Problem 1.
Let $F$ denote an ordered field and let $R$ be a nontrivial ordered $F$-linear space as defined in (7.2). Construe $R$ as a model-theoretic structure for the language $L_F = \{<, 0, +\} \cup \{\lambda \cdot : \lambda \in F\}$ of ordered abelian groups augmented by a unary function symbol $\lambda \cdot$ for each $\lambda \in F$, to be interpreted as multiplication by the scalar $\lambda$. Prove:

1. The subsets of $R^m$ definable in the $L_F$-structure $R$ using constants are exactly the semilinear sets in $R^m$.

2. The maps $R \to R$ that are additive and definable using constants are exactly the scalar multiplications by elements of $F$. A map $f$ is additive iff

$$\forall r_1, r_2 \in R : f(r_1 + r_2) = f(r_1) + f(r_2).$$