Exercise HW 5 (integral of rotationally symmetric function) :

Let $r_0 > 0$ and $\widetilde{f} : [0, r_0] \to \mathbb{R}$ be a continuous function. We define

$$f : \overline{B}^3_{r_0} \to \mathbb{R}, \quad f(x) := \widetilde{f}(\|x\|).$$

(i) Find a formula for $\int_{\overline{B}^3_{r_0}} f(x)\, dx$ in terms of $\widetilde{f}$.

(ii) Use your formula to calculate the volume of $\overline{B}^3_{r_0}$.