**Babylonisch rekenen (1)**
**tablet YBC 7344**

Probeer aan de hand van dit nagetekende kleitablet uit te vinden hoe de Babyloniërs getallen schreven en wat er op dit kleitablet staat.

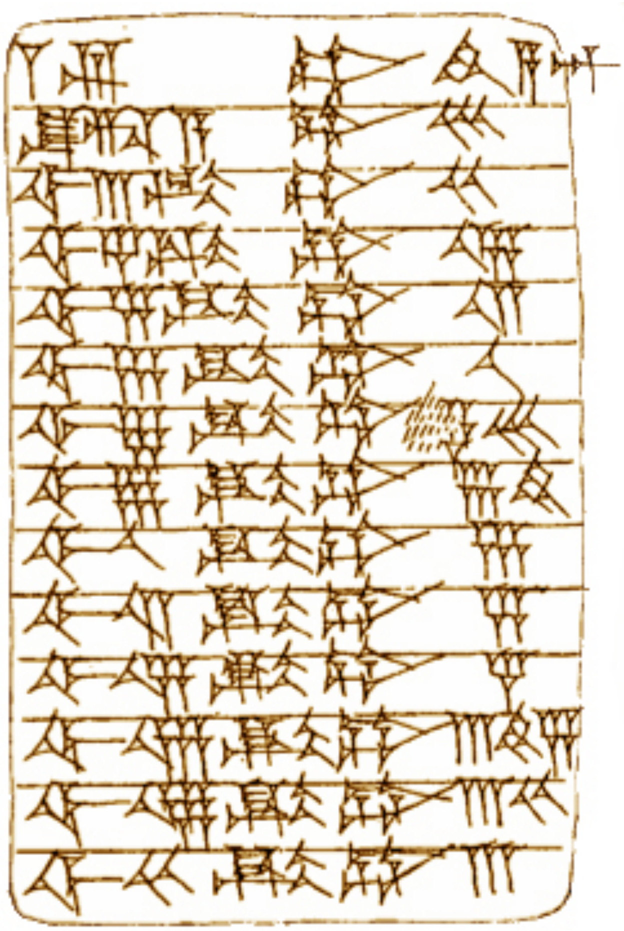
Tip:  is een woord, geen getal.

Bron: Babylonische wiskunde – A. Van der Roest en M. Kindt, Zebra-deeltje, Epsilon Uitgaven

**Babylonisch rekenen (2)**

We gaan in deze opgave het onderstaande kleitablet onderzoeken. Op dit kleitablet staan naast wat getallen ook veel woorden. Die kun je natuurlijk niet lezen. Maar de getallen wel, en aan de getallen kun je een patroon zien.

