THE 4D/5D CONNECTION BLACK HOLES and HIGHER-DERIVATIVE COUPLINGS

Mathematics and Applications of Branes in String and M-Theory

Newton Institute, Cambridge

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Bernard de Wit

Nikhef Amsterdam

Utrecht University

Motivation:

What is the relation between:

microscopic/statistical entropy ⟷ macroscopic/field-theoretic entropy

- microstate counting ⟶ entropy $S_{\rm micro} = \ln d(q,p)$
- supergravity: Noether surface charge *Wald, 1993*
  first law of black hole mechanics *(BH thermodynamics)*

A reliable comparison requires a (super)symmetric setting!

Ideal testing ground: *supergravities with 8 supercharges*

- *D=4 space-time dimensions with N=2 supersymmetry*
- *D=5 space-time dimensions with N=1* supersymmetry

To clarify previous results on 4D and 5D black hole solutions, to further the understanding between them, and to obtain new results.

- *Higher-derivative terms (4D and 5D)*
  Chern-Simons terms (5D)

  Bergshoeff, de Roo, dW, 1981
  Cardoso, dW, Mohaupt, 1998
  Hanaki, Ohashi, Tachikawa, 2007
  dW, Katmadas, van Zalk, 2009, 2010

- *5D black holes (and black rings)*

  Castro, Davis, Kraus, Larsen, 2007, 2008
  dW, Katmadas, 2009

- *the 4D/5D connection*

  Gaiotto, Strominger, Yin, 2005
  Behrndt, Cardoso, Mahapatra, 2005

- *more recent developments*

  Sen, 2011
  Banerjee, dW, Katmadas, 2011

Methodology: superconformal multiplet calculus

- off-shell irreducible supermultiplets
  in superconformal gravity background
- extra superconformal gauge invariances
- gauge equivalence (compensating supermultiplets)

4D *dW, van Holten, Van Proeyen, et al., 1980-85*

5D *Kugo, Ohashi, 2000*
Bergshoeff, Vandoren, Van Proeyen, et al., 2001-04
Fujita, Ohashi, 2001
Hanaki, Ohashi, Tachikawa, 2006

6D/5D off-shell connection

BPS black holes in four space-time dimensions

N=2 supergravity: *vector multiplet sector* (Wilsonian effective action)

vector multiplets contain scalars X^I

projectively defined: $X^I \longrightarrow Y^I$ (residual scale invariance)

Lagrangian encoded in a holomorphic homogeneous function

$$F(\lambda Y) = \lambda^2 \, F(Y)$$

with near-horizon geometry: $\mathrm{AdS}_2 \times S^2$

Subleading corrections to Bekenstein-Hawking area law:

extend with one 'extra' complex field, originating from pure supergravity

$$\Upsilon$$

$$F(Y) \longrightarrow F(Y, \Upsilon)$$

↑ Weyl background

homogeneity: $$F(\lambda Y, \lambda^2 \Upsilon) = \lambda^2\, F(Y, \Upsilon)$$

Υ-dependence leads to terms $\propto (R_{\mu\nu\rho}{}^{\sigma})^2$ in effective action

BPS: supersymmetry at the horizon

$$Y^I - \bar{Y}^I = \mathrm{i} p^I \qquad \text{magnetic charges}$$

$$F_I - \bar{F}_I = \mathrm{i} q_I \qquad \text{electric charges}$$

Ferrara, Kallosh, Strominger, 1996
Cardoso, dW, Käppeli, Mohaupt, 2000

covariant under dualities!

refers to the full function *F* !

Furthermore $\Upsilon = -64$ leads to subleading corrections

The Wilsonian effective action is not sufficient to realize all dualities. When integrating out the massless modes, so as to obtain the 1PI action, one encounters non-holomorphic corrections. For the BPS near-horizon region one has a natural infrared cut-off provided by the S^2 radius.

In Minkowski space-time the integration over massless modes is problematic !

