Opg 1 Zij $z = e^{ix}$. Laat zien $\frac{z^2 - z + 1}{z}$ reëel is.

Eerst herschrijven we: $\boxed{z - 1 + \frac{1}{z}}$

We substitueren $z = e^{ix}$

$e^{ix} - 1 + e^{-ix}$

$= \cos x + i \sin x - 1 + \cos x + i \sin(-x)$

We weten

cos is even functie, dus $\cos -x = \cos x$

sin is oneven functie, dus $\sin -x = -\sin x$

$\cos x + i \sin x - 1 + \cos -x + i \sin -x$

$= \cos x + \cancel{i \sin x} - 1 + \cos x - \cancel{i \sin x}$.

$= 2\cos x - 1$

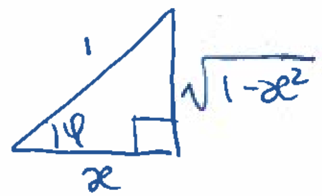
Conclusie $\frac{z^2 - z + 1}{z}$ met $z = e^{ix}$ is reëel opg.

2

$\tan(\arccos x)$

MAP!

$\arccos x = \varphi$ $\cos\varphi = \frac{x}{1}$



pytagoras

$$x^2 + \text{over}^2 = 1$$

$$\text{over} = \sqrt{1-x^2}$$

$$\tan(\varphi) = \frac{\sqrt{1-x^2}}{x}$$

OPG3

$$f(x)=\log(\sqrt{1+x^2}\;4-x)$$

$$-f(x) \overset{?}{=} f(-x)$$

$$-\log(\sqrt{1+x^2}-x) \overset{?}{=} \log(\sqrt{1+x^2}+x)$$

$$\log\left(\frac{1}{\sqrt{1+x^2}-x}\right) \overset{?}{=} \log(\sqrt{1+x^2}+x)$$

$$\frac{1}{\sqrt{1+x^2}-x} \overset{?}{=} \sqrt{1+x^2}+x$$

$$1 \overset{?}{=} (\sqrt{1+x^2}+x)(\sqrt{1+x^2}-x)$$

$$1 \overset{?}{=} 1+x^2-x^2=1$$

$$a\log(x)=\log(ax^a)$$

$$(a-b)(a+b)=a^2-b^2$$

$\boxed{\text{OPG. 4}}$

$$f(x) = \sqrt{1-x^2}$$

$$\cancel{T_4(x) = f(x) + f'(x) + \frac{1}{2} f''(x) + \frac{1}{6} f'''(x) + \frac{1}{24} f''''(x)}$$

$$T_4(x) = f(0) + f'(0)x + \frac{1}{2} f''(0)x^2 + \frac{1}{6} f'''(0)x^3 + \frac{1}{24} f''''(0)x^4$$

$$f(0) = \sqrt{1-0} = 1$$

$$f'(x) = \frac{1}{2\sqrt{1-x^2}} \cdot (-2x) = -\frac{x}{\sqrt{1-x^2}} = -x\,(1-x^2)^{-\frac{1}{2}}$$

$$f'(0) = 0$$

$$f''(x) = -(1-x^2)^{-\frac{1}{2}} - x\left(-\frac{1}{2}(1-x^2)^{-1\frac{1}{2}} \cdot (-2x)\right)$$

$$= -(1-x^2)(1-x^2)^{-1\frac{1}{2}} - x^2(1-x^2)^{-1\frac{1}{2}}$$

$$= (1-x^2)^{-1\frac{1}{2}} \cdot (-1) = -(1-x^2)^{-1\frac{1}{2}}$$

$$f''(0) = -(1-0)^{-1\frac{1}{2}} = -1.$$

$\cancel{f''''(x)}$

$f''(x) = -(1-x^2)^{\cancel{-1\frac{1}{2}}}$

$f'''(x) = -(-1\frac{1}{2})(1-x^2)^{-2\frac{1}{2}} \cdot (-2x)$

$= \underbrace{-3x}(1-x^2)^{-2\frac{1}{2}}$

$f'''(0) = 0$

$f''''(x) = \cancel{-3x} -3(1-x^2)^{-2\frac{1}{2}} - 3x(-2\frac{1}{2}(1-x^2)^{-3\frac{1}{2}} \cdot (-2x))$