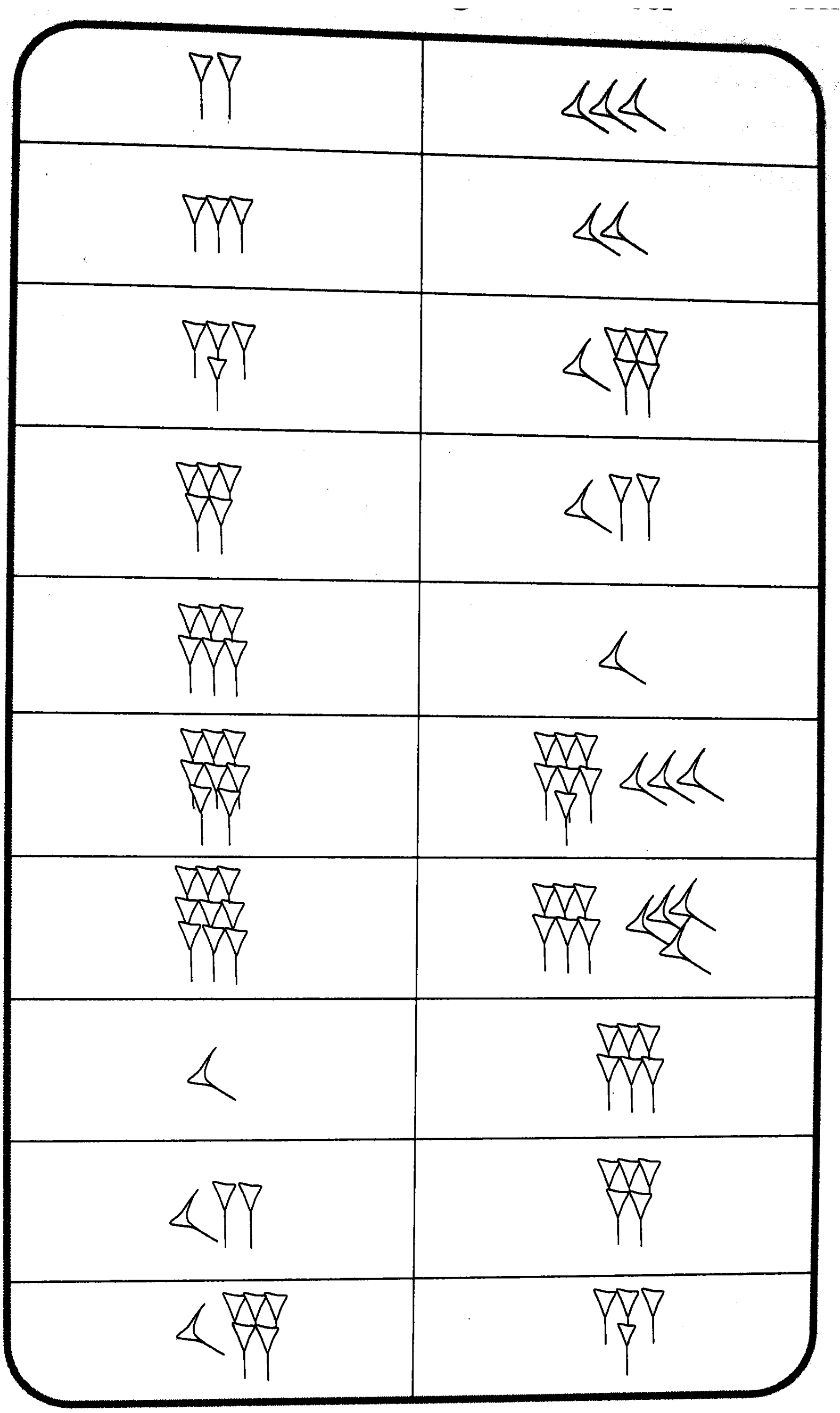
Welke getallen zie je in deze tabel? Als dat nog lastig is: op de achterkant van dit werkblad zijn de getallen op regels 2 t/m 11 nagetekend, kijk of je die getallen ook op het moeilijkere plaatje terug kunt vinden.



1) Schrijf deze tabel eens op met gewone getallen. Zie je een patroon? Waarvoor zou deze tabel gebruikt kunnen worden?

2) Als wij in ons getalsysteem een tabel zouden maken met dezelfde bedoeling, hoe zou die er dan kunnen uitzien?

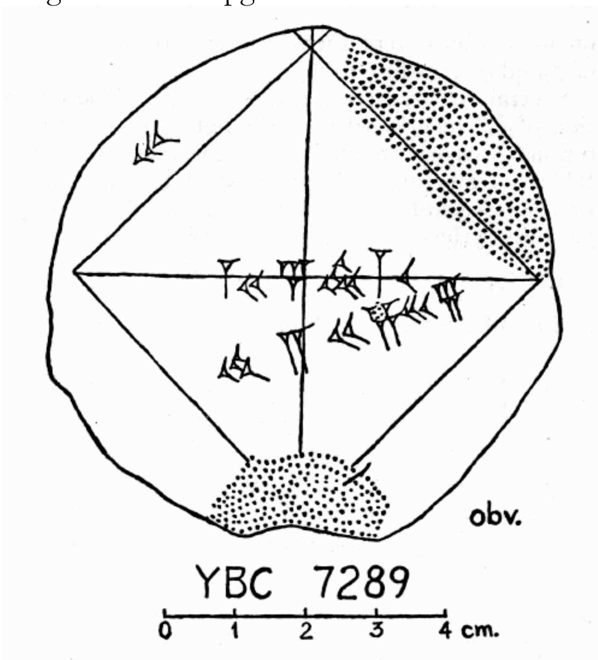
3) Kun je bedenken waarom bepaalde getallen ontbreken in de linkerkolom?



Bron van dit plaatje: Babylonische wiskunde – A. Van der Roest en M. Kindt, Zebra-deeltje, Epsilon Uitgaven

**Babylonisch rekenen (3)**
**tablet YBC 7289: Vierkant met diagonalen**

We gaan in deze opgave het onderstaande kleitablet onderzoeken.