It is possible to determine the short-distance corrections that depend logarithmically on the S^2 radius. *Sen, 2011*

Surprisingly enough these are consistent with the terms found from a study of degeneracies of BPS states of D-branes on compact Calabi-Yau manifolds. *Denef, Moore, 2007*

There is another issue related to the coupling to Gauss-Bonnet terms, which has been used to obtain the entropy for heterotic black holes. However, their supersymmetric form is not known.

Sen, 2005

BPS black holes and rings in five space-time dimensions

two different supersymmetric horizon topologies !

- S^3 **(SPINNING) BLACK HOLE** *Breckenridge, Myers, Peet, Vafa, 1996*
- $S^1 \times S^2$ **BLACK RING** *Elvang, Emparan, Mateos, Reall, 2004*

with near-horizon geometry: $\mathrm{AdS}_2 \times S^3$ *or* $\mathrm{AdS}_3 \times S^2$

(this result does not *depend on the details of the Lagrangian)*

5D vector supermultiplets contain $\begin{cases} \text{scalar fields:} & \sigma^I \\ \text{vector fields:} & W_\mu{}^I \end{cases}$

abelian field strengths $F_{\mu\nu}{}^I = \partial_\mu W_\nu{}^I - \partial_\nu W_\mu{}^I$

supergravity (Weyl) multiplet contains

(auxiliary) tensor $T_{\mu\nu} \to T_{ab} \begin{cases} T_{01} \\ T_{23} \end{cases}$

$$v^2 \equiv (T_{01})^2 + (T_{23})^2$$

supersymmetry + partial gauge choice

$\sigma^I = \text{constant}$ (remain subject to residual (constant) scale transformations!)

$T_{\mu\nu}$ *conformal Killing-Yano tensor*

$$\mathcal{D}_\rho T_{\mu\nu} = \tfrac{1}{2} g_{\rho[\mu}\, \xi_{\nu]} \qquad \xi^\mu T_{\mu\nu} = 0$$

Killing vector associated with the fifth dimension ψ

$$\xi^\mu \propto e^{-1}\, \varepsilon^{\mu\nu\rho\sigma\tau}\, T_{\nu\rho}\, T_{\sigma\tau}$$

$\mathrm{AdS}_2 \times S^2$

$$ds^2 = \frac{1}{16\, v^2}\Big(- r^2 \mathrm{d}t^2 + \frac{\mathrm{d}r^2}{r^2} + \mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\varphi^2 \Big) + \mathrm{e}^{2g} \big(\mathrm{d}\psi + \sigma\big)^2$$

$$\sigma = -\frac{1}{4\, v^2}\, \mathrm{e}^{-g}\, \big(T_{23}\, r\, \mathrm{d}t - T_{01}\, \cos\theta\, \mathrm{d}\varphi\big)$$

Two distinct cases:

- $T_{01} \neq 0$ **SPINNING BLACK HOLE** *Breckenridge, Myers, Peet, Vafa, 1996*

  angular momentum $\propto \dfrac{T_{23}}{T_{01}}$

- $T_{01} = 0$ **BLACK RING** *Elvang, Emparan, Mateos, Reall, 2004*

Additional horizon condition and 'magnetic' charges

$$F_{\mu\nu}{}^{I} = 4\,\sigma^{I}\,T_{\mu\nu} \qquad \mathcal{Q}_{\mu\nu} = \partial_\mu\sigma_\nu - \partial_\nu\sigma_\mu$$

$$F_{\theta\varphi} \longrightarrow p^{I} = \frac{\sigma^{I}}{4\,v^2}\,T_{23}$$

$$\mathcal{Q}_{\theta\varphi} \longrightarrow p^{0} = \frac{\mathrm{e}^{-g}}{4\,v^2}\,T_{01}$$

attractor equations

Scale invariance (residue of conformal invariance)

