus $\int_{-\infty}^{+\infty} e^{-|x|}dx = 2$

Convergent

Gonio substituties

boek § 6.3 (daar staat nog veel meer in. o.a. hyperbolische f-ies overslaan)

Onthoud: volgende 3 situaties

(i) integraal met $\sqrt{a^2-x^2}$: denk aan subs $\frac{x}{a} = \sin u$

(ii) $\sqrt{x^2-a^2}$: $x = \frac{1}{\cos u}$

(iii) a^2+x^2 : $x = \tan u$

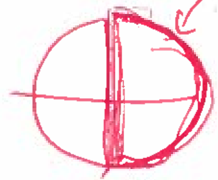
achteraf: ik had beter $a=1$ kunnen nemen

Voorbeelden

(i) • $\int \frac{dx}{\sqrt{1-x^2}}$ subs $x = \sin u$, $dx = \cos u \, du$ invullen geeft:

$-1 \leq x \leq +1$

u op deze helft



$$\int \frac{\cos u \, du}{\sqrt{1-\sin^2 u}} = \int \frac{\cos u \, du}{\cos u} = \int du = u + c$$

terugsubs: $u = \arcsin x$

Conclusie: $\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + c.$

- $\int \sqrt{25-(x)^2}\,dx = 5\int\sqrt{1-\left(\frac{x}{5}\right)^2}\,dx$

Subs' $x = 5\sin u$, $dx = 5\cos u\,du$, invullen:

$$5\int\sqrt{1-\sin^2 u}\,(5\cos u\,du)$$

$= 25\int \cos^2 u\,du$ verdub. form: $\cos 2u = 2\cos^2 u - 1$

dus $\cos^2 u = \frac{1}{2}(\cos 2u + 1)$

$= \frac{25}{2}\int \cos 2u + 1\,du$ etc.

$= \frac{25}{4}\sin 2u + \frac{25}{2}u + c$ terugsubs: als $x = 5\sin u$

dan is $\arcsin\frac{x}{5} = u$

~~Invullen~~:

$\sin 2u = 2\sin u\cos u$

$= 2\sin u\sqrt{1-\sin^2 u}$

met $\sin u = \frac{x}{5}$ geeft

$2\frac{x}{5}\sqrt{1-\frac{x^2}{5^2}}$

$$\boxed{\frac{25}{4}\left(\frac{2}{5}x\sqrt{1-\left(\frac{x}{5}\right)^2}\right) + \frac{25}{2}\arcsin\frac{x}{5} + c}$$

(ii) Voorbeeld.

$$\int \frac{dx}{\sqrt{x^2-1}} \qquad \text{subs.} \quad x = \frac{1}{\cos u} \quad \text{en} \quad dx = \frac{\sin u}{\cos^2 u}\, du \quad \text{invullen:}$$

$$\int \frac{1}{\sqrt{\frac{1}{\cos^2 u} - 1}} \left(\frac{\sin u}{\cos^2 u}\, du \right) \qquad \text{merk op:} \quad \sqrt{\frac{1}{\cos^2 u} - 1} = \sqrt{\frac{1-\cos^2 u}{\cos^2 u}}$$

$$\|$$

$$\int \sqrt{\frac{\cos^2 u}{1-\cos^2 u}}\, \frac{\sin u}{\cos^2 u}\, du = \int \sqrt{\frac{1}{1-\cos^2 u}}\, \frac{\sin u}{\cos u}\, du$$

$$= \int \frac{1}{\sin u}\, \frac{\sin u}{\cos u}\, du = \int \frac{1}{\cos u}\, du \rightarrow \text{inleveropg} \rightarrow \text{~~[illegible]~~}$$

$$\rightarrow \text{terugsubs} \rightarrow \log\left| x + \sqrt{x^2-1} \right| + c.$$

(iii) $\int \frac{dx}{\sqrt{4+x^2}} = \frac{1}{2}\int \frac{dx}{\sqrt{1+\left(\frac{x}{2}\right)^2}}$ subs. $\frac{x}{2} = \tan u$

of $x = 2\tan u$

$$dx = 2\left(\frac{1}{\cos^2 u}\right) du$$
$$= 2(1+\tan^2 u)\, du$$

Merk op: $\sqrt{1+\left(\frac{x}{2}\right)^2} = \sqrt{1+\tan^2 u} = \sqrt{\frac{1}{\cos^2 u}} = \frac{1}{\cos u}$

$$\frac{1}{2}\int \frac{dx}{\sqrt{1+\left(\frac{x}{2}\right)^2}} = \frac{1}{2}\int \frac{2(1+\tan^2 u)\, du}{\sqrt{1+\tan^2 u}}$$

of $= \frac{1}{2}\int \cos u \left(2\frac{1}{\cos^2 u}\, du\right)$

$= \int \frac{du}{\cos u}$ etc.

Intermezzo (geen stof)

$$\int \frac{dx}{1-x^2} \longrightarrow \frac{1}{2}\int \frac{1}{1-x} + \frac{1}{1+x}\,dx$$

$$= \frac{1}{2}\Big(-\log|1-x| + \log|1+x|\Big) + c$$

$$= \boxed{\frac{1}{2}\log\left|\frac{1+x}{1-x}\right|} + c$$

HUH?!

$$\int \frac{dx}{1+x^2} = \arctan x$$

$$\frac{1}{1+x^2} = \frac{1}{(1+ix)(1-ix)} = \frac{1}{2}\left(\frac{1}{1+ix} + \frac{1}{1-ix}\right)$$

$$\int \frac{dx}{1+x^2} = \frac{1}{2}\left(\int \frac{dx}{1+ix} + \int \frac{dx}{1-ix}\right) = \frac{1}{2i}\Big(\log(1+ix) - \log(1-ix)\Big)$$

$$= \boxed{\frac{1}{2i}\log\frac{1+ix}{1-ix}}$$

Geavanceerde theorie
geen Infi-stof!

§ 3.3. Logaritmisch differentiëren.

dwz: eerst log nemen, daarna diff.

Vb: Zij $f(x) = (1+x)(1+2x)^2(1+3x)^3$

Gevraagd: bepaal $f'(0)$

Oplossing: Log. diff.

neem eerst log: $\log f(x) = \log(1+x) + 2\log(1+2x) + 3\log(1+3x)$

nu diff: $$\frac{f'(x)}{f(x)} = \frac{1}{1+x} + \frac{4}{1+2x} + \frac{9}{1+3x}$$

dus $$f'(x) = f(x)\left(\frac{1}{1+x} + \frac{4}{1+2x} + \frac{9}{1+3x}\right)$$

en $$f'(0) = f(0)(1+4+9) = 14.$$

Zelf: vind de afgeleide van $f(x) = x^x$ mbv. log. diff.