**a)** Er zijn drie getallen te zien op dit kleitablet. Schrijf zestigtallig op welke getallen je afleest.

Om er achter te komen welke getallen precies bedoeld zijn (waar de "komma" hoort), gaan we ook eens naar het plaatje kijken. Je ziet een vierkant met twee diagonalen.
**b)** Eén van de getallen staat bij een zijde van het vierkant. Welk getal zou daar kunnen staan, tientallig? Geef de twee waarschijnlijkste opties.

**c)** Het bovenste van de andere twee getallen is best lang. Neem aan dat in dit getal de "komma" na de eerste 1 staat. Schrijf dit getal in het tientallig stelsel.
* Ken je dit getal ergens van? (Zo nee, geeft niks, ga gewoon verder met het volgende onderdeel.)

**d)** Bereken ook het derde getal in het tientallig stelsel, geef een paar mogelijke oplossingen.

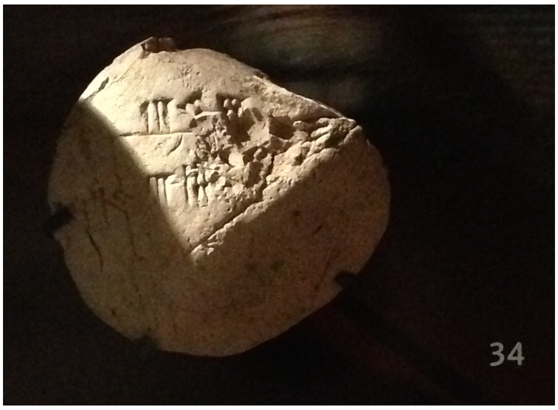
**e)** Kies nu één van de opties van vraag b). Stel dat de zijde van het vierkant inderdaad die lengte heeft, hoe lang is dan de diagonaal van het vierkant? Geef duidelijk aan hoe je aan je antwoord komt. Zie je dit getal eerder in deze opgave terug?

**f)** Stel dat je een vierkant hebt met onbekende zijden van lengte *a*. Wat moet je met dat getal *a* doen om de lengte van de diagonaal te krijgen?

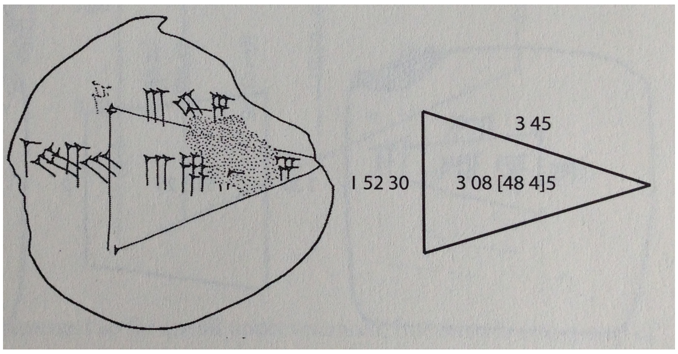
**g)** Zie je een verband tussen de drie getallen en het plaatje? Leg uit wat er volgens jou op dit kleitablet staat.

**Babylonisch rekenen (5): rekenfout**

In het Ashmolean Museum in Oxford is het volgende kleitablet te zien:




Het is een beetje lastig te lezen wat er precies op staat, maar Eleonor Robson heeft een transcriptie gemaakt[1].




De opdracht voor de student die dit kleitablet maakte was: vind de oppervlakte van een rechthoekige driehoek waarvan twee rechthoekszijdes gegeven zijn. Het correcte antwoord zou zijn: 3;45 × 1;52,30 × 0;30 = 3;30,56,15.
a) Waarom wordt er vermenigvuldigd met 0;30?

Deze student heeft echter een fout gemaakt in zijn berekening.
b) Bereken cijferend zestigtallig de volgende vermenigvuldiging:

```
      3 45
 × 1 52 30
 _________
```

c) Schrijf deze twee getallen en je uitkomst om naar het tientallig stelsel en controleer of je uitkomst klopt.
d) Halveer je uitkomst zestigtallig. Als het goed is, kom je inderdaad op 3;30,56,15.

e) Deze student heeft in de eerste vermenigvuldiging een fout gemaakt: de 45 is in de verkeerde "kolom" terechtgekomen. Laat zien dat er in dat geval inderdaad het foute antwoord 3;8,48,45 uitkomt.