$\sigma^I \,, T_{ab} \,, v \,, \mathrm{e}^{-g}$ scale uniformly

the metric is scale dependent

Action in *5* space-time dimensions consists of two cubic invariants, each containing a Chern-Simons term:

$$\mathcal{L} \propto C_{IJK}\, \varepsilon^{\mu\nu\rho\sigma\tau}\, W_\mu{}^I F_{\nu\rho}{}^J F_{\sigma\tau}{}^K$$

$$\mathcal{L} \propto c_I\, \varepsilon^{\mu\nu\rho\sigma\tau}\, W_\mu{}^I\, R_{\nu\rho}{}^{ab} R_{\sigma\tau\, ab}$$

Hanaki, Ohashi, Tachikawa, 2006

dW,Katmadas, 2009

The Chern-Simons terms cause non-trivial complications in the determination of entropy, electric charges and angular momenta !

An example: *5D electromagnetism with CS term*

$$\mathcal{L}^{\rm total} = \mathcal{L}^{\rm inv}(F_{\mu\nu}, \nabla_\rho F_{\mu\nu}, \psi, \nabla_\mu\psi) + \varepsilon^{\mu\nu\rho\sigma\tau} A_\mu F_{\nu\rho} F_{\sigma\tau}$$

The Noether potential associated with the abelian gauge symmetry takes the form

$$\mathcal{Q}^{\mu\nu}_{\rm gauge}(\phi,\xi) = 2\,\mathcal{L}_F^{\mu\nu}\,\xi - 2\,\nabla_\rho\mathcal{L}_F^{\rho,\mu\nu}\,\xi + 6\,e^{-1}\varepsilon^{\mu\nu\rho\sigma\tau}\,\xi A_\rho F_{\sigma\tau}$$

local gauge parameter

where $\delta\mathcal{L}^{\rm inv} = \mathcal{L}_F^{\mu\nu}\,\delta F_{\mu\nu} + \mathcal{L}_F^{\rho,\mu\nu}\,\delta(\nabla_\rho F_{\mu\nu}) + \mathcal{L}_\psi\,\delta\psi + \mathcal{L}_\psi^\mu\,\delta(\nabla_\mu\psi)$

$$\partial_\nu\,\mathcal{Q}^{\mu\nu}_{\rm gauge} = J^\mu_{\rm Noether} = 0$$

definition

for $\delta_\xi\phi = 0$

$^*\mathcal{Q}_{\rm gauge}$ is a closed (*d*-2)-form for symmetric configurations!

Electric charge is defined as

$$q = \int_{\Sigma_{\rm hor}} \varepsilon_{\mu\nu}\, \mathcal{Q}^{\mu\nu}_{\rm gauge}(\phi, \xi)$$

bi-normal

(This definition coincides with the definition based on the field equations!)

The electric charge now contains the integral over a 3-cycle of the CS term!

This poses no difficulty for black holes for which the gauge fields are globally defined.
However, the mixed CS term leads to the integral over a gravitational CS term, which is problematic.

For black rings the gauge fields are not globally defined. Both CS terms are therefore problematic.

Entropy and angular momentum

Entropy (based on first law of black hole mechanics

$$\mathcal{S}_{\rm macro} = -\pi \int_{\Sigma_{\rm hor}} \varepsilon_{\mu\nu}\, \mathcal{Q}^{\mu\nu}(\xi)\Big|_{\nabla_{[\mu}\xi_{\nu]}=\varepsilon_{\mu\nu}\,;\,\xi^\mu=0}$$

Wald, 1993

$\xi^\mu\partial_\mu = \partial/\partial t$ timelike Killing vector

bi-normal

Angular momenta

$$J(\xi) = \int_{\Sigma_{\rm hor}} \varepsilon_{\mu\nu}\, \mathcal{Q}^{\mu\nu}(\xi)$$

ξ^μ periodic Killing vector

Evaluating the CS terms for black rings

The correct evaluation of the CS term for the ring geometry yield

$$Q_I^{\rm CS} \propto \oint_\Sigma C_{IJK}\, W^J \wedge F^K \propto C_{IJK}\, a^J p^K$$

Hence, integer shifts of the Wilson line moduli induce a shift in the integrated CS term

For concentric rings, one finds

$$Q_I^{\rm CS} - 6\, C_{IJK} P^J P^K = -12\, C_{IJK} \sum_i (a^J + \tfrac{1}{2} p^J)_i\, p^K{}_i$$

with $P^I = \sum_i p^J{}_i$

The integrated CS terms are not additive!

