Exercise HW4 = 3.48

On $X = \mathbb{R}$ we consider the following families of subsets:

$$\mathscr{B} := \{(-p, p) : p \in \mathbb{Q}, p > 0\}, \qquad \mathscr{T} := \{(-a, a) \mid 0 \leq a \leq \infty\}.$$

In particular, by taking $a = 0, \infty$ we see that $\emptyset$ and X belong to $\mathscr{T}$.

(a) Show that $\mathscr{B}$ is a topology basis.

(b) Show that $\mathscr{T}$ is the topology associated to $\mathscr{B}$.

(c) Is the sequence $x_n = (-1)^n + \frac{1}{n}$ convergent in $(X, \mathscr{T})$? If so, to what?

(d) Find the interior and the closure of $A := (-1, 2)$ in $(X, \mathscr{T})$.

(e) Show that any continuous function $f : X \to \mathbb{R}$ is constant. Here X is equipped with the topology $\mathscr{T}$ and $\mathbb{R}$ with the Euclidean topology.

(f) For the topological space $(X, \mathscr{T})$ decide whether it is

 (1) Hausdorff;

 (2) 1st countable;

 (3) Metrizable.

 For all three questions, prove the correctness of your answer.