$\cancel{= 2 - 3(1-x^2)}$

$f''''(0) = -3 \cdot 1 \quad \cancel{+} \quad = -3$

$T_4(x) = f(0) + f'(0)x + f''(0) \cdot \frac{1}{2}x^2 + f''' \cdot \frac{1}{6}x^3 + f'''' \cdot \frac{1}{24}x^4$

$T_4(x) = 1 - \frac{1}{2}x^2 - \frac{1}{8}x^4$

$$\sqrt{1-x^2}$$

$$\frac{1}{2}\sqrt{3} = \sqrt{\frac{3}{4}} = \sqrt{1-\frac{1}{4}} = \sqrt{1-\left(\frac{1}{2}\right)^2} \approx T_4\left(\frac{1}{2}\right)$$

$$= 1 - \frac{1}{2}\cdot\left(\frac{1}{2}\right)^2 - \frac{1}{8}\left(\frac{1}{4}\right)^4$$

$$= 1 - \frac{1}{8} - \frac{1}{128}$$

$$= \frac{128}{128} - \frac{16}{128} - \frac{1}{128} = \boxed{\frac{111}{128}}$$

OPG 5

Onderzoek of $\lim_{x \to \infty} \frac{4x - 3\cos(2\sqrt{x+1})}{5-6x}$ bestaat.

Herschrijven

$$\frac{4 - \frac{3\cos(2\sqrt{x+1})}{x}}{\frac{5}{x} - 6}$$

dit beest ??

die weet je!

$$\lim_{x \to \infty} \frac{3\cos(2\sqrt{x+1})}{x} = ?$$

insluitstelling!

$$-1 \leq \cos x \leq 1$$

$$-1 \leq \cos(2\sqrt{x+1}) \leq 1$$

$$-3 \leq 3\cos(2\sqrt{x+1}) \leq 3$$

$$\underbrace{-\frac{3}{x}}_{} \leq \underbrace{\frac{3\cos(2\sqrt{x+1})}{x}}_{g(x)} \leq \underbrace{\frac{3}{x}}_{h(x)}$$

$$f(x) = -\frac{3}{x}$$

$$\lim_{x \to \infty} f(x) = 0 \qquad \lim_{x \to \infty} h(x) = 0 \qquad \lim_{x \to 0} g(x) \text{ dan ook } 0$$

invullen

$$\frac{4 - 0}{0 - 6} = \boxed{-\frac{2}{3}}$$

OPG. 6.

$y = \sqrt{x}^{\sin x}$, bereken y'.

$$\ln(y) = \ln(\sqrt{x}^{\sin x})$$
$$= \sin(x) \cdot \ln(\sqrt{x})$$
$$= \tfrac{1}{2}\sin(x) \cdot \ln(x)$$

$$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}\left(\tfrac{1}{2}\sin x \cdot \ln x\right)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{2}\left(\sin x \cdot \frac{1}{x} + \cos x \cdot \ln x\right)$$
$$= \frac{1}{2}\frac{\sin x}{x} + \frac{1}{2}\cos x \cdot \ln x$$

$$\frac{dy}{dx} = y\left(\frac{1}{2}\frac{\sin x}{x} + \frac{1}{2}\cos x \cdot \ln x\right)$$
$$= \sqrt{x}^{\sin(x)} \cdot \left(\frac{1}{2}\frac{\sin x}{x} + \frac{1}{2}\cos x \ln x\right)$$
$$= \frac{1}{2}\frac{\sin x}{x} \cdot \sqrt{x}^{\sin x} + \frac{1}{2}\cos x \cdot \ln x \cdot \sqrt{x}^{\sin x}.$$