Hanaki, Ohashi, Tachikawa, 2007

Confirmed by explicit results for global solutions.

Gauntlett, Gutowski, 2004

Additive charges take the following form
(upon solving the Wilson line moduli in terms of the charges)

$$q_I - 6\, C_{IJK}\, p^J p^K$$

5D black ring versus 4D black hole:

$$\mathcal{S}^{\rm BR}_{\rm macro} = \frac{4\pi}{\phi^0}\left[C_{IJK}\, p^I p^J p^K + \frac{1}{4} c_I p^I\right] \qquad D=5$$

$$q_I = -12\, C_{IJK}\, p^J a^K$$

$$J_\varphi = p^I (q_I - \tfrac{1}{6} C_{IJ} p^J)$$

$$J_\psi - J_\varphi - \frac{1}{24} C^{IJ} (q_I - 6 C_{IK} p^K)(q_J - 6 C_{JL} p^L) = \frac{2}{{\phi^0}^2}\left[C_{IJK}\, p^I p^J p^K + \frac{1}{4}\, c_I p^I\right]$$

up to calibration

$$\mathcal{S}^{\rm BH}_{\rm 4D} = -\frac{2\pi}{\phi^0}\left[D_{IJK}\, p^I p^J p^K + 256\, d_I p^I\right] \qquad D=4$$

$${q_I}^{\rm 4D} = \frac{6}{\phi^0} D_{IJK}\, p^J \phi^K$$

$${\hat{q}_0}^{4D} \equiv {q_0}^{4D} + \tfrac{1}{12} D^{IJ} q_I q_J = \frac{1}{{\phi^0}^2}\left[D_{IJK}\, p^I p^J p^K + 256\, d_I p^I\right]$$

COMMENTS:

Confirmation from near-horizon analysis in the presence of higher-derivative couplings. Partial results were already known (but somewhat disputed at the time).

Bena, Kraus, etc

The Wilson line moduli are defined up to integers. This implies that the electric charges and angular momenta are shifted under the large gauge transformations (spectral flow) induced by these integer shifts.
Indeed, under

Bena, Kraus,Warner, Cheng, de Boer, etc

$$a^I \rightarrow a^I + k^I$$

one finds,

$$\begin{aligned} q_I &\rightarrow q_I - 12\, C_{IJK} p^J k^K \\ J_\varphi &\rightarrow J_\varphi - 12\, C_{IJK} p^I p^J k^K \\ J_\psi &\rightarrow J_\psi - q_I k^I - 6\, C_{IJK} p^I p^J k^K + 6\, C_{IJK} p^I k^J k^K \end{aligned}$$

These transformations are in agreement with the corresponding *4D* black holes where the above transformations correspond to a duality invariance!

4D/5D connection: difference resides in the contributions from the Chern-Simons term !

The spinning black hole

$$\mathcal{S}^{\rm BH}_{\rm macro} = \frac{\pi\, {\rm e}^g}{4\, v^2}\left[C_{IJK}\sigma^I\sigma^J\sigma^K + 4\, c_I\sigma^I\, {T_{23}}^2\right]$$

$$q_I = \frac{6\, {\rm e}^g}{4\, T_{01}}\left[C_{IJK}\sigma^J\sigma^K - c_I {T_{01}}^2\right]$$

$$p^0 = \frac{{\rm e}^{-g}}{4\, v^2}\, T_{01}$$

$$J_\psi = \frac{T_{23}\, {\rm e}^{2g}}{{T_{01}}^2}\left[C_{IJK}\sigma^I\sigma^J\sigma^K - 4\, c_I\sigma^I\, {T_{01}}^2\right]$$