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[1] Katz (ed.), The Mathematics of Egypt, Mesopotamia, China, India, and Islam - A source book, Princeton University Press, 2007, p. 143

**Inleveropdracht GvdW: $\sqrt{2}$.** **deadline di 13/2/24 13:15**

In bijgaande kopie (bron: [1, p. 30–32]) geeft C.F. Boyer op p. 31 een mogelijk algoritme waarmee de Mesopotamiërs $\sqrt{2}$ zouden kunnen hebben benaderd. Voer het algoritme uit in het sexagesimale getalstelsel en ga na of het inderdaad de juiste uitkomst geeft.

Beschouw je berekening kritisch in het licht van de in het college behandelde stof, en probeer aan die beschouwing conclusies te verbinden ten aanzien van het door Boyer voorgestelde algoritme.

Verwerk je resultaten in een "ingezonden brief" aan een denkbeeldig Nederlandstalig vaktijdschrift over geschiedenis van de wiskunde. In de brief moet je in ieder geval het algoritme uitvoeren en je conclusies motiveren. Alleen de brief hoeft ingeleverd te worden.

# Referenties

[1] C. F. Boyer, *A History of Mathematics*, New York 1968.

a numeral was missing. From that time on, as long as cuneiform was used, the number 𒐖 𒑱 𒐖, or $2(60)^2 + 0(60) + 2$, was readily distinguishable from 𒐖 𒐖, or $2(60) + 2$.

The Babylonian zero symbol apparently did not end all ambiguity, for the sign seems to have been used for intermediate empty positions only. There are no extant tablets in which the zero sign appears in a terminal position. This means that the Babylonians in antiquity never achieved an absolute positional system. Position was relative only; hence the symbol 𒐖 𒐖 could represent $2(60) + 2$ or $2(60)^2 + 2(60)$ or $2(60)^3 + 2(60)^2$ or any one of indefinitely many other numbers in which two successive positions are involved.

3 Had Mesopotamian mathematics, like that of the Nile Valley, been based on the addition of integers and unit fractions, the invention of the positional notation would not have been of great significance at the time. It is not much more difficult to write 98,765 in hieroglyphic notation than in cuneiform, and the latter is definitely more difficult to write than the same number in hieratic script. The secret of the clear superiority of Babylonian mathematics over that of the Egyptians undoubtedly lies in the fact that those who lived "between the two rivers" took the most felicitous step of extending the principle of position to cover fractions as well as whole numbers. That is, the notation 𒐖 𒐖 was used not only for $2(60)+2$, but also for $2+2(60)^{-1}$ or for $2(60)^{-1} + 2(60)^{-2}$ or for other fractional forms involving two successive positions. This meant that the Babylonians had at their command the computational power that the modern decimal fractional notation affords us today. For the Babylonian scholar, as for the modern engineer, the addition or the multiplication of 23.45 and 9.876 was essentially no more difficult than was the addition or multiplication of the whole numbers 2345 and 9876; and the Mesopotamians were quick to exploit this important discovery. An Old Babylonian tablet from the Yale Collection (No. 7289) includes the calculation of the square root of two to three sexagesimal places, the answer being written 𒐕 𒌍𒐘 𒐏𒐕 𒌋. In modern characters this number can be appropriately written as 1;24,51,10, where a semicolon is used to separate the integral and fractional parts and a comma is used as a separatrix for the sexagesimal positions. This form will generally be used throughout this chapter to designate numbers in sexagesimal notation. This Babylonian value for $\sqrt{2}$ is equal to approximately 1.414222, differing by about 0.000008 from the true value. Accuracy in approximations was relatively easy for the Babylonians to achieve with their fractional notation, the best that any civilization afforded until the time of the Renaissance.

4 The effectiveness of Babylonian computation did not result from their system of numeration alone. Mesopotamian mathematicians were skillful

in developing algorithmic procedures, among which was a square-root process often ascribed to later men. It sometimes is attributed to the Greek scholar Archytas (428–365 B.C.) or to Heron of Alexandria (ca. 100); occasionally one finds it called Newton's algorithm. This Babylonian procedure is as simple as it is effective. Let $x = \sqrt{a}$ be the root desired and let $a_1$ be a first approximation to this root; let a second approximation $b_1$ be found from the equation $b_1 = a/a_1$. If $a_1$ is too small, then $b_1$ is too large, and vice versa. Hence the arithmetic mean $a_2 = \frac{1}{2}(a_1 + b_1)$ is a plausible next approximation. Inasmuch as $a_2$ always is too large, the next approximation $b_2 = a/a_2$ will be too small, and one takes the arithmetic mean $a_3 = \frac{1}{2}(a_2 + b_2)$ to obtain a still better result; the procedure can be continued indefinitely. The value of $\sqrt{2}$ on Yale table 7289 will be found to be that of $a_3$, where $a_1 = 1;30$. In the Babylonian square-root algorithm one finds an iterative procedure that could have put the mathematicians of the time in touch with infinite processes, but scholars of the time did not pursue the implications of such problems.