OPG 7A

$$\int_t^{\infty} \cancel{4}t e^{-tx} dx =$$

$$\lim_{R \to \infty} -e^{-tR} + e^{-t \cdot t}$$

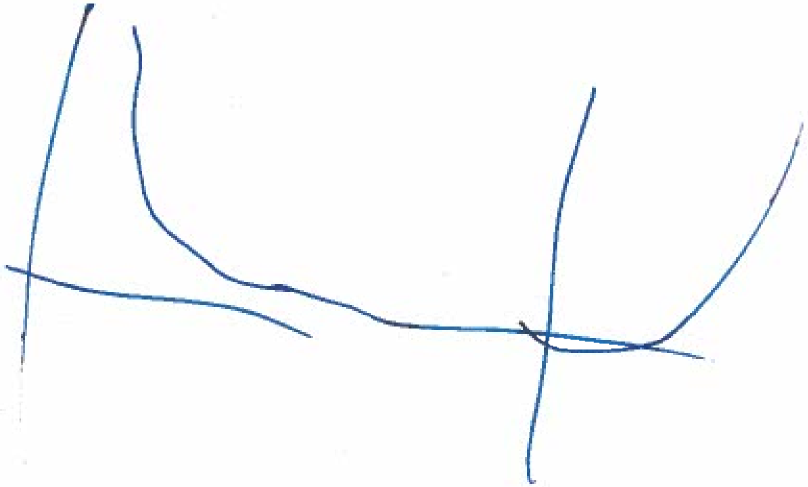
3 geval:

$t = 0$: 0

$t > 0$: $0 + e^{-t^2} = \frac{1}{e^{t^2}}$

$t < 0$: divergent, Limiet bestaat niet

$$\int \cancel{4}te^{-tx} dx = -e^{-tx} + C$$



7b) $\int \frac{x^2}{\sqrt{2-x^2}}\,dx$

$$\boxed{\begin{aligned} x &= \sqrt{2}\,\sin\varphi \\ dx &= \sqrt{2}\,\cos\varphi\,d\varphi \end{aligned}}$$

$\sin(\varphi) = \frac{x}{\sqrt{2}} = \frac{1}{2}\sqrt{2}x$

$\varphi = \arcsin\left(\frac{1}{2}\sqrt{2}x\right)$

$$\int \frac{(\sqrt{2}\sin\varphi)^2}{\sqrt{2-2\sin^2\varphi}}\sqrt{2}\cos\varphi\,d\varphi$$

$$\int \frac{2\sin^2\varphi\,\sqrt{2}\cos\varphi}{\sqrt{2-2\sin^2\varphi}}\,d\varphi$$

$$\int \frac{}{\sqrt{2}\,\cos\varphi}\,d\varphi$$

$$\int 2\sin^2\varphi\,d\varphi$$

$$\boxed{\begin{aligned} &\cancel{1-\sin^2\varphi = \cos^2\varphi} \\ &\cos 2\varphi = \cos^2\varphi - \sin^2\varphi \\ &\qquad\;\; = 1 - 2\sin^2\varphi \\ &\sin^2\varphi = \tfrac{1}{2} - \tfrac{1}{2}\cos 2\varphi \end{aligned}}$$

$$2\int \frac{1}{2} - \frac{1}{2}\cos(2\varphi)\,d\varphi$$

$$\int 1\,d\varphi - \int \cos 2\varphi\,d\varphi$$

$$\varphi - \frac{1}{2}\sin(2\varphi) + C$$

$$\varphi - \sin\varphi\cos\varphi + C$$

$$\arcsin\left(\tfrac{1}{2}\sqrt{2}x\right) - \left(\tfrac{1}{2}\sqrt{2}x\right)\left(\sqrt{1-\tfrac{x^2}{2}}\right) +$$

$$\arcsin\left(\tfrac{1}{2}\sqrt{2}x\right) - \tfrac{1}{2}x\sqrt{2-x^2} + C$$

$$\varphi - \sin\varphi\left(\sqrt{1-\sin^2\varphi}\right) + C'$$

7c) $\int x \arctan(2x)\, dx$

$k = 2x \qquad dk = 2dx$

$\int x \arctan(2x) = \frac{1}{2}\cdot\frac{1}{2}\int k \cdot \arctan(k)\, dk$

$u' = k \qquad v = \arctan(k)$

$u = \frac{1}{2}k^2 \qquad v' = \frac{1}{1+k^2}\, dk$

$\frac{1}{4}\int k \cdot \arctan(k)\, dk = \frac{1}{4}\left(\frac{1}{2}k^2 \arctan(k) - \frac{1}{2}\int \frac{k^2}{1+k^2}\, dk\right)$