choose scale invariant variables

$$\phi^I = \frac{\sigma^I}{4T_{01}}$$

$$\phi^0 = \frac{e^{-g}T_{23}}{4v^2} = \frac{p^0\, T_{23}}{T_{01}}$$

$$\mathcal{S}^{\rm BH}_{\rm macro} = \frac{4\pi p^0}{({\phi^0}^2 + {p^0}^2)^2}\left[{p^0}^2 C_{IJK}\phi^I\phi^J\phi^K + \frac{1}{4}\, c_I\phi^I\, {\phi^0}^2\right]$$

$$q_I = \frac{6\, p^0}{{\phi^0}^2 + {p^0}^2}\left[C_{IJK}\phi^J\phi^K - \frac{1}{16}\, c_I\right] \qquad D = 5$$

$$J_\psi = \frac{4\phi^0 p^0}{({\phi^0}^2 + {p^0}^2)^2}\left[C_{IJK}\phi^I\phi^J\phi^K - \frac{1}{4}\, c_I\phi^I\right]$$

up to calibration

relative factor $\frac{4}{3}$

agrees!

$$\mathcal{S}^{\rm BH}_{4D} = \frac{2\pi p^0}{({\phi^0}^2 + {p^0}^2)^2}\left[{p^0}^2 D_{IJK}\phi^I\phi^J\phi^K + 256\, d_I\phi^I{\phi^0}^2\right]$$

$${q_I}^{4D} = -\frac{3\, p^0}{{\phi^0}^2 + {p^0}^2}\left[D_{IJK}\phi^J\phi^K - \frac{256}{3}\, d_I\right] \qquad D = 4$$

$${q_0}^{4D} = \frac{2\phi^0 p^0}{({\phi^0}^2 + {p^0}^2)^2}\left[D_{IJK}\phi^I\phi^J\phi^K - 256\, d_I\phi^I\right]$$

Comments:

Agrees with microstate counting for $J_\psi = 0$

Vafa, 1997
Huang, Klemm, Mariño, Tavanfar, 2007

Two important differences with the literature!

Castro, Davis, Kraus, Larsen, 2007,2008

The expression for J_ψ is rather different! We have $J_\psi = q_0$.
The electric charges receives different corrections from the higher-order derivative couplings:

$$3\,c^I \Leftrightarrow \left[3 - \frac{{\phi^0}^2}{{p^0}^2}\right] c_I$$

identical when $J_\psi = 0$.

It turns out that the discrepancy between 4D/5D results resides in the mixed Chern Simons term.

Banerjee, dW, Katmadas, 2011

To exclude the possibility that contributions from other terms could be present one has to make a detailed comparison of *4D* and *5D* supergravity.

➠ *the off-shell 4D/5D connection*

The off-shell 4D/5D connection also enables one to understand why the field equations of the *5D* and the *4D* theory are different (in the bulk), as has been observed in the literature.

Upon dimensional reduction the invariant Lagrangians with at most two derivatives lead to the corresponding *4D* Lagrangians. There will be no qualitative differences.

However, the higher-order derivative term in *5D* leads upon reduction to three separate *4D* invariant Lagrangians.

One of these Lagrangians is known to give no contribution to the entropy and the electric charges of *4D* BPS black holes, because of a non-renormalization theorem.

dW, Katmadas, van Zalk, 2010

Off-shell dimensional reduction

Weyl multiplet, vector multiplet, hypermultiplet

First Kaluza-Klein ansatz with gauge choices:

$$e_M{}^A = \begin{pmatrix} e_\mu{}^a & B_\mu \phi^{-1} \\ 0 & \phi^{-1} \end{pmatrix} , \qquad e_A{}^M = \begin{pmatrix} e_a{}^\mu & -e_a{}^\nu B_\nu \\ 0 & \phi \end{pmatrix} , \qquad b_M = \begin{pmatrix} b_\mu \\ 0 \end{pmatrix}$$

There are compensating transformations to preserve this form.

