Principles of Magnetic Resonance Imaging.
Exercise session 2

Exercise 1

Prove that:

$$f(x) = e^{-|x|} \Rightarrow \hat{f}(\omega) = \frac{2}{1 + 4\pi^2\omega^2}$$

Exercise 2: the shift theorem

Take

```
N = 1024;
dt = 1/N;
t = -0.5:dt:0.5-dt;
omega = -N/2:N/2-1;
```

and `f3 = -abs(t)+1;`. Compute its Fourier transform, `F3`. Multiply `F3` by a *linear phase* term `exp(i*2*pi*omega*t0);` with `t0=0.2`. Apply Inverse Fourier Transform to the resulting function. What do you notice? Give explanations.
Hint: to compute the inverse DFT use `fftshift(ifft(fftshift(....)))` (replace the dots with the function)

Exercise 3: the convolution theorem

Take `f4 = zeros(1,N);f4(find(abs(t)<a))=1;` with `a = 0.05`. Compute the DFT and plot it:

```
F4 = fftshift(fft(fftshift(f4)));
figure;plot(omega,real(F4))
```

Now construct a boxcar function, $W(\omega)$ in the *frequency* domain, for instance:

```
BW=200;
W=zeros(1,N);W(find(abs(omega)<BW))=1;
figure;plot(omega,real(W))
```

Multiply `F4` by `W` and denote the product by `F4w` . Transform back `F4w` the to the time domain. What do you observe? Can you explain this phenomenon at the hand of the Convolution theorem?

Exercise 4: k-space and image space

Load the test file `head2k.mat` . This is the k-space data of a physicist's head.

```
load('head2k.mat');
[k1 k2] = size(head2k);
kx = -k1/2:k1/2-1;
ky = -k2/2:k2/2-1;
figure(1); imagesc(kx,ky,log10(abs(head2k))); colormap gray; axis image;
xlabel('K_x');
ylabel('K_y');
```

Display the physicist's head in the image domain (use `ifft2`).
Set equal to zero the k-space coefficients corresponding to the 10 lowest frequencies, positive and negative, in both directions, that is:

$$F(k_x, k_y) \leftarrow 0, \quad \text{for } |k_x| \leq 10 \text{ and } |k_y| \leq 10.$$

F denotes the k-space coefficient, i.e. the data stored in the matrix `head2k`.
Reconstruct the image.
Try also the opposite, that is, set equal to zero all k-space coefficients, except those ones corresponding to the 10 lowest frequencies, that is:

$$F(k_x, k_y) \leftarrow 0, \quad \text{for } |k_x| > 10 \text{ or } |k_y| > 10.$$

What does a low-frequency and high-frequency look like in an image? Can you see the logic in discarding the high-frequency?
What is the effect of discarding k-space data during the MRI scan?
What happens if the k-space is *under-sampled* in such a way that the data corresponding to the odd coefficients of k_y are *not* acquired?
Hint: represent the under-sampled k-space data by `USdata = head2k; USdata(2:2:end,:)=[];`
This particular aliasing effect is called *fold over*.

Exercise 5: A k-space trajectory

Compute the gradients $(G_x(t), G_y(t))$ for the k-space trajectory shown in Fig. 1: three lines parallel to the k_x axis are traversed in the following order: A, B and C. The trajectory goes back to the origin every time a line is traversed. Each line comprises seven equidistant points (the distance between the points is given by α. Assume that each segment takes the same amount of time, given by Δ_t. The whole trajectory lasts thus $27\Delta_t$, that is: $t \in [0, 27\Delta_t]$.

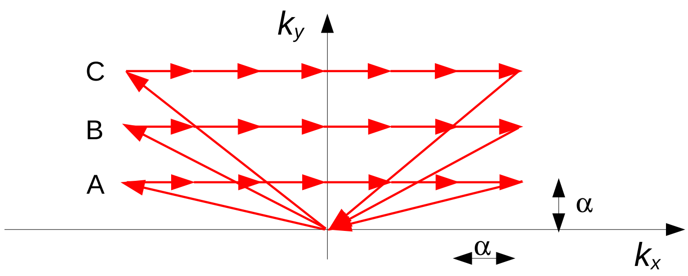

Figure 1: A k-space trajectory.