Exercise HW4

Let $A \subset \mathbb{R}^n$ be a bounded set. Prove that the following assertions are equivalent.

(a) the set A is (Jordan) negligable.

(b) for every $\varepsilon > 0$ there exists a finite collection of rectangles $R_1, \ldots, R_k$ such that

$$A \subset \cup_{j=1}^{k} R_j \quad \text{and} \quad \sum_{j=1}^{n} \mathrm{vol}_n(R_j) < \varepsilon.$$

(c) for every $\varepsilon > 0$ there exists a finite collection of rectangles $R_1, \ldots, R_k$ such that

$$A \subset \cup_{j=1}^{k} \mathrm{int}(R_j) \quad \text{and} \quad \sum_{j=1}^{n} \mathrm{vol}_n(R_j) < \varepsilon.$$