Upon the reduction the *5D* Weyl multiplet decomposes into the *4D* Weyl multiplet and a Kaluza-Klein vector multiplet.

Continue with the other fields and then consider the action of the five-dimensional supersymmetry transformations.

In this case, there is no conflict between conformal invariance and dimensional reduction.

Banerjee, dW, Katmadas, 2011

The supersymmetry transformations turn out to satisfy a uniform decomposition:

$$\delta_{\rm Q}(\epsilon)\big|_{5D}^{\rm reduced}\Phi = \delta_{\rm Q}(\epsilon)\big|_{4D}\Phi + \delta_{\rm S}(\tilde\eta)\big|_{4D}\Phi + \delta_{\rm SU(2)}(\tilde\Lambda)\big|_{4D}\Phi + \delta_{\rm U(1)}(\tilde\Lambda^0)\big|_{4D}\Phi\,.$$

compensating transformation

The Kaluza-Klein vector multiplet has a real scalar field, rather than a complex one. Moreover, the R-symmetry remains restricted to $\mathrm{SU}(2)$ rather than extending to $\mathrm{SU}(2)\times \mathrm{U}(1)$!

Explanation: It turns out the the result takes the form of a gauge-fixed version with respect to $\mathrm{U}(1)$!

Weyl multiplet 5D ➡ 4D (reducible)

$$\phi = 2|X^0|$$
$$B_\mu = W_\mu{}^0$$

KK vector multiplet
missing pseudoscalar

$$V_{Mi}{}^j = \begin{cases} V_{\mu i}{}^j = \mathcal{V}_\mu{}^j{}_i - \frac{1}{4}\varepsilon_{ik}\, Y^{kj\,0}\,|X^0|^2 W_\mu{}^0 \\ V_{5i}{}^j = -\frac{1}{4}\varepsilon_{ik}\, Y^{kj\,0}|X^0|^2 \end{cases}$$

$$T_{a5} = \tfrac{1}{12}\mathrm{i}e_a{}^\mu\Big(\frac{\mathcal{D}_\mu X^0}{X^0} - \frac{\mathcal{D}_\mu \bar{X}^0}{\bar{X}^0}\Big)$$

$$T_{ab} = -\frac{\mathrm{i}}{24\,|X^0|}\Big(\varepsilon_{ij}T_{ab}{}^{ij}\,\bar{X}^0 - F^-_{ab}(B)\Big) + \mathrm{h.c.}$$

gauge fixed formulation w.r.t. *U(1)*

vector multiplet 5D ➡ 4D

$$\sigma = -\mathrm{i}|X^0|\,(t-\bar{t})$$

$$W_M = \begin{cases} W_\mu = & W_\mu - \frac{1}{2}(t+\bar{t})\,W_\mu{}^0 \\ W_5 = & -\frac{1}{2}(t+\bar{t}) \end{cases}$$

$$Y^{ij} = -\tfrac{1}{2}Y^{ij} + \tfrac{1}{4}(t+\bar{t})\,Y^{ij\,0}$$

$$t = \frac{X}{X^0}$$

5D vector multiplet action

$$8\pi^2\mathcal{L}_{\rm vvv} \;=\; 3\,E\,C_{ABC}\,\sigma^A\Big[\tfrac{1}{2}\mathcal{D}_M\sigma^B\,\mathcal{D}^M\sigma^C + \tfrac{1}{4}F_{MN}{}^B F^{MNC} - Y_{ij}{}^B Y^{ijC} - 3\,\sigma^B F_{MN}{}^C T^{MN}\Big]$$
$$-\tfrac{1}{8}\mathrm{i}\,C_{ABC}\varepsilon^{MNPPQR}\,W_M{}^A F_{NP}{}^B F_{QR}{}^C$$
$$-E\,C_{ABC}\sigma^A\sigma^B\sigma^C\,\Big[\tfrac{1}{8}R - 4\,D - \tfrac{39}{2}T^{AB}T_{AB}\Big]$$