$= \frac{1}{8}k^2 \arctan(k) - \frac{1}{8}\int \frac{k^2}{1+k^2}\, dk$

$\int \frac{k^2}{1+k^2}\, dk$

$k = \tan(w) \qquad dk = \tan^2(w) + 1\, dw$

$\int \frac{\tan^2(w) \cdot (\tan^2(w)+1)\, dw}{1+\tan^2(w)} = \int \tan^2(w)\, dw$

$\cos^2(w) = \frac{1}{\tan^2(w)+1}$ aus $\tan^2(w) = \frac{1}{\cos^2(w)} - 1$

$\int \frac{1}{\cos^2(w)} - 1\, dw = \int \frac{1}{\cos^2(w)}\, dw - \int 1\, dw$

$= \tan(w) - w + C$

$$= k - \arctan(k) + C$$

$$\frac{1}{8}k^2 \arctan(k) - \frac{1}{8}\int \frac{k^2}{1+k^2}\,dk$$

$$= \frac{1}{8}k^2 \arctan(k) - \frac{1}{8}k + \frac{1}{8}\arctan(k) + C$$

$$= \frac{1}{2}x^2 \arctan(2x) - \frac{1}{4}x + \frac{1}{8}\arctan(2x) + C$$

OPG 8

$$\ddot{x} + 4\dot{x} + 4x = e^{-3t}$$

$$x(0) = 0$$
$$\dot{x}(0) = 4$$

$$x(t) = x_H + x_p$$

$$\ddot{x} + 4\dot{x} + 4x = e^{-3t}$$

$$\lambda^2 + 4\lambda + 4 = 0$$

$$\lambda_{1,2} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 4}}{2} = \frac{-4 \pm \sqrt{0}}{2} = -2$$

$$x_H = (A + Bt)e^{\lambda t} \quad A \text{ en } B \text{ constant}$$

$$x_H = (A + Bt)e^{-2t}$$

$$x_p(t) = C_1 e^{-3t}$$
$$\dot{x}_p(t) = -3 \cdot C_1 e^{-3t}$$
$$\ddot{x}_p(t) = 9 \cdot C_1 e^{-3t}$$

$$9 \cdot C_1 e^{-3t} + 4 \cdot -3 \cdot C_1 \cdot e^{-3t} + 4 \cdot C_1 e^{-3t} = e^{-3t}$$

$$(9 - 12 + 4) C_1 \cdot e^{-3t} = e^{-3t}$$

$$C_1 \cdot e^{-3t} = e^{-3t}$$

$$C_1 = 1$$

$$x_p(t) = e^{-3t}$$

$$x(t) = x_H + x_p$$

$$x(t) = (A + Bt)e^{-2t} + e^{-3t}$$

$$x(0) = 0$$

$$(A+B\cdot 0)e^{-2\cdot 0}+e^{-3\cdot 0}=0$$

$$A+1=0$$

$$A=-1$$

$$X(t)=(-1+Bt)e^{-2t}+e^{-3t}$$

$$\dot{X}(t)=-2(-1+Bt)e^{-2t}+B\cdot e^{-2t}-3e^{-3t}$$

$$\dot{X}(0)=4$$

$$-2(-1+B\cdot 0)e^{-2\cdot 0}+B\cdot e^{-2\cdot 0}-3e^{-3\cdot 0}=4$$

$$2+B-3=4 \qquad B=5$$

$$\underline{X(t)=(5t-1)e^{-2t}+e^{-3t}}$$