The algorithm just described is equivalent to a two-term approximation to the binomial series, a case with which the Babylonians were familiar. If $\sqrt{a^2 + b}$ is desired, the approximation $a_1 = a$ leads to $b_1 = (a^2 + b)/a$ and $a_2 = (a_1 + b_1)/2 = a + b/(2a)$, which is in agreement with the first two terms in the expansion of $(a^2 + b)^{1/2}$ and provides an approximation found in Old Babylonian texts. Despite the efficacy of their rule for square roots, the Mesopotamian scribes seem to have imitated the modern applied mathematician in having frequent recourse to the ubiquitous tables that were available. In fact, a substantial proportion of the cuneiform tablets that have been unearthed are "table texts," including multiplication tables, tables of reciprocals, and tables of squares and cubes and of square and cube roots written, of course, in cuneiform sexagesimals. One of these, for example, carries the equivalents of the entries shown in the table below. The product

| | |
|---|---|
| 2 | 30 |
| 3 | 20 |
| 4 | 15 |
| 5 | 12 |
| 6 | 10 |
| 8 | 7,30 |
| 9 | 6,40 |
| 10 | 6 |
| 12 | 5 |

of elements in the same line is in all cases 60, the Babylonian number base, and the table apparently was thought of as a table of reciprocals. The sixth line, for example, denotes that the reciprocal of 8 is $7/60 + 30/(60)^2$. It will be noted that the reciprocals of 7 and 11 are missing from the table, because the reciprocals of such "irregular" numbers are nonterminating sexagesimals, just as in our decimal system the reciprocals of 3, 6, 7, and 9 are infinite when expanded decimally. Again the Babylonians were faced by the problem of infinity, but they did not consider it systematically. At one point, however, a Mesopotamian scribe seems to give upper and lower bounds for the reciprocal of the irregular number 7, placing it between 0;8,34,16,59 and 0;8,34,18. With their penchant for multipositional computations, it is tantalizing not to find among them a recognition of the simple three-place periodicity in the sexagesimal representation of $\frac{1}{7}$, a discovery that could have provoked considerations of infinite series.

It is clear that the fundamental arithmetic operations were handled by the Babylonians in a manner not unlike that which would be employed today, and with comparable facility. Division was not carried out by the clumsy duplication method of the Egyptians, but through an easy multiplication of the dividend by the reciprocal of the divisor, using the appropriate items in the table texts. Just as today the quotient of 34 divided by 5 is easily found by multiplying 34 by 2 and shifting the decimal point, so in antiquity the same division problem was carried out by finding the product of 34 by 12 and shifting one sexagesimal place to obtain $6\frac{48}{60}$. Tables of reciprocals in general furnished reciprocals of "regular" integers only—that is, those that can be written as products of twos, threes, and fives—although there are a few exceptions. One table text includes the approximations $\frac{1}{59}$ = ;1,1,1 and $\frac{1}{61}$ = ;0,59,0,59. Here we have sexagesimal analogues of our decimal expressions $\frac{1}{9} = .1\overline{1}$ and $\frac{1}{11} = .0\overline{909}$, unit fractions in which the denominator is one more or one less than the base; but it appears again that the Babylonians did not notice, or at least did not regard as significant, the infinite periodic expansions in this connection.[2]

One finds among the Old Babylonian tablets some table texts containing successive powers of a given number, analogous to our modern tables of logarithms, or, more properly speaking, of antilogarithms. Exponential (or logarithmic) tables have been found in which the first ten powers are listed for the bases 9 and 16 and 1,40 and 3,45 (all perfect squares). The question raised in a problem text, to what power must a certain number be raised in order to yield a given number, is equivalent to our question, what is the logarithm of the given number in a system with the certain number as base. The chief differences between the ancient tables and our own, apart from matters of language and notation, are that no single number was systematically used as a base in varied connections and that the gaps between entries in the ancient tables are far larger than in our tables. Then, too, their "logarithm tables" were not used for general purposes of calculation, but rather to solve certain very specific questions.

[2] In addition to the references cited in footnote 1, see also Kurt Vogel, *Vorgriechische Mathematik*, Vol. II, *Die Mathematik der Babylonier* (1959).