5D higher-derivative action

$$8\pi^2\,\mathcal{L}_{\rm vww} \;=\; \tfrac{1}{4}\,E\,c_A Y_{ij}{}^A\,T^{CD}\,R_{CDk}{}^j(V)\,\varepsilon^{ki}$$
$$+E\,c_A\sigma^A\,\Big[\tfrac{1}{64}R_{CD}{}^{EF}(M)\,R_{EF}{}^{CD}(M) + \tfrac{1}{96}R_{MNj}{}^i(V)\,R^{MN}{}_i{}^j(V)\Big]$$
$$-\tfrac{1}{128}\mathrm{i}\varepsilon^{MNPQR}\,c_A W_M{}^A\;\big[R_{NP}{}^{CD}(M)\,R_{QRCD}(M) + \tfrac{1}{3}\,R_{NPj}{}^i(V)\,R_{QRi}{}^j(V)\big]$$
$$+\tfrac{3}{16}E\,c_A\big(10\,\sigma^A T_{BC} - F_{BC}{}^A\big)R(M)_{DE}{}^{BC}\,T^{DE} + \cdots$$

yields the following 4D actions:

$$F(X, ({T_{ab}}^{ij}\varepsilon_{ij})^2) = -\frac{1}{2}\frac{C_{ABC}X^A X^B X^C}{X^0} - \frac{1}{2048}\,\frac{c_A\,X^A}{X^0}\,({T_{ab}}^{ij}\varepsilon_{ij})^2$$

corresponding to the chiral superspace density invariant

$$\mathcal{H}(X,\bar{X}) = \tfrac{1}{384}\mathrm{i}c_A\left(\frac{X^A}{X^0}\,\ln\bar{X}^0 - \frac{\bar{X}^A}{\bar{X}^0}\,\ln X^0\right)$$

corresponding to a full superspace density invariant, described in terms of a Kähler potential. This class of actions does not contribute to the electric charges and the entropy of BPS black holes.

dW, Katmadas, van Zalk, 2010

Supersymmetric *4D* Lagrangian arising from dimensional reduction of five-dimensional supergravity:

$$
\begin{aligned}
e^{-1}\mathcal{L} = {} & \\
& \mathcal{H}_{IJ\bar K\bar L}\Big[\tfrac14\big(F_{ab}^{-\,I}\,F^{-ab\,J} - \tfrac12 Y_{ij}{}^{I}\,Y^{ij\,J}\big)\big(F_{ab}^{+\,K}\,F^{+ab\,L} - \tfrac12 Y^{K\,ij}\,Y^{L}{}_{ij}\big) \\
& \qquad + 4\,\mathcal{D}_a X^I\,\mathcal{D}_b \bar X^K\big(\mathcal{D}^a X^J\,\mathcal{D}^b \bar X^L + 2\,F^{-\,ac\,J}\,F^{+\,b}{}_{c}{}^{L} - \tfrac14 \eta^{ab}\,Y_{ij}^{J}\,Y^{L\,ij}\big)\Big] \\
& + \Big\{\mathcal{H}_{IJ\bar K}\Big[4\,\mathcal{D}_a X^I\,\mathcal{D}^a X^J\,\mathcal{D}^2 \bar X^K - \big(F^{-ab\,I}\,F_{ab}^{-\,J} - \tfrac12 Y_{ij}^{I}\,Y^{Jij}\big)\big(\Box_{\rm c} X^K + \tfrac18 F_{ab}^{-\,K}\,T^{abij}\varepsilon_{ij}\big) \\
& \qquad + 8\,\mathcal{D}^a X^I F_{ab}^{-\,J}\big(D_c F^{+\,cb\,K} - \tfrac12 \mathcal{D}_c \bar X^K T^{ij\,cb}\varepsilon_{ij}\big) - \mathcal{D}_a X^I\,Y_{ij}^{J}\,\mathcal{D}^a Y^{K\,ij}\Big] + \text{h.c.}\Big\} \\
& + \mathcal{H}_{I\bar K}\Big[4\big(\Box_{\rm c} \bar X^I + \tfrac18 F_{ab}^{+\,I}\,T^{ab}{}_{ij}\varepsilon^{ij}\big)\big(\Box_{\rm c} X^K + \tfrac18 F_{ab}^{-\,K}\,T^{abij}\varepsilon_{ij}\big) + 4\,\mathcal{D}^2 X^I\,\mathcal{D}^2 \bar X^K \\
& \qquad + 8\,\mathcal{D}_a F^{-\,abI}\,\mathcal{D}_c F^{+c}{}_{b}{}^{K} - \mathcal{D}_a Y_{ij}{}^{I}\,\mathcal{D}^a Y^{K\,ij} + \tfrac14 T_{ab}{}^{ij}\,T_{cdij}\,F^{-ab\,I}F^{+cd\,K} \\
& \qquad + \big(\tfrac16 R(\omega,e) + 2\,D\big)Y_{ij}{}^{I}\,Y^{ij\,K} + 4\,F^{-ac\,I}\,F^{+}{}_{bc}{}^{K}\,R(\omega,e)_a{}^{b} \\
& \qquad - \big[\varepsilon^{ik}\,Y_{ij}{}^{I}\,F^{+ab\,K}\,R(\mathcal{V})_{ab}{}^{j}{}_{k} + \text{h.c.}\big] \\
& \qquad - \big[\mathcal{D}_c \bar X^K\big(D^c T_{ab}{}^{ij}\,F^{-\,I\,ab} + 4\,T^{ij\,cb}\,D^a F_{ab}^{-\,I}\big)\varepsilon_{ij} + \text{h.c.}\big] \\
& \qquad + 8\big(R^{\mu\nu}(\omega,e) - \tfrac13 g^{\mu\nu} R(\omega,e) + \tfrac14 T^{\mu}{}_{b}{}^{ij}\,T^{\nu b}{}_{ij} + \mathrm{i}R(A)^{\mu\nu} - g^{\mu\nu} D\big)\mathcal{D}_\mu X^I\,\mathcal{D}_\nu \bar X^K\Big]
\end{aligned}
$$

Surprise: there is one more higher-derivative coupling emerging from *5D* !

Some characteristic terms:

$$
\begin{aligned}
8\pi^2\,\mathcal{L}_{\rm vww} \;\rightarrow\; & -\tfrac{1}{384}\mathrm{i}c_A t^A\,\big[\tfrac{2}{3}\mathcal{R}_{ab}\mathcal{R}^{ab} + R(\mathcal{V})^{+i}_{ab\,j}\,R(\mathcal{V})^{+abj}{}_i\big] \\
& -\tfrac{1}{768}\mathrm{i}c_A(t^A-\bar{t}^A)\,(X^0)^{-1}\varepsilon_{ij}T^{cdij}\,R(M)^{ab}{}_{cd}\,F^{-0}_{ab} \\
& +\mathrm{h.c.}
\end{aligned}
$$

where $\mathcal{R}_{\mu\nu}$ is the Ricci tensor

Its structure is not yet known. Neither is it known whether this coupling is subject to some non-renormalization theorem. It will probably have some bearing on the supersymmetric Gauss-Bonnet invariant.

Conclusions/comments

Off-shell dimensional reduction enables one to get a precise comparison, also in the presence of higher-order invariants, between complicated Lagrangians in 4 and 5 dimensions.

The known higher-derivative invariant coupling in 5D gives rise to three independent higher-derivative couplings in 4D. The indication is that only one of them contributes to electric charges and entropy of BPS black holes. One is not yet explicitly known, and seems to be related to a supersymmetric coupling to the Gauss-Bonnet term.

The difference in the near-horizon behavior between 4- and 5-dimensional solutions is exclusively related to non-trivial aspects of Chern-Simons terms.

The off-shell dimensional reduction method can be used in many more situations. For instance, in performing the *c-map* of